Geotechnical Engineering II Foundation Engineering
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Lecture - 09
Shallow Foundation and Bearing Capacity

Good morning, once again I welcome you to this Foundation Engineering lectures in lecture. Today is the a lecture number 9 and previously I have discussed about various aspect of foundation shallow foundation, shallow foundation and what is shallow foundation what are the different types of shallow foundation etcetera.

Today, I will try to discuss how the shallow foundation different computation; that means, by bearing capacity etcetera. How to find out about the theory of bearing capacity? I will discuss. And maybe in this lecturer I will be able to discuss one or two aspects there are several aspects we will be discuss subsequently.

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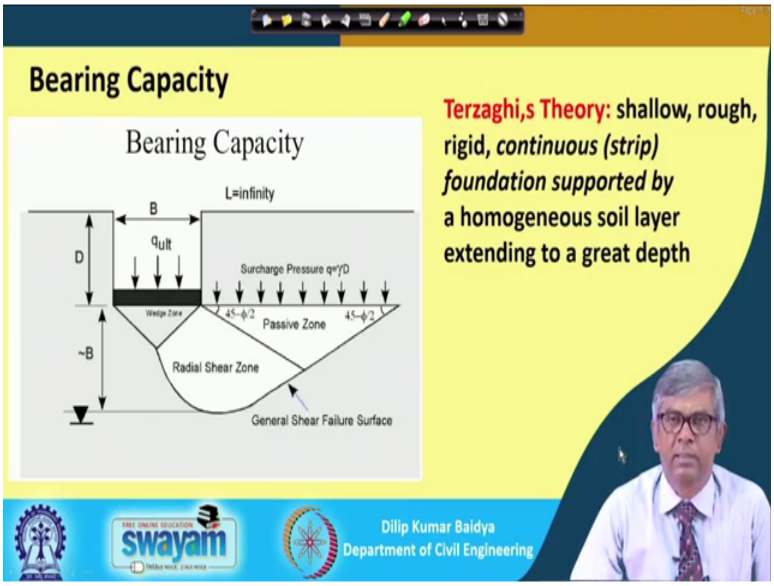

So, let me go to next one that I have already discussed that a strip footing if it is loaded and then go up to the failure, then it is observed by tertiary that below the footing. There are 3 distinct zones will be formed and they are actually one is this, that is called a Elastic zone or Wedge zone this is called Radial Shear Zone. And this is actually Rankines Passive Zone. And this 3 zones and the this is also geometrically perfect that here is it was shown 45 degree minus phi by 2, here actually angle was phi and that phi is as phi of the soil.

And so for developing this bearing Terzaghi theory Terzaghi assumed several things that one is that is this is a shallow footing and it is a base is a rough, and this footing is rigid and it is a continuous that mean strip type footing and then finally, supported on a homogeneous soil layer extending to a great depth.

That means, that there will be some influence zone so because of that assume that there will be up to a sufficient depth the soil is homogeneous and based on these assumption that bearing capacity theory was developed.

And I can tell you one or two things that when it is loaded from here and slowly increased, then the this footing will have to tendency to go downward and finally, it may fail. And while doing, so what will happen until and unless this wedge goes down that will not happen and when this wedge will try to go down then what will be there? This will be like a one part and it will have some resistance here and resistance here because of this adhesion between the surface and also some pressure from both side.

So, that is a mechanism; that means, it will have an before failure it will have an equilibrium because of this resistance, here resistance here and the weight of the soil at the load all three together it will be in equilibrium position.

Now, let me go to the next slide where, I will show that wedge only and I will not show other parts and how the equilibrium of the wedge considered in terms of this is not shown, but in terms of their effect some force are shown and then finally, I will consider the equilibrium. And from there I will try to develop generalized bearing capacity equation and you can see here that this is the wedge as shown here.

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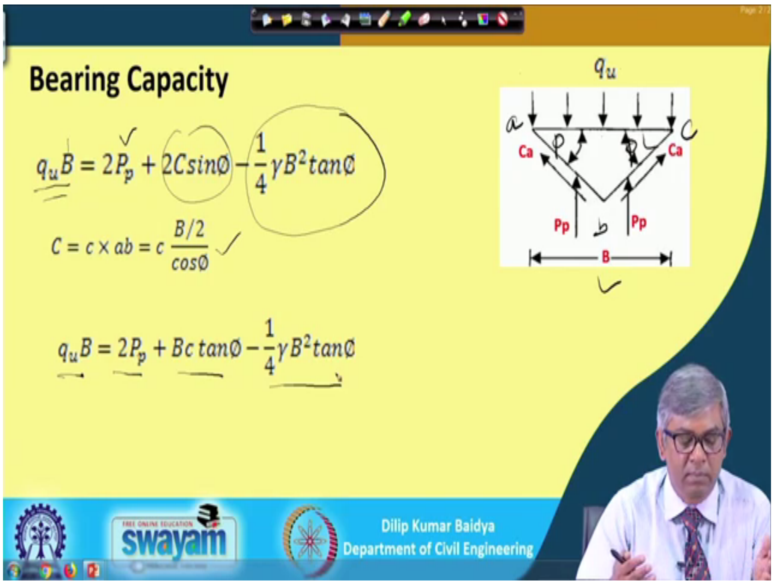

And you can see I can mark this one as a, this one as b and this one as c. And this the a b c triangular wedge and q is applied here, and suppose that failure condition at just before failure, I am consider in the equilibrium so this q become ultimate q. And then this from the surface there is a adhesion and this there will be resistance vertical resistance and this angle is phi; this angle is phi then and this width of the footing is B. And so, that is this to this is a; that means, over the width B the load pressure load or pressure is applied.

And other 2 zones are there which are no which are not shown, but their effects are shown by these two forces. Now, if we consider the vertical equilibrium of this system, then you can see that q u is applied over a over and over width B. So, q u multiplied by B and if I consider the unit length, perpendicular to this I take unit length of strip footing then q into B into one you can say that become force. And P p from here P p from here two are there so 2 P p.

P p And this one C force along this so if I take vertical component and then it become c sin phi and since from 2 sides are there so 2 C sin phi. And if I consider this triangle the volume of the triangle can be obtained and that will be can be obtained easily because half of this can be and then half base into height is the a area and then multiplied by 2 then become the entire value entire area multiplied by unit length and then multiplied by the gamma.

And if you do that then the weight of the soil in this ways become 1 by 4 gamma B square tan phi so; that means, this is actually acting downward and this is down acting downward. So, this should be in same sign and these two are acting upward. So, this is the way vertical equilibrium of the system are considered.

Now, C actually c in to a b and that actually capital C that actually multiplied again it can be converted in terms for the geometry c multiplied B by 2 by cos phi. And if I substitute in these then q in to B become 2 P p plus B c tan phi minus 1 by 4 gamma B square tan phi. So, this is one equation I have got now I have to see still there are not in a convenient form.

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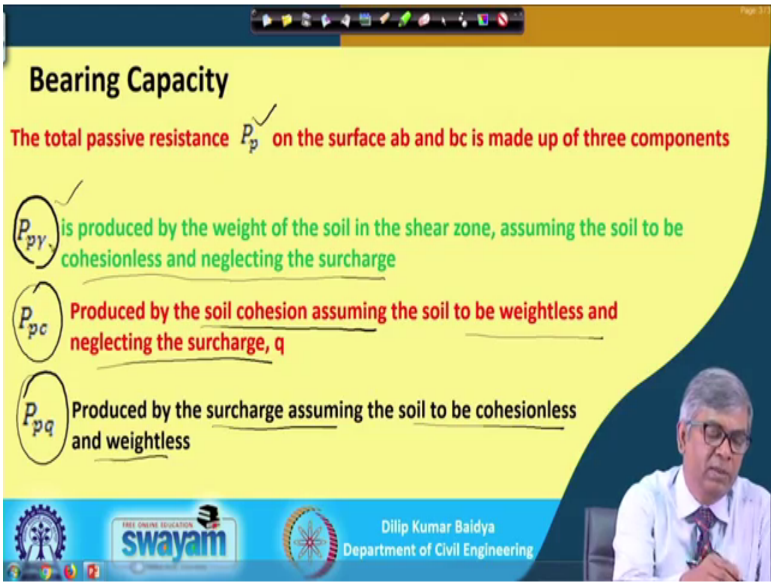

So, I will see next slide that you can see here that total 4 passive force or total passive resistance P p on the surface a b and b c both side made up of the 3 components. And that 3 components are one is actually called P p gamma, another is P p c, another is P p q, what is this P p gamma?

This is actually is the force component which is coming from the shear zone because of the weight of the soil, this is coming from the weight of the soil in the shear zone and while I am considering this assuming the soil to be cohesionless and neglecting the surcharge.

So; that means, when I will consider that part that soil weight then I will ignore other part so 3 part separately can be done. Similarly, P p c is the term which is actually coming from cohesion of the soil. And while assuming the cohesion the soil to be weightless and neglecting the surcharge this is also as assumption.

So, only cohesion to be considered and then only P p c when only weight is considered then P p gamma and this is actually P p q it is coming from the surcharge and when I am considered a surcharge then I will assume soil to be cohesionless and weightless.

So, independently 3 parts I can do, but finally, after doing that I can superimpose and then because of the superimposing, that error will not be much so that concept can be utilised. So, I can visualise P p in three component P p gamma P p c and P p q and then if I substitute in that equation what I have got then it will become something like that.

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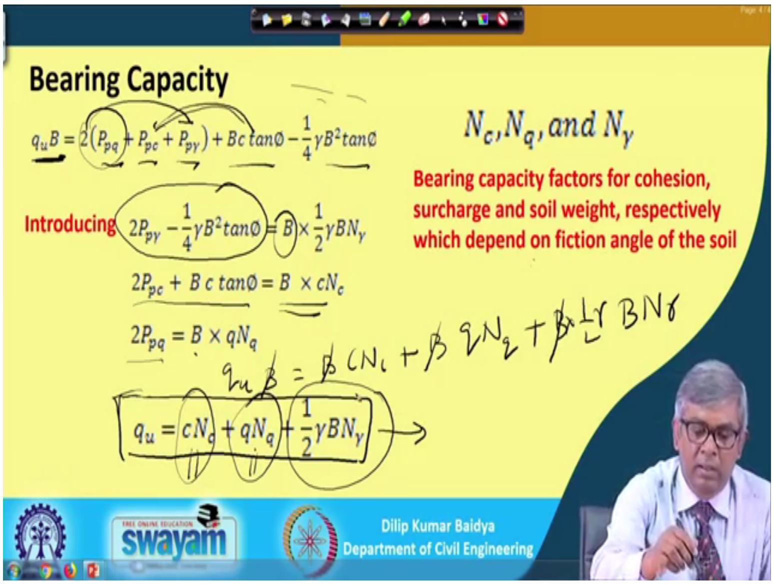

You can see q u in to B it will be remain unchange and this P p actually converted in to P p q plus P p c plus P p p gamma and these are kept same. Now, I am introducing a new term that I will take 2 P p gamma minus this 2 P p gamma 2 primes P p gamma minus this that I will take together one part and that I am giving a new term taking B one B outside and half gamma B N gamma.

Similarly, I am taking 2 P p c; that means, this plus this these two together 2 P p c, but this I am giving a different name B multiplied by c N c and 2 P p q this part only I am giving another name B multiplied by q N q. So, like that if I 3 parts ultimately this entire part; that means, I have divided into 3 parts and 3 part; that means, if I do that then it equation will come something like that q u B equal to your B multiplied by C N c plus B multiplied by q N q plus B multiplied by; B multiplied by half gamma B N gamma. So, it will come like this so, B everywhere common.

So, if I ignore or remove from this then the equation ultimately q equation equal to it is comes in the form C N c plus q N q plus half gamma B N gamma. So, this is the actually in between I have skipped several steps because, the if you see the development of bearing capacity equation P p c N c N q N gamma from the Terzaghi theory, it will be a very elaborate and lengthy procedure full of mathematics. Perhaps we will not be able go so much of details here.

So, I will just try to given you the visualisation how it has come and finally, what is the final form of bearing capacity; that means, q ultimate equal to c N c plus q N q plus half gamma B N gamma; that means, for this is applicable for strip footing because we have assume the footing is a strip footing.

And it is a an homogeneous soil and of course, raft footing then there are several assumption was there so based on that you have got this. That means, q ultimate will be of summation of 3 component one is the c N c another is q N q another is half gamma B N gamma. So, this 3 parts are called this is a cohesion contribution this is called surcharge contribution and this is for weight unit weight contribution so 3 parts.

So, this is the Terzaghi bearing capacity equation for strip footing. Now, at this 3 N c, N q, N gamma whatever we have taken they are actually called a bearing capacity factors N c, N q, N gamma. They are called bearing capacity factor and one is N c is bearing capacity factor for cohesion, N q is the bearing capacity for surcharge and N gamma is a bearing capacity for unit weight. So, that is a thing bearing capacity factor for cohesion, surcharge and soil weight, respectively which depend on fiction again this N c, N q, N gamma their relationship phi also given by Terzaghi. So, if you know the phi of the soil then we can find out the N c, N q, N gamma and then using this equation I can find out the ultimate bearing capacity of the soil.

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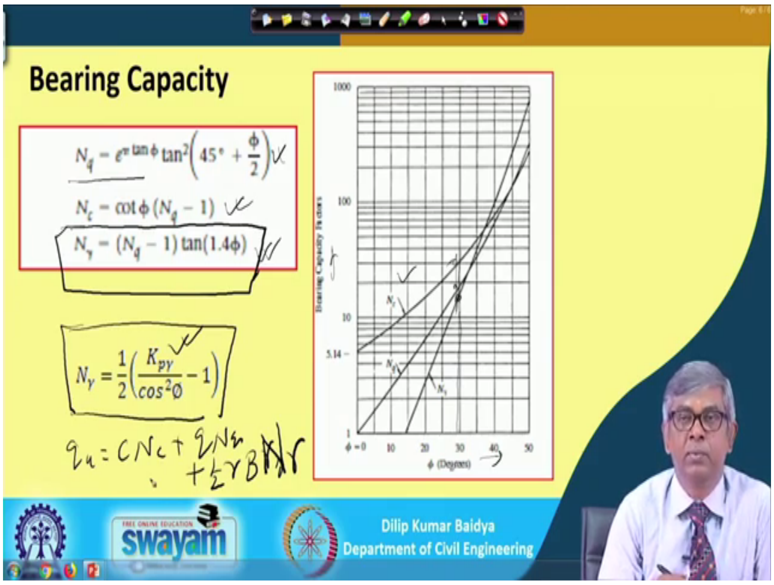

And these are the final equation form which is given by Terzaghi actually N q is e to the power pi tan phi multiplied by tan square 45 degree plus phi by 2; that means, if you just use phi in this equation, I will get some of phi is 10 degrees, phi is 20 degrees, then what is the value of N q I can find out from here. Similarly, N c is cot phi into minus 1 so if they again phi equal to 10 degrees or 20 degrees from there you will get N q, N q minus 1 multiplied by cot phi will be your N c.

And N gamma actually this is the one given later on by some other investigator N q minus 1 tan 1.4 phi. And this is actually not by Terzaghi this was given later on by someone, Terzaghi initially given by the equation for N gamma by this expression.

And this expression contents there is a K p gamma, which is very difficult to estimate actually it is a passive earth pressure coefficient depends on phi actually as not given any some guidelines also given. But by using these whatever value we get still Terzaghi value is little bit of conservative, because of that later on several investigator did and then they have expressed N gamma in this simple form.

So, we will come later on and of course, by using this we can find out N q N gamma and N c based on phi value, but Terzaghi so when Terzaghi developed this actually it has we had difficulty to compute even calculator was not there. So, he has followed some procedure and he has given the value of N c, N q, N gamma for different values of phi.

In the graphical form and in the also the tabular form this is graphical form, that is also may not be exactly by Terzaghi is given by somebody, but it is also similar way see one axis is phi other axis is a bearing capacity factor and 3 curves are for 3 curves are for 3 different aspect that is N c, N q and N gamma.

And what is when phi equal to 0 what is the value of N c etcetera, will come later on, but these are if you use graphical form; that means, you know the value of phi, then how to find out the N c, N q, N gamma then suppose phi equal to 30 degrees.

I will produced on this on the 3 curves read this curve all 3 curves and from there I can get the N c, N q, N gamma and then ensure that N c, N q, N gamma I can put it in the previous equation which is written as q equal to C N c; that means, q u equal to C N c plus q N q plus half gamma B N gamma sorry half gamma B N gamma this and if I put then I will get the ultimate bearing capacity.

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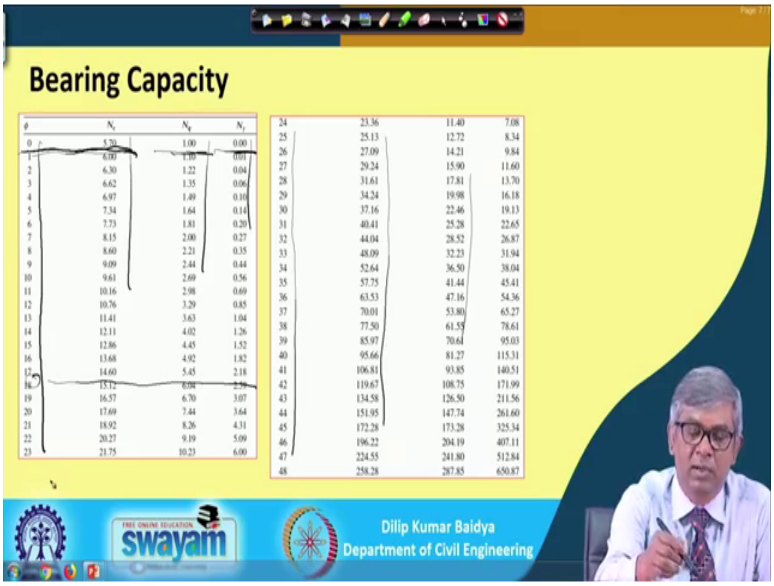

Also, this is the graphical form then there is a in the form of chart this is actually given by Terzaghi and you can see for different values of phi and different values of N c, N q, N gamma is given you can see when phi equal to 0 as per Terzaghi N c equal to 5.7 N q equal to 1 and N gamma is 0. So, this is the value actually sometime we have to remember though suppose for a particular value of phi suppose 18 degrees you cannot remember this 3 values.

You have to provide the chart, if you want to calculate bearing capacity, but when phi equal to 0, what is the value of N c, what is the value of N q what is the value of N gamma, you cannot expect that this should be given. So, in the table at top of the table when phi equal to 0; that means, N c equal to 5.7, N q equalto1; that means, what? Phi equal to 0 means what? It is a purely cohesive soil and when cohesive soil then your N c value will be there, N q will be only 1 and N gamma will be 0. That is a thing one has to remember.

And subsequently if the phi value changes then how your N c, N q value changes that is the thing is given up to 50 degrees actually is given here I have shown up to 48 degrees.

So, at different books if you see this table as also given some books is given in the interval of 5 degrees, some were given 2 degrees here actually every degrees values are given, so this can be used for calculation purpose.

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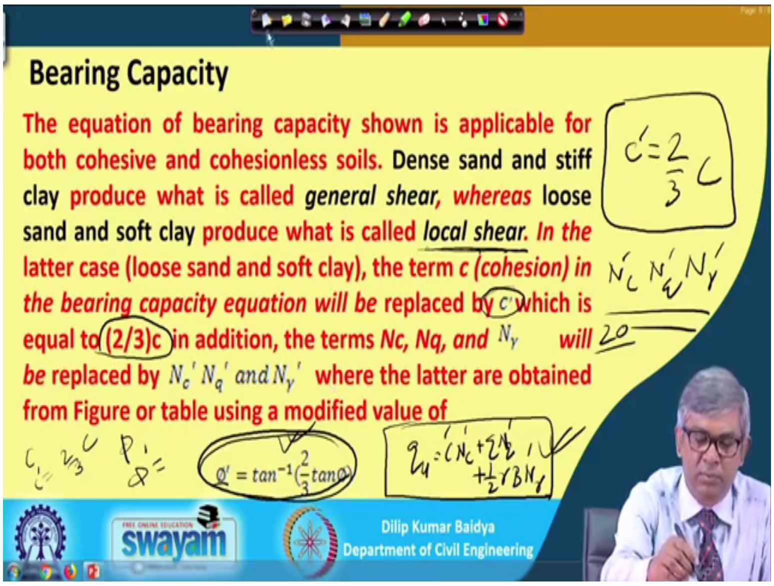

And the bearing capacity are actually see the equation of bearing capacity is shown is applicable for both cohesive and cohesionless. So generalized equation we have given that is whatever equation once again I can write here that q equal to C N c plus q N q plus half gamma B N gamma. So, this is actually generalized equation and you can see dense sand and stiff clay as I have showed previously dense sand I have not discussed whether it is a stiff clay.

For dense sand actually it has it generally fail in general shear failure, whereas loose sand and soft clay it fail in local shear failure. And so, this is actually for this equation developed for general shear failure. So, if local shear failure happens that mean when the soil is loose, then what equation will be used here there also the same equation will be used, but replacing some of the parameters by something.

So, this in the later case that means you loose soil the term c will be replaced by a c dash c will be replaced by c dash, and that c dash will be what 2 third of c when the soft soil then same general shear bearing capacity will be used, but c dash will be 2 third c, this is the modified value will be taken.

Similarly, N c, N q, N gamma there are 3 terms those N c N instead of N c, N q, N gamma that will be taken as N c gamma, N c dash, N q dash and N gamma dash. So, these 3 factors will be used and this factors also can be obtained from the same chart with modified value of phi.

Suppose the soil has phi equal to 20 degrees then what I will take, the phi will be new phi phi dash will be calculated by this equation. That means, same bearing capacity equation only thing what I have to do? If the soil has a value of C, then this C will be reduced to 2 3rd C dash will be that will be and if the soil has value phi, then phi will be reduced to phi dash which will be equal to given by this equation tan inverse 2 by 3 tan phi.

So, after getting the reduced value of phi dash they refer to the either to the table or chart and get the value of corresponding N c dash, N q dash and N gamma dash and then you same value same bearing capacity equation; that means, it will be then c dash, N c dash, N q dash, N gamma dash. And that to be obtained and then you will get the whatever value you will get; obviously, the value alternate bearing capacity will be is much smaller than if I use the actual value of phi.

So, this is the one way actually, so separately for local shear failure what will be the bearing capacity equation is not developed theoretically only thing based on general shear failure, some modification is introduced to find out the bearing capacity of local shear failure that is only and what is this modification? C will be reduced to a 2; 2 3rd C and phi will be reduced to phi dash which will be equation given by this equation. So, that is the only difference which and you can be used.

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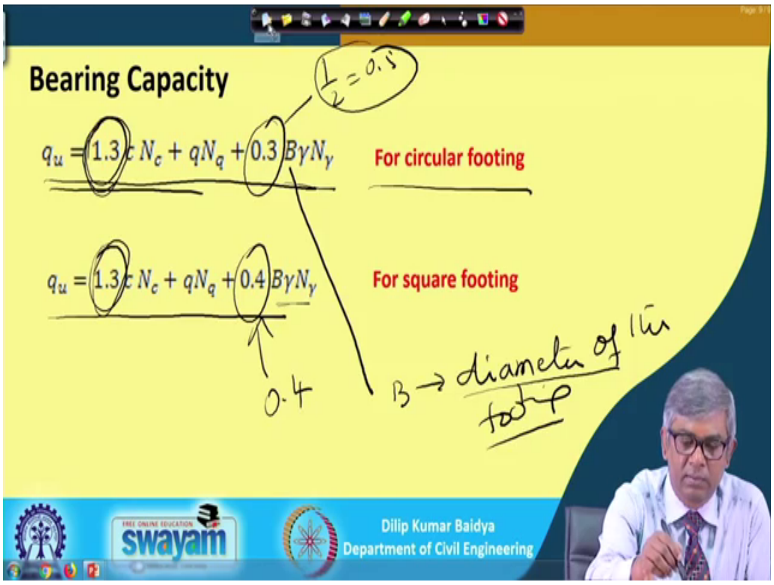

Now, you can see as I have mentioned that the equation what we have suggested or whatever developed by Terzaghi originally it is taken by initially footing was strip footing; that means, that bearing capacity equation was for strip footing.

But this bearing capacity also was extended to different other footings like when the footing is circular then your bearing capacity equation was 1.3 c N c. So, this is the one term added here and here 0.3 B gamma N gamma here actually it was point half it was half means it was 0.5 is so, 0.5 become 0.3 and when it was a strip footing the B was width, but when the circular footing B is the diameter of the footing. So, there are changes in two places one is this one factor 1.3 in the quotient part has come and instead of 0.5 it came 0.3. The 0.5 reduced to 0.3 and the B is width of the footing instead of that it is become diameter of the footing that is for circular footing.

Again same bearing capacity equation extended to square footing also here also same thing 1. Is introduced at this quotient part and here instead of 0.5 again it is 0.4 so; that means, 0.3 for circular, 0.4 for square and 0.5 for strip. This is the easiest way to remember, otherwise B gamma, N gamma and when this square footing B is the dimension of the footing both sides are same so dimension of the footing. So, these are the modification for other shape of the footing for circular and square of course, for rectangular footing sometime, it is a too long footing then strip footing mission can be used and when it is a suppose rectangular footing or aspect ratio 2 or 3 or 4, then rectangle equation can be used. So, with some modification that I will discuss later on.

And in some book of course, some time instead of 10.3 they use 1.2 actually. Sometime to make it conservative actually instead of 1.3 sometime people use 1.3 so 1.2; that means, we can if you reduce the bearing capacity in the conservative site.

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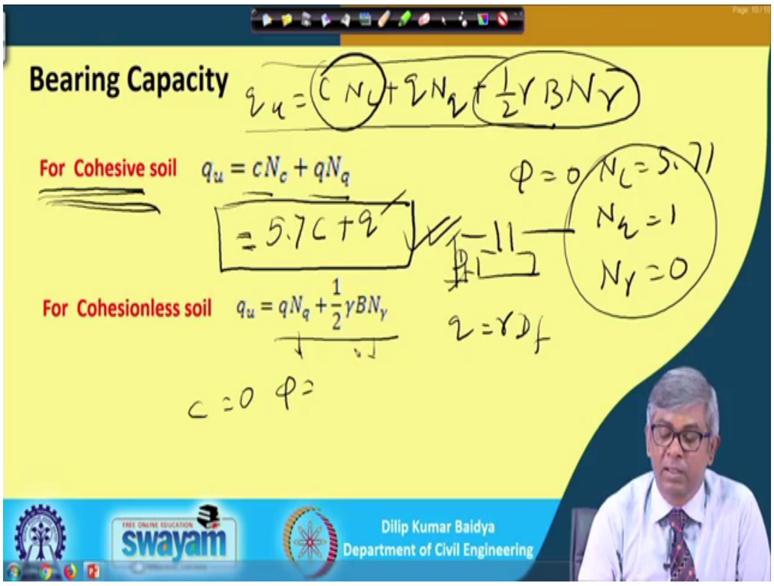

Now, I will try to see some more aspect, you can see for we have got the generalized equation q u equal to c N c plus q N q plus half gamma B N gamma so this is the general equation.

Now, I will try to modify it for different soil suppose this is a purely cohesive soil then what will happen? Purely cohesive soil; that means, phi equal to 0 and there I have seen that N c equal to 5.71, N q equal to 1 and N gamma equal to 0. So, when N gamma is 0 that component become 0 so equation reduced to c N c plus q N q and further this equation can be reduced to c means it will be N c means it is 5.7 c plus q N q means again q N q is 1. So, ultimately you can say the equation will be reduced to this form.

So, when the soil on the footing is squared on a strip footing on purely cohesive soil then ultimate bearing capacity can be strict, can be either one you can go by this; go by this universal or generalized bearing capacity equation and modify for based on phi equal to 0 put this value and then come to this or directly you can remember that is 5.76 plus q.

This is the ultimate bearing capacity on cohesive soil. Similarly, when there is a cohesionless; that means, c equal to 0, but phi is having some finite value. In that case when c equal to 0 then this part will become 0 so, that will not then your equation will be reduced to q N q half gamma B N gamma. Here actually depending on value of phi you can find out what is the value of N q? What is the value of N gamma? And then you can find out the value of bearing capacity.

Now, further modification can be done suppose the footing initially was footing was here; that means, it is there was a D depth then that was q equal to gamma times D f. So, q can be replaced like this, that is the equation can be used here, but now I can imagine that I can imagine footing is on the surface of the ground surface of the ground.

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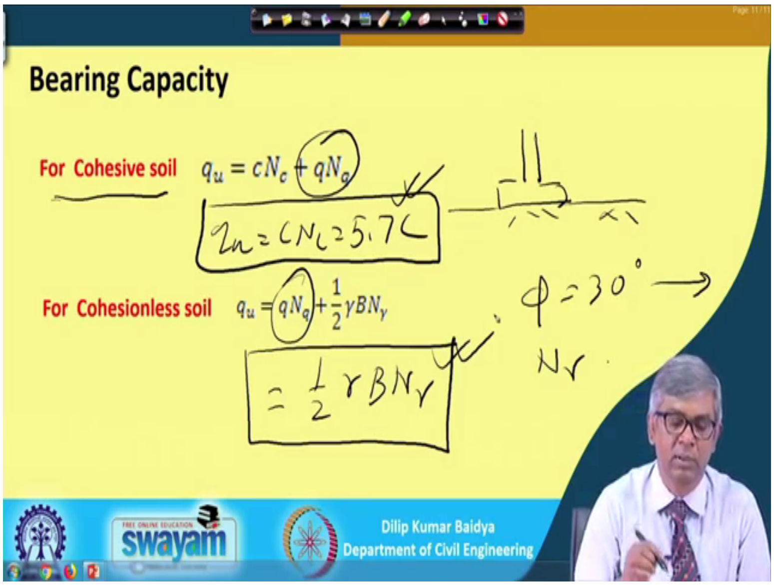

In that case for cohesive soil, this component also become 0. So, ultimately for surface footing on cohesive soil your q u equal to c N c or 5.7 c. Similarly, if it is a surface footing on sand, then your c N c become 0, but these 2 parts will be there and you can see then again if it is a surface footing q is 0 so this part also will be there.

So, this equation will be reduced to half gamma BN gamma so; that means, a footing on the surface of a cohesive soil then directly bearing capacity is this. And footing directly on the cohesionless soil on the surface of the cohesionless soil, then directly you can remember the bearing capacity equation for strip footing is half gamma B N gamma.

Now, if phi equal to suppose 30 degrees then refer the table find out the N gamma then put it in this then you will get the ultimate bearing capacity. So, these are the; that means, generalized bearing capacity if you remember, then you can modify, depending upon the condition given suppose if it is a cohesive then c will be there phi will be 0 and corresponding N c, N q, N gamma you know.

Similarly, if it is on the surface again surcharge will be 0. Then further modification will be there so, all tool is reduced to this and similarly on the sand it will be reduce to this. So, like that everyone has to remember.

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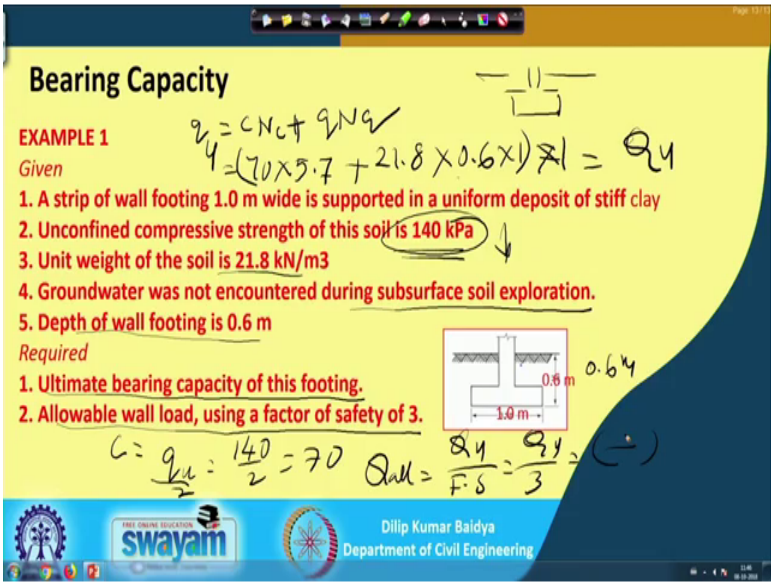

Now, let me take one problem you can see a strip footing is given suppose and it is embedded at a depth of 0.6 meter and width is 1 meter and unconfined compressive strength of this soil is 140 kPa and unit weight is 20 1.8 kilo Newton per meter ground water I forgot, I missed that point actually. When the Terzaghi bearing capacity theory was developed, then they have considered water table is at a great depth. So, that it is effect will be ignored so, water table effect is not there.

So, it is here also groundwater was not encountered during sub soil exploration that means, water table is much deeper and depth of the footing is 0.6. Now. we have to find out the ultimate bearing capacity of this footing an allowable wall load using factor safety of 3. So, there are different types of bearing capacity I have already discussed ultimate bearing capacity, there are net bearing capacity there are allowable bearing capacity, we will discuss later on; for the timing I will try to find out ultimate bearing capacity and then if I introduce factor of safety 3 then what is the bearing capacity that I will try to show.

Now, for this footing actually you can see since it is a strip deposit of clay; that means, your phi is 0 then your q become c N c plus c N c plus q N q this equation is there.

So, your c is how much? When unconfined compressive strength is given that is actually unconfined compressive strength q u and what is the relation with the c? C will be equal to q u by 2. So, that mean one 40 by 2; that means, 70. So, this become 70 and your N c become 5.7 and plus q become how much? 21.8 is the unit weight gamma multiplied by depth equal to 0.6 and N q a N q is 1.

So, if you calculate this one, we will get the value of ultimate bearing capacity. And I have not calculated that you can calculate that. Once you get that ultimate bearing capacity, then we can find out allowable wall load, allowable wall load means with respect to face. So, if it is the ultimate load suppose ultimate bearing capacity and it is if I consider unit length then; that means, these multiplied by 1, this multiplied by 1 will be load ultimate load on the wall. This is ultimately ultimate load on the wall ultimate load on the wall.

And then what is the allowable ultimate load of the wall? that I can say, this is multiplied this become Q u and what is the Q allowable; will be Q u divided by factor of safety; that means, Q u divided by 3 whatever value it comes that will be your allowable load on the wall per metre length.

So, this is the one this is the simplest possible problem I have taken I will take some more problems some other aspect also I will discuss one by one. So, this is one actually so, ultimate bearing capacity of strip footing and then for various condition how to modify all those things I have discussed. And with this I will stop today, maybe in the next class I will discuss some other aspect of the bearing capacity.

Thank you.