

Geotechnical Engineering II/ Foundation Engineering
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Lecture – 58
Introduction to machine foundation (Contd.)

So, already I have discussed various aspects of briefly of course, various aspects of vibration and solution also have how to use them. I have taken one set of problem and once again, I have also mentioned what are the dynamic profile parameters involved for the analysis of machine foundation and how to determine, all I have discussed.

Now, one more set of problem I will take to understand better.

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Introduction to Machine Foundation

A body weighing 600 N is suspended from a spring which deflects 12 mm under the load. It is subjected to a damping effect adjusted to a value 0.2 times that required for critical damping. Find the natural frequency of the undamped and damped vibrations, and in the latter case, determine the ratio of successive amplitudes

$D = \frac{C}{C_c} = 0.2$
 $W = 600 \text{ N}$
 $m = \frac{600}{9.81} \approx 60$
 $K = \frac{W}{\delta} = \frac{600}{12} = 50 \text{ N/mm}$

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So, let me go to the problem number 1 for today's problem you can see a body weighing 600 Newton is suspended from a spring which deflects 12 millimeter under the load. It is subjected to a damping effect adjusted to a value 0.2 times that required for critical damping. Find the natural frequency of the undamped and damped vibration, and in the latter case, determine the ratio of successive amplitude.

You can see here that this problem mass is given mass of the body is given and suppose W is 600 Newton and then you can find out m 600 by g 9.81 which we generated at 600 by 10, ok; so it will be 60.

And, you can see that instead of spring stiffness when this mass is suspended from the spring how much deflected is given. From this what we can get actually? The load actually is given and deflection is given actually stiffness definition of stiffness actually load by deflection so; that means, your load deflection is 600 and your deflection is 12, ok. So, it will be it will give you 50 Newton per millimeter. Is it not?

And, you can see one once again that your K become this and I know the damping will be equal to C over C C. So, this C is given 0.2 times of C C. So, that means, so, it is 0.2 times C C divided by C C; that means, damping is 0 nothing, but 0.2.

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The slide contains the following handwritten calculations:

- $K = 50 \text{ N/mm}$ and $K = \frac{W}{\delta} = \frac{600}{12}$
- $\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{50 \times 1000 \times 10}{600}} = 28.86$ and $D = 0.2$
- $W = 600$
- $\omega_d = \omega_n \sqrt{1 - D^2} = 28.27$
- $S = \frac{2\pi D}{\omega_d} = \ln\left(\frac{21}{22}\right) \left(\frac{21}{22}\right)^{2\pi D} = 3.51$

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And, find the natural frequency. So, we have got this spring stiffness. So, let me go to the next page your K become K become 50 Newton per millimeter and D become 0.2 and W become 600. And, so, your natural frequency omega equal to under root K by m.

So, it will be under root K is actually 50 Newton per millimeter square 50 multiplied by 1000 Newton per meter and m here actually is 600 and divided by g, so, multiplied by 10 here. So, if we do this 600 by 9.81. So, you get 50 multiplied by 1000 multiplied by 10 divided by 600 under root this gives you 28.8. It is actually 28 point it is 28.86.

Now, your damped natural frequency ω_d is actually equal ω_n under root 1 minus D square; so, D is 0.2. So, 1 minus 0.2 square and that to under root answer, so, that become these multiplied by 28.86. So, it become 28.27 28.27 this is the portion ares

actually natural frequency damped natural frequency and also it is asked ah to find out that is given ratio of successive peak, ok. So, δ actually equal to δ equal to nothing, but $2\pi D$ and nothing, but $\ln Z_1$ by Z_2 , is it not?

So, D already is known and so, your Z_1 by Z_2 will be equal to e to the power $2\pi D$. So, it will be equal to 2 multiplied by π multiplied by 0.2 this and it will be e to the power sorry, 2 multiplied by π multiplied by point to that become this 1256.

So, e to the power e to the power so, it will become 3.51; that means, the in that problem where a 600 Newton weight is hanging from a spring and the spring stiffness is not given instead of that when you hang how much deflection to place that is mentioned 12 millimeters, if it is given then from there I got the w_k equal to w by δ from their actually it is 600 and this is actually 12 from there I got this.

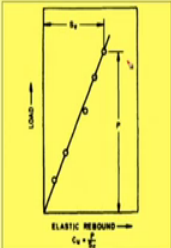
And then from there actually ω_n once K and m is known then we can find out ω and damping also is given 0.2, then ωD I can find out and further last point was asked actually to find out the ratio of successive peak and we know the ratio of successive peak actually when you put it a logarithm the log that is called logarithmic decrement and that is actually $2\pi D$ and. So, δ equal to $2\pi D$ equal to $\ln Z_1$ by Z_2 . So, Z_1 by Z_2 will be e to the power $2\pi D$ and if you put their damping value here then we are getting Z_1 by Z_2 directly 3.5 51.

That means actually first peak will be 3.51 times of second peak, ok; so this is the problem number 1.

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Introduction to Machine Foundation

In a cyclic plate load test on a plate of $0.60 \text{ m} \times 0.60 \text{ m}$ size settles 0.65 mm under a pressure of 20 kN/m^2 . On unloading observed plate settlement was 0.60 mm . Determine the value of coefficient of elastic uniform compression of the soil



$p = 20 \text{ kN/m}^2$
 0.65 mm
 $0.60 \rightarrow$
 $\delta_e = 0.05 \text{ mm}$
 $P = 20 \times 1000$
 $C_u = \frac{P}{\delta_e} = \frac{20}{0.05} = 400 \text{ kN/m}^2$

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Let us go to the next problem and in a cyclic played load test actually you can see that the pressure applied actually 600 K sorry P is actually 20 kilo Newton per meter square and initially was 0.65 meter sorry, millimeter and then finally, after rebound on unloading observed plate settlement was 0.60 ok. Then, elastic will be delta e will be equal to 0.05 millimeter. So, your C u will be equal to P by delta e. So, that will be 0 point sorry 20 divided by 0.05.

And, and this is millimeter and so, this millimeter 20 you have to make 1000. So, this way it will be 20 divided by 0.05 multiplied by 1000. So, it gives you 40000 kilo Newton per meter cube ok.

So, this is a very simple problem whatever I have discussed about the is code, directly application of that particular loading, what is the elastic settlement is given then the ratio is actually your coefficient of compression elastic compression.

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Let me go to the next problem you can see a mass attached to a spring of 5 Newton per millimeter has a viscous damping device and when the mass was displaced a release the period of vibration was found to be 2 second, ratio of the consecutive amplitude was 10 by 3. Determine the damping factor and natural frequency of the system. Determine also the amplitude of motion when a force $3 \sin 4t$ acts on the system.

So, here actually you can see spring K is given 5 Newton per millimeter square or 5; so 5 kilo Newton per meter, ok. And, you can see the period T is given 2 second which is nothing, but 2π by ω . So, you from here we get ω equal to ω ω equal to 2π by 2, so; that means, 3.14, ω become three ω n natural frequency you get 3.14.

And, then the ratio of the consecutive amplitude was 10 by 3. So, it was δ equal to δ equal to $\ln 10$ by 3 equal to $2\pi D$, is it not? So, ratio of successive peak is this one, if you take log that is actually δ and that δ we know this $2\pi D$. So, from there I can get D equal to 1 by $2\pi \ln 10$ by 3. So, from here I will get D equal to 19.16. Actually you will get initially 0.1916, but if I put in percent; so it become 19.16 percent.

Now, actually you have to find out also let me so, we have done that determine the damping factor D already damping fact or natural frequency already obtained natural frequency and damping factor all already obtained.

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Introduction to Machine Foundation

A mass attached to a spring of 5 N/mm has a viscous damping device. When the mass was displaced and released, the period of vibration was found to be 2 s and ratio of the consecutive amplitudes was 10/3. Determine the damping factor and natural frequency of the system. Determine also the amplitude of motion when a force of $3 \sin 4t$ N acts on the system.

Handwritten solution:

$$F = 3 \sin 4t$$

$$F_0 = 3 \text{ N}$$

$$\omega = 4$$

$$\omega_n = \frac{4}{3.14} = 1.274$$

$$D = 0.191$$

$$u = \frac{F_0}{K} \cdot \frac{1}{\sqrt{(1 - \frac{\omega^2}{\omega_n^2})^2 + (2D \frac{\omega}{\omega_n})^2}}$$

$$= \frac{3}{5 \times 10^3} \times 1.263$$

$$= 0.758 \text{ mm}$$

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So, I will clean it and for second determined also the amplitude of motion when a force of so, f actually is given $3 \sin 4t$. So, that means, as I have mentioned f naught actually 3 Newton and ω actually 4. And, your expression for u u equal to F naught by K and 1 over under root 1 minus ω over ω_n square whole square plus $2D \omega$ over ω_n whole square if I do this and ω over ω_n you can see ω actually 4, ω over ω_n equal to 4 divided by 3.14 it is nothing, but 1.274. 1.274 and D become 19 point or 0.191.

And, if you put these and this in this expression F naught equal to 3 and K actually already I have shown that K actually 5 kilo Newton per meter K actually 5 kilo Newton per meter. If I put all those in this expression then you will get 3 divided by 5 multiplied by since both are in expression Newton multiplied by this expression comes 1.263. So, if I put this there it come become 0.75 sorry 0.758 millimeter.

So, amplitude of this vibration when this system is system will vibrate then with a force equal to $3 \sin 4t$ then your amplitude will become 0.758 millimeter, So, this is actually problem number 3. Let me go to the last problem.

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Introduction to Machine Foundation

Plot the frequency-amplitude response curve and determine (i) the stiffness and (ii) the damping ratio of the foundation soil system for the following data:

Frequency (rpm)	1400	1500	1525	1575	1625	1675	1710	1770	1825	1925
Amplitude (micron)	18	30	40	60	88	85	72	55	45	34

Weight of the foundation block including machine weight is 8 kN and eccentric moment, $w_e = 0.156$ N-m. Determine the coefficient of uniform compression and damping as per IS 5249.

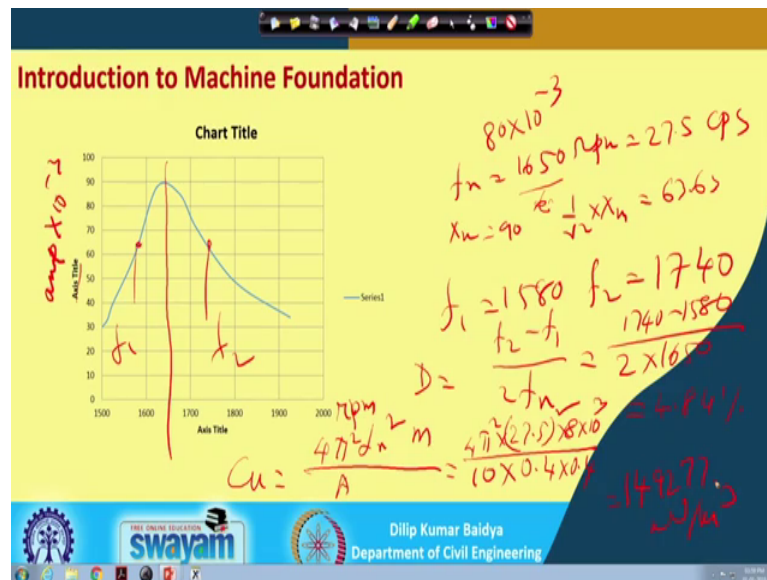
$A = 0.4 \times 0.4$

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So, this is the last problem. You can see here that plot the frequency amplitude response curve and determine the steepness and the damping ratio of the foundation solid system for the following data; that weight of the foundation block including machine weight is 8 kilo Newton and the eccentric moment is 0.156 Newton millimeter. Determine the coefficient of uniform compression and damping as per IS 5254. So, this is it is end actually a is missing. So, this is one and here one data is missing actually foundation size is missing. So, foundation always test was conducted a equal to 0.4 by 0.4.

So, this data is there. So, these data has to be plotted. So, amplitude versus frequency you have to plot like this and from there you have to find out resonance f_m and then $1/\sqrt{2}$ times f_m and this is X_m and then you have to find out $1/\sqrt{2}$ times X_m . So, one point here one point here you can find out.

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So, like that if I do let me go to the next page and you can see that if I plot this one frequency versus amplitude these are actually this is amplitude multiplied by 10 to the power minus 3. Suppose if it is 50, so, if I get 80. So, 80 multiplied by minus 3 is the amplitude and here actually amplitude in rpm, ok. So, from these actually from this plot I can find find out f_n equals to 1650 rpm or if I cps if I plot then it become 27.5 cps. What is cps actually if I divide by 60 then it becomes cps.

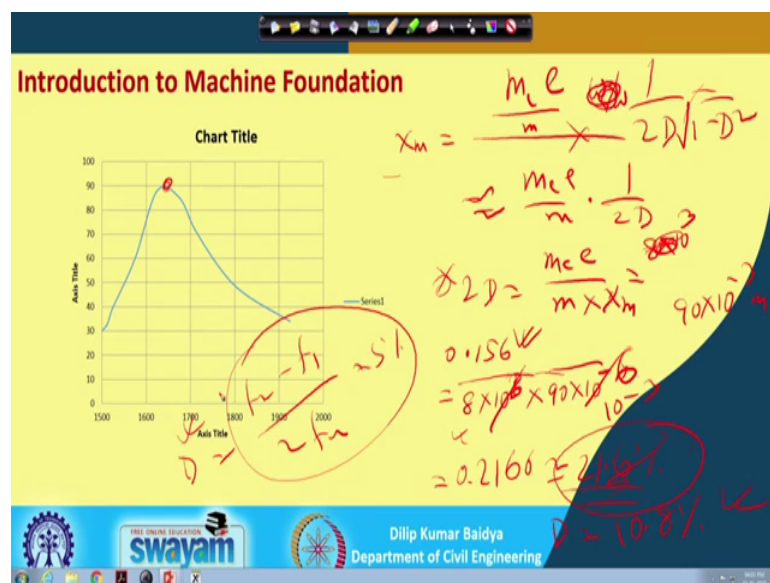
f_n and X_m actually your become 90 and so, 1 by root 2 times X_m that become 1 by root 2 time X_m actually 63.63 and for that somewhere here and somewhere here and this is if it f_1 and this is f_2 then f_1 comes as 1580 and f_2 comes as 1740, ok; so based on that you can find out damping. So, damping will be equal to f_2 minus f_1 by 2 f_n ok. So, that means, how much it is? 1740 minus 1580 divided by 2 multiplied by f_n actually is it is 1650. So, if I do this then you are getting damping value actually 4.84 percent; this is the damping.

And, if you want to find out C_u ; C_u actually $4\pi^2 f_n^2 m$ divided by A . So, you can see that $4\pi^2$ square multiplied by f_n actually here your cycle per second you have to put 17 point 27.5 whole square multiplied by m actually 8 kilo Newton is the weight. So, 8 multiplied by 10 cube is the Newton divided by 10 actually taken g and then divided by area is 0.4 multiple by 0.4.

So, these calculation if you do this calculation if you do then you may get a value something like 4 multiplied by multiplied by 8 multiplied by 1000 divided by 10 divided by 0.4 divided by 0.4. So, this value actually 149277 kilo Newton per meter cube. This is the value of C u. So, this is thing asked for this and for finding out the damping I can do another way that is from the peak amplitude I can find out the damping.

So, that is another way of finding damping; so let me try that one. Let me remove this one whatever asked actually asked for IS code I have done.

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Now, let me do the other part. We know that X_m equal to $m_c e$ over $m \omega$ over ωn no divided by multiplied by 1 by $2D$ under root 1 minus D square or it can be $m_c e$ over m multiplied by 1 by $2D$. So, approximately; if D is small these approximation can be done.

So, I know X_m , so, that means, I can say $2D$ so, $2D$ will be equal to $m_c e$ into m multiplied by X_m . So, here actually 8 kilo Newton in converted Newton sorry, this is not 8 kilo Newton let me $m_c e$ actually 0.156 0.156. So, this is in Newton this is also. So, here actually 8 into 10 to the power sorry 10 cube this one. So, since both are mass. So, if I multiply g both sides so, it will get cancelled. So, directly w into e and this is w I have written and here actually X_m actually is 90 that is actually multiplied by 10 to the power minus 3. So, this is actually your $2D$.

So, if I calculate this one $2D$ comes 0.156 multiplied by 1000 divided by 8 divided by 90 , this will be 10 to the power 6 because millimeter to it was 90 multiplied by 10 to the power minus 3 in millimeter, but if I want to meter then it will become 10 to the power 6 . So, these and these will get; so it will be the 10 to the power minus 3 . So, 0.156 multiplied by 1000 sorry 0.156 multiplied by thousand divided by 8 divided by 90 , sorry that become that become 0.2166 or equivalent to 21.6 percent.

So, you can see from the peak amplitude we are getting if I take directly from the peak using the peak amplitude load you can find out the damping this is the way. So, it is becoming 21.6 percent whereas, from the other way; that means, damping equal to f_2 minus f_1 by $2 f_n$ from that if I calculate then there actually you are getting only 5 percent. Oh sorry. Oh sorry this is actually $2D$ this is actually $2D$. So, your D will become 10.8 percent or so 21.6 divided by 2 .

21.6 divided by 2 it become sorry, 21.6 divided by 2 that become 10.8 percent so; that means, from the peak amplitude when you are calculated; that means, from this equation we are getting 10.8 percent, but if you use this expression you are getting 5 percent. So, this is actually this is a reason actually from the peak amplitude if I determine the damping it is over estimates and whereas, if I use this concept that you get the average value of damping. So, this is better actually, but I have explained both the method you can use any of this there is any in the in the exam.

So, I think I have covered enough now about the machine foundation, simple machine foundation, even rotating mass system or constant mode system, free force vibration and their relevant application and then I have very little left. So, I will give you the summary in the next class.

Thank you.