

Geotechnical Engineering II / Foundation Engineering
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Lecture - 57
Introduction to machine foundation (Contd.)

Let me continue with machine foundation Introduction to machine Foundation. Machine foundation is a full course, but very preliminary portion we will include in this and already we have discussed vibration theories like force vibration, free vibration again force damped force undamped, free damped, free undamped. And they are corresponding governing equation how to develop the equation and how to solve and also finally, what is their solution all we have discussed.

And in fact, I have taken a few problems also, to make you understand how to use them. And once again I will summarize those final form and in addition to that I will try to show what are the soil parameters, dynamic soil parameter is required. And then finally, how to determine them through IS following IS Code Indian Standard Code.

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The slide displays two systems and their corresponding equations:

- Constant Force System:**

$$u = \frac{F_0/k}{\left[\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + (2D\frac{\omega}{\omega_n})^2 \right]^{1/2}}$$

$$u_{max} = \frac{F_0}{K} \frac{1}{2D\sqrt{1-D^2}}$$
- Rotating mass system:**

$$u = \frac{\frac{m_e e}{m} (\omega/\omega_n)^2}{\left[\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + (2D\frac{\omega}{\omega_n})^2 \right]^{1/2}}$$

$$u_{max} = \frac{m_e e}{m} \frac{1}{2D\sqrt{1-D^2}}$$

Handwritten notes include: $F = m_e e \omega^2 \sin \omega t$, $F_0 = m_e \omega^2 e$, and $\omega_n = \sqrt{K/m}$. A note at the bottom right says "mass of eccentric part".

So, let me go to the first slide, and you have seen here that though we have two types of motion at the free vibration or free transient motion and force vibration bar sinusoidal motion. And there and free vibration of course, there can be many thing like have a

foundation or if I during or any if you strike blasting then slowly the initial amplitude will decrease, that is all our free vibration.

And now if you have the force vibration and that too sinusoidal type then for that we have also discussed two types of force system can be there, one is actually that constant force system. That means, in these of course, there will be $\sin \omega t$ can be there, here also there can be a $\sin \omega t$, but we will not, if I do not take these do not take these then this is actually giving for a particular frequency what is the maximum value. So, because of that we do not consider this, we consider this as amplitude in millimeter.

So, now in this constant force system means what F equal to $F \sin \omega t$. One example suppose it is $3 \sin 40$ if this is the equation then your F naught become 3 and your ω become 4. So, this is one type of forcing system; that means, 3 it has some unit maybe 3 Newton of force at a frequency, at a speed ω it is subjecting to have any foundation then it will produce some vibration. So, this is the force, in this F naught is this and speed of the from there actually you can find out frequency of course, and that actually given by this. So, while solving problem we will try to explain this. So, this is one type of; that means, constant force that with this constant force 3 Newton or 3 constant force is repeating at a particular frequency. So, that is one thing.

Another thing I have mentioned that rotating mass system; that means, as I have told shown before the many times that if there is in a system there is a mass, a static mass it is rotating with respect to some radius with some center that with this mass will produce some centrifugal force which will be equal to $m e \omega^2$ or I can write $m e$ into r into ω^2 . This r is actually radius; that means, and which is nothing, but eccentricity also sometimes we can write a $m e$ into $u \omega^2$; that means, this is eccentric distance or you can write simply r because along the radius r it is rotating.

And small $m e$ is actually a mass of eccentric rotating part and r and r or e is the distance from the center along which it is rotating and ω is the speed. If so, this type of it that is in the machine this type of unbalanced part rotates with some center with respect to some center that also creates some vibration and that actually if that is the one then in that case we write force will be equal to $m e \omega^2 \sin \omega t$ ok. So, this is the one this is the force.

Now, this force here actually F_0 was constant it is repeating at a particular frequency here, the force also magnitude of force dynamic force also depends on the speed of ω . So, if the ω changes then the force also changes. So, with for this type of forcing system and this type of forcing system the solution, corresponding solution steady state solution is this and for this type of forcing system steady state solution is this.

Now, we have got this solution and this is actually at any frequency and when u_{\max} when you get displacement maximum when actually ω / ω_n equal to 1 or at resonance ok, it may not be exactly 1. So, for this what is the value ω_n what is the value for this ω / ω_n what is the value I have to shown last class so, in the last lecture. So, I am not repeating that that what actually this is actually general amplitude for constant force system at any frequency that ω is here ω can be 2 4 8 anything.

And similarly for rotating mass system this is the steady state solution for any frequency at this is ω is there, but at resonance that amplitude become maximum and expression for a maximum is F_0/k multiplied by this term only damping will be there. This is already there here also it is there additionally instead of this big expression only this expression will be there.

Similarly, for rotating mass system u_{\max} is this more this thing is there already and instead of this such a big expression we have got this. So; that means, if I want to find out at any frequency for constant force system, I will use this equation if I want to find our resonant amplitude for constant force system that I will use this expression. Similarly, if I want to find out amplitude at any frequency for rotating mass system, I will use this expression and if I want to find out the resonant amplitude for rotating mass system then I will use this expression. So, these actually have to be very very familiar with.

Now I will just better whatever I written I just clean it and once again I write few important thing.

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Introduction to Machine Foundation

Constant Force System

$u = \frac{F_0/k}{\left[\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2D \frac{\omega}{\omega_n}\right)^2 \right]^{1/2}}$

$u_{max} = \frac{F_0}{K} \frac{1}{2D\sqrt{1-D^2}}$

Handwritten notes: F_0/k → stiffness; $2D \frac{\omega}{\omega_n}$ → Damping; ω_n → natural frequency; ω → operating frequency.

Rotating mass system

$u = \frac{\frac{m_e}{m} (\omega/\omega_n)^2}{\left[\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + \left(2D \frac{\omega}{\omega_n}\right)^2 \right]^{1/2}}$

$u_{max} = \frac{m_e}{m} \frac{1}{2D\sqrt{1-D^2}}$

Handwritten notes: m_e → mass ecc rotating part; e → eccentricity → r .

Diagram: A small sketch of a rotating mass m with an eccentricity e .

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Firstly, I have already told that constant force magnitude and this k actually this is capital K can be there, k is the stiffness and your D is the damping, ω_n is the natural frequency and your ω and F_0/k and k is the stiffness already I have mentioned constant ω and natural frequency and this is there. And when you come here all those things are there additionally there are 3 terms.

So, m_e into e that is mass of eccentric rotating part and e is the eccentricity, sometime it is r radius along which actually it is rotating and your m capital m is the mass of foundation plus machine. That means, if I imagine a foundation like this and that is a machine here then because of this machine this entire machine and foundation will vibrate together. So, these two together mass is m . So, m is the mass of foundation plus machine together. So, all those terms are known now, and ω is the operating frequency and which can be varied ok. So, depending upon your requirement; so, all terms are explained now.

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Introduction to Machine Foundation

Constant Force System *Shear modulus* Rotating mass system *D → damping*

$$u = \frac{F_0/k}{\left[\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + (2D \frac{\omega}{\omega_n})^2 \right]^{1/2}}$$

$$u = \frac{\frac{m_e e (\omega/\omega_n)^2}{m}}{\left[\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + (2D \frac{\omega}{\omega_n})^2 \right]^{1/2}}$$

$$u_{max} = \frac{F_0}{K} \frac{1}{2D\sqrt{1-D^2}}$$

$$u_{max} = \frac{m_e e}{m} \frac{1}{2D\sqrt{1-D^2}}$$

Handwritten notes:
 $K = \frac{4Gr}{1-\mu}$
 Radius of the foundation
 Poisson's ratio

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Now, if I want to find out this amplitude or natural frequency from this what are the things you require? You need to find out k which will be equal to $4Gr$ by $1 - \mu$ for vertical vibration I will discuss only vertical vibration. And in these G is the shear modulus and your r is the radius of the foundation of the foundation and μ is the Poisson's ratio ok.

So, all the k is required, m naught will be known; then another thing is damping D is the damping that has these are actually damping and shear modulus is the soil properties and shear modulus and size of the footing together is actually stiffness. So, these are the things actually you need to find out; that means, you want to find out the amplitude. So, how to determine them? So, that actually there are some Indian Code actually there are of course, many places they do not follow the similar method, but Indian codes still follow that. So, we have to I want to discuss that and you can go through that and accordingly we will take some problem also.

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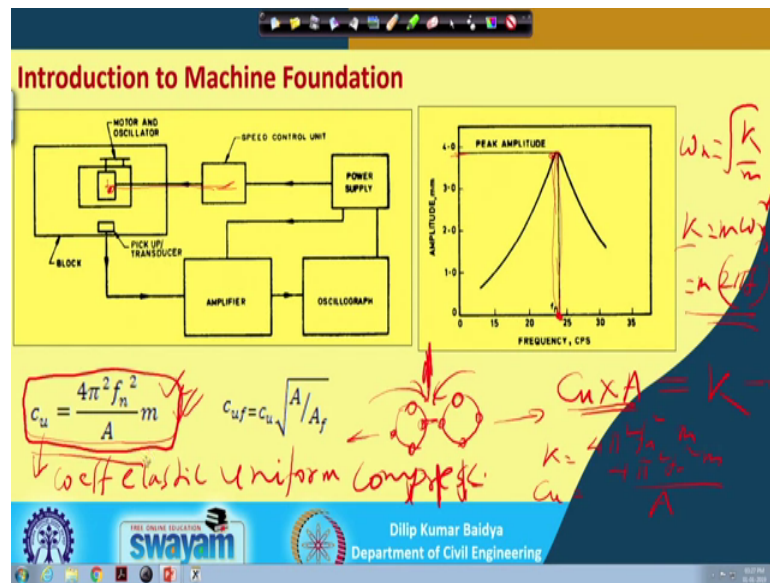
The slide is titled "Introduction to Machine Foundation". It features a diagram of a concrete block (C.C. BLOCK) and a pick-up mechanism (PICK-UPS) with a meter. The diagram shows two blocks of different widths, labeled d_1 and d_2 . Handwritten notes in red ink include "IS 5249", "1999", "depth of foundation", and "1m x 1m x 1.5m". The slide also contains logos for Swamyam and Dilip Kumar Baidya, Department of Civil Engineering.

So, let me go to the next slide, you can see here that as per IS code IS 5249, and it has several versions. So, 1999 was the last code revised and there actually that things are given that you have to make a pit of sufficiently long, and sufficiently big and that pit should be at this depth actually at what depth actually will be it will be actually depth of foundation, depth of foundation. How deep it should be if the depth of foundation recommended. So, 2 meter then you have to make the pit of 2 meter deep and then appropriate size, size also mentioned.

And then you have to make a block of 1 meter by 1 meter by 1 meter by 1.5 meter; that means, one by one cross section and 1.5 meter height and if the soil is stiffer then size can be made smaller so that the recommendation also there. So, one concrete block has to be constructed and then were that you have to put a oscillator and then the oscillator can be operated through electricity and then this block will be vibrated and then at different frequencies amplitude can be observed and that amplitude versus frequency versus amplitude can be plotted. And finally, from that plot one can find out many things, that I will discuss now in the next slide.

So, this is the actually experimental arrangement that we have, you have to make a pit of certain depth that depth equal to depth of foundation.

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Next let me, see this is another arrangement actually you can see here this is actually given the planned view how it will be this is the pit actually, on the pit there will be block mode and on the mode on that actually motor and oscillator this is a motor and oscillator and from there is the speed control unit. Through that through these actually it will be rotated directly in there is no connection.

And then power supply etcetera will be there and then there will be some pickup to measure the amplitude and frequency. So, this is actually the arrangement of test system, once you change the frequency and in this vibrator actually this oscillator that rotating mass type of system is there to counter rotating mass is there as I have shown before also that to counter rotating mass, rotating mass is there. And based on that, when if it is here then if particle vibration is there if it goes here then sorry.

If it is this way then it will come here this will come and so they will face each other get canceled when it will be coming here then it will be this direction will be coming again these direction this. So, it will be canceled. So, only produce this vertical vibration. Similar type of things are there in this oscillator and then you change the frequency and observe the amplitude on the foundation and then if you finally, plot amplitude frequency versus amplitude you get a typical curve like this from this curve, what we will do? You have find out the peak amplitude and then from the peak amplitude you measure what is

the value and then where the peak we are getting is dropped to the x axis and then read the x axis that is actually natural frequency.

So, natural frequency resonance it is a little differences if the damping is very high the natural frequency and your resonant frequency is slightly different, actually at resonant frequency t is a maximum. So, it is actually truly speaking this is resonant frequency and resonance frequency will be little different from the natural frequency it is damping correction will be there, but if the damping is small then natural frequency can be approximated as a resonant frequency.

So; that means, this I can take as the amplitude the frequency corresponding to maximum amplitude will be taken as natural frequency. And if you know the natural frequency, and if you know that area of the foundation on which you have conducted the test then using this equation I can find out the coefficient of elastic uniform compression. So, this is as per code coefficient of elastic uniform compression ok.

So, this is actually your c_u and your a multiplied by a it gives you the k . And how we are getting actually? Your f_n or ω_n equal to $\sqrt{k/m}$ and; that means, k equal to $m \omega_n^2$ and; that means, m into $2\pi f$ whole square. So, if I do this you can see it is coming this one, k is coming this and then c_u actually divided by you can see k become k will be equal to $4\pi^2 f_n^2 m$ and then c_u will be equal to $4\pi^2 f_n^2 m$ divided by a because your c_u multiplied by a is k . So, that is the way it is this expression given in the code itself. So, one can find out coefficient of uniform elastic compression which is a parameter as per IS Code.

Now, if I conduct a test on a small footing, but actual footing size is big, then actually whatever from the test value we are getting that cannot be used and there is a there is a need for correction. And that correction if the foundation in size actual foundation size is f and test foundation is A then actual c_u to be used for the foundation c_u foundation actually will be corrected can be corrected like this c_u multiplied by \sqrt{A} over A foundation. So, like that it has to be corrected.

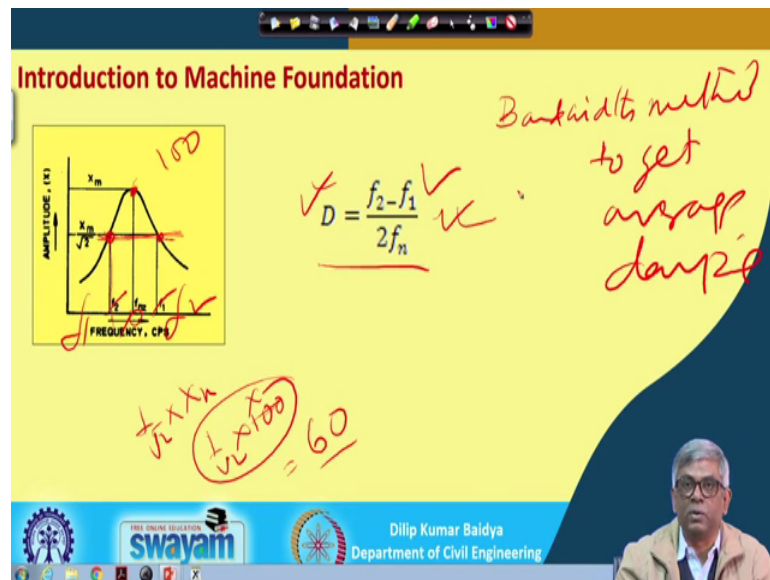
So, if the foundation size is 2 meter by 2, it if you actually you are tested suppose 1 meter by 1 meter, but your foundation size actually 5 meter by 5 meter then it has to be corrected taking f equal to 25.

But I have given an extreme example, actually if your foundation size is too big more than 10 meter square then A_f to be used as 10 meter square. If it is suppose instead of 5 by 5 meter if it is 2 by 2 meter or 3 by 3 meter then A_f will be taken 9, and if it is suppose 5 by 5 then 5 by 5 become 25, but limiting value can be used actually 10, maximum 10 can be for correction you can use this equation up to foundations area are 10 meter square.

So, if it is 2 by 2 you can use actual foundation area if it is 3 by 3 you can use actual foundation, but when it cross 4 by 4 then it become more than 10. So, you have to use only 10. So, that is the correction so; that means, by observing, by conducting the test we will obtain this as response curve, from this response curve you just identify what is the maximum value amplitude and from there you produce then you will get the what is the natural frequency.

Once you get the natural frequency put it in this equation to get the coefficient of uniform compression and if it is the foundation size is different then you correct the found coefficient of elastic uniform compression for design by using this equation.

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And then for determining the damping. So, if this is the response curve as I have shown before previous one, similarly you may get like this. So, that is called bandwidth method or to get average damping, actually see you can see here how we have got if it is maximum amplitude x_m then you multiply by $1/\sqrt{2} \times x_m$. Suppose if it is suppose a 100, then $1/\sqrt{2}$ multiplied by 100, whatever value it comes suppose it

is suppose some 60 or whatever maybe it is x and that is 60. Then I will read 60 here, I read 6 60 you will have two frequency corresponding to my amplitude sixty we may get two frequencies.

So, lesser than before resonance whatever frequency I get that is called f 2 and beyond this one we are getting suppose f 1 and then from this or you can write f 1 and this is f 2, this is better and in that case if it will be expression of damping will be f 2 minus f 1 by 2 f n. So, f n is here and f 2 is this one; that means, whatever x m you are getting multiplied by 1 by root 2 then that corresponding to that amplitude you will get two frequencies. Before frequent resonance whatever you are getting that is called f 1, after resonance whatever you are getting that is suppose f 2. Then read from this graph f 1 f 2 and f n and then put in this equation to get that damping ok.

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So, this is actually from the block vibration test. Now actually this is the one actually free vibration I have shown and we have the expression for this oscillatory things actually we have given. And it is it can be shown that if you take this is the peak, successive peak ratio of successive peak if you take and take logarithmic that is called logarithmic decrement. That is log of successive, log of ratio of successive peak is called logarithmic decrement and if you do take ratio of successive if you take log which is defined as logarithmic delta and using the expression we can prove that that value finally,

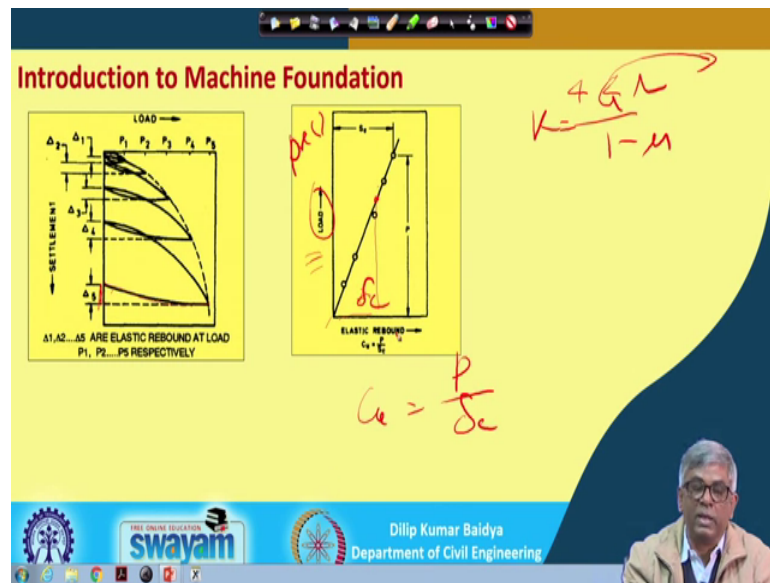
comes like the $2\pi d$ under root $1 - d^2$. If D is small then this can be approximated as $2\pi D$.

And now then we can write you can see then D can be $\frac{1}{2\pi} \ln \frac{x_n}{x_{n-1}}$. So, a successive peak between 1 and 2 can be that always the delta will be constant, similarly between 2 and 3 the ratio will be log of ratio will be same. So, because of that in general I can write the expression D equal to $\frac{1}{2\pi} \ln \frac{x_n}{x_{n-1}}$ otherwise I could have written $\frac{x_1}{x_2}$, that is also could have been written, but to make generalized I have written $\frac{x_n}{x_{n-1}}$ by $\frac{x_n}{x_{n-1}}$ log of $\frac{x_n}{x_{n-1}}$. So, this is the expression when I will consider successive peak.

But some time to catch the successive peak very difficult, some time 0 peak I catch and then I end number n th number of peak I will catch. In that case how to use this one you can see if I write x_0 by x_n that is nothing x_0 by x_1 x_1 by x_2 x_2 by x_3 x_3 by x_n . And you can see these are actually nothing, but what? This is all ratio of successive peak and from this expression you can see x_1 by x_2 is nothing, but e to the power δ . So, that way this is e to the power δ , this is another e to the power δ this is another e^2 the power δ this is another e to the power δ so; that means, and it gets cancelled actually. So, ultimately x_0 by x_n will be remaining, but ultimately if I put individual value it become e to the power $n\delta$.

And; that means, $\ln \frac{x_1}{x_n}$ by x_n is nothing, but if I take log now then it becomes $n\delta$ and δ is already, I have shown that $2\pi D$. So, $2\pi D$, n it become then your D become $\frac{1}{2\pi}$ and $\ln \frac{x_0}{x_n}$; that means, if I measure a peak here another peak somewhere here after a long time and suppose number is n and then the ratio of that that amplitude x_0 by x_n if you take the log that value multiplied by $\frac{1}{2\pi}$ into n will be actually damping value. So, this is another method that is by free vibration test one can find out the damping by this method, that is called logarithmic decrement method.

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Now, for finding out the stiffness there are methodological method actually put $G r$ by $1 - \mu$, if you know the G you can see the stiffness can be obtained $4 G r$ by $1 - \mu$ already I have told. And from if you know the G , then we can find out that, but here actually to find out c_u that is a method is given you can see if I do load versus settlement and loading unloading if you do if I P_1 load is applied and then remove then it will not go to the original position, there will be some the final deflection δ_1 .

So, final deflection that is δ_1 corresponding to $P_1 P_2 P_3$ that $\delta_1 \delta_2 \delta_3 \delta_4$ whatever it is shown it is final here and rebound is here, then what is the this is called δ_5 or whatever elastic settlement. So, whatever rebound is technically that is actually elastic settlement. So, that one actually if you take so, the correspondence of that load. So, this is the elastic settlement δ_e and this side actually load actually nothing, but load is not load actually this is pressure actually it is pressure.

And then if you plot them you will get it like that and finally, your c_u become P by. So, you will get a straight line and you can take any point over that and find out by measuring P by δ_e will be your c_u from this also test one can, by play load test one can do it.

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The slide is titled "Introduction to Machine Foundation". It features a graph on the left with "TIME OF TRAVEL, s" on the y-axis (0 to 0.15) and "DISTANCE, m" on the x-axis (0 to 60). A straight line is plotted through several data points. A point is marked at approximately (40, 0.12). Below the graph, the text "AVERAGE VELOCITY $v_c = \frac{s}{t}$ m/s" is written, with v_c circled in red. To the right of the graph, two equations are boxed in red: $E = \rho v_c^2 \frac{(1+\mu)(1-2\mu)}{(1-\mu)}$ and $E = 2G(1+\mu)$. Below these equations is a simple hand-drawn diagram of a rectangular block with horizontal lines representing internal layers or stress distribution. At the bottom of the slide, there is a logo for "swayam" and the name "Dilip Kumar Baldya, Department of Civil Engineering". A small video inset of the speaker is visible in the bottom right corner.

Similarly there is another test actually that hammer test, if I take a block if I take a block and then if I strike it then vibration will be there and if I measure at different position actually arrival time distance are known. So, the travel time of travel versus distance you plot then we get an approximate straight line and from there I can find out the velocity and v_c by this equation, v_c equal to s by t . And once you get the v_c you can use this equation one can find out the elastic modulus E and after getting the elastic modulus E ; after getting the elastic modulus E one can after getting the elastic modulus E one can find out G also by using this equation.

So, these are the actually some methods given in IS 5249 and there are so many others actually whatever method we have discussed before also that is during the soil exploration and that actually come your, there are different methods find to find out the G value. So, that method is also applicable in addition to that S 5249 whatever given I have just discussed and with this perhaps I will stop here. About the your determination of dynamic properties whatever given in the IS code briefly I have discussed. You know do not require everything, but some G I have covered; with this I will stop here.

Thank you.