Geotechnical Engineering II / Foundation Engineering
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Lecture - 56
Introduction to machine foundation (Contd.)

So, let me, I have discussed several aspects of free vibration, force vibration and their equation their solution. And now I will try to take a few problem application actually so that it will be helpful to you. So, few problems I will taking this module and try to show you how to solve them different aspect to find out different parameters.

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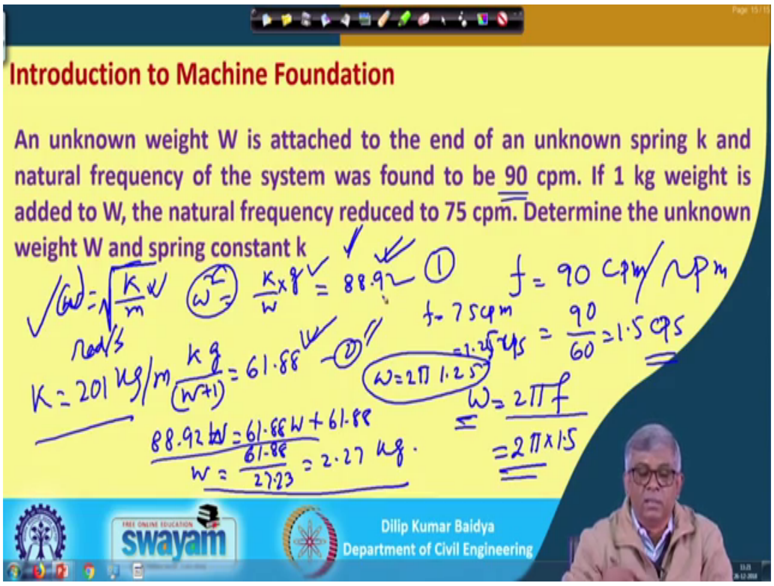

And you can see here let me see the first problem and you can see and unknown weight W is attached to the end of an unknown spring k; that means, both mass and spring is unknown and the natural frequency of the system was found to be 90 cpm cycle per minute. If 1 kg weight is added to W the natural frequency reduced to 75 cycle per minute.

Determine the unknown weight W and the spring constant k. So, this is a problem and actually main thing what you what actually to be used in this? We know that your this is actually given frequency of the system cycle per second f, f equal to given 90 cycle per minute or it will be 90 divided by 60, it will be 1.5 cps cycle per second that is the unit test to be used and you know omega equal to twice 5 times f.

So, this will be equal to 2 multiplied by pi multiplied by 1.5; that means, W omega and we know that your omega equal to under root k over m ok; under root. This is the only thing you used here and if I do that and then you can see that you will be if you do this then omega square will be equal to under root k over W multiplied by g suppose.

And so that will be equal to if x square this one it comes 88.92 and another case actually when actually f equal to 75 cycle per second then if that will be cpm that will be called to 1.25 rpm no 1.25 cps and then omega will be equal to 2 pi multiplied by 1.25. So; that means, I can do another k g divided by 1 plus 1 that will be called to your this square.

So, that will be equal to 61.88. So, these two equations you have got you can see so, this is one equation one and this is equation 2 and you can see we can find out that equating these two equation we can get equating equation 1 and 2 I can see I can equate I can find out k from here.

I can find out k from here this I will equate and then I will get actually 88.92 minus equal to 81 point W equal to 61.88 W plus 61.88 I will get equating this; where simplifying this using this two equation from here I will get W equal to 61.88 divided by 77.23 that will give you 2.27 kg and if I put this W equal to 2.27 kg, then I will get k equal to 201 kg per meter.

So, this is a simple problem actually in the in the vibration any vibration problem the natural frequency is defined as omega is equal to under root k by m, but here actually your natural frequency given in cycle per minute, and that to be converted into cycle per second and from there actually omega actually these omega actually is nothing in radian per second radian per second.

So, I have to convert f to omega; and what is the relationship between omega and f? Omega equal to 2 pi f and that I will do and from the given from the from the given condition 1, I will get this is the equation and from the given condition 1 I will get this is the equation, from this two using these two equation I can said this equation from there I will get W equal to this k equal to this. Simple thing we have done; that means, I have not done anything other than under root k by m or natural frequency equal to under root k by m.

And simultaneously you have to keep in mind that that natural frequency under root k by m is expressed in radian per second that has to be remembered. And when it is expressed either in cycle per minute or revelation per minute cpm or it can be also rpm; that means, that has to be expressed first cycle per second. And if this cycle expression cycle per second, then what is the relationship between omega and f? f actually in cycle per second and omega in radian per second.

And what is their relationship? Omega equal to 2 pi f. So, that to be used in addition to this to be used and then condition 1 this is the equation, condition 2 this is the equation, then solving this I can get the both two unknowns I have got two equations. So, I will get that ok.

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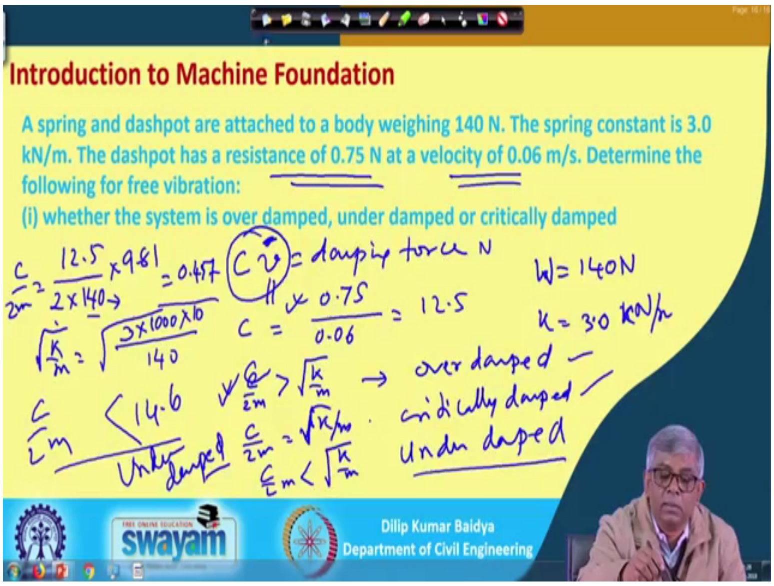

So, let me go to the next problem and you can see a spring and dashpot are attached to a body and body weighing 140 Newton and the spring constant is 3 kilo Newton per meter the dashboard has a resistance of 0.75 Newton at velocity of 0.06 meter per second, determine the following determine the following for free vibration ok.

So, free vibration actually you can see two things are given here, your W is given W is given your W equal to 1 wait, actually given 140 Newton and your spring k is given 3 kilo Newton per 3 kilo Newton per meter. So, now, you can see and I know that that damping force is given 0.74; that means, damping value is not given instead of that it is given damping force.

So, damping force actually C times velocity C times velocity equal to damping force. So, velocity is given 0.06. So, what is the value of C actually C will be equal to 0.75 Newton divided by 0.06 ok. So, that become this is actually it was in Newton and this is actually meter per second. So, this will be 0.75 by that 12.5 ok, C become 12.5 and I have to expect whether the system will over damped under damped or critically damped. So, you actually see that when it is C over 2 m greater than under root k over m then that is actually over damped, that already I have done. And C over 2 m equal to under root k over m then that is called critically damped and C over 2 m less than under root k by m that is under damped.

So, we can find out this value C over 2 m, C is 12.5 and 2 C over 2 is W is 140 multiplied by 9.81. So, this was W. So, I have I have actually C over 2 m ok. So, 2 W actually here C over 2 W. So, W I can divide by g then only we should g will here that value comes actually 0.437 and under root k by m under root k by m it comes actually it is under root 3 multiplied by 1000 multiplied by 10 divided by 140.

So, 3 kilo Newton per meter; so, 3000 Newton per meter multiplied by 10 because of this. So, if I do this we can say 3 multiplied by 3 kilo Newton per meter, I think that is wrong anyway; that means, 3000 multiplied by 10 divided by 140 and under root 3000 multiplied by 10 divided by 140 equal to 214 point that is actually 214.3 21.4.3 under root 214.3 that become 14.6.

So, that become 14.6. So, it is actually much bigger this is much bigger than this one you can see under; that means, this is under root k by m become much bigger this become C over 2 m. So, this is actually; that means, it is under damped. So, this system is under damped. So, this is a problem that is simple thing what you have done, we have tried to explain that what is the relationship between damping coefficient and damping force. Actually velocity multiplied by damping coefficient is the damping force, from there I have got the damping and then we know that different relationship C over 2 m greater than under root k by m then over damped C by 2 under root equal that critically damped and C over 2 m less than under root k m then under damped.

So, then I have computed C by 2 m and then under root k by m then we can see that this system is under damped; under damped. What is the meaning of it? Under damped means most of the system is under damned actually it will be that the oxidation will be there.

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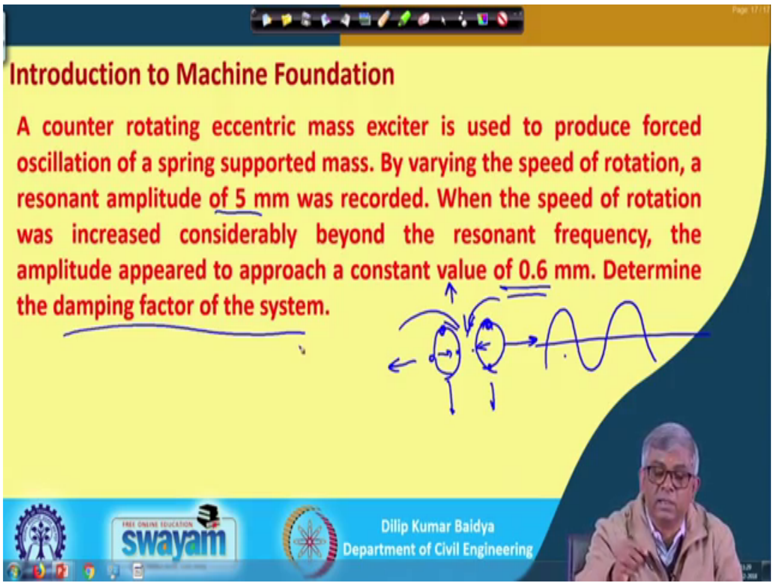

Next go to the sorry 3rd problem ok, this is the problem actually you can see a counter rotating eccentric mass excite are used to produce force oscillation of spring supported mass by varying the actually, counter rotating means actually this like this as I have explained before also.

If the mass is like that and it is rotating like that then because of that when the mass is here both are going downward when the mass is here then they are facing each other. So, no vibration when it is here it is going downward when the mass is here it is going here it gets cancelled so; that means, it is called producing and vibration like this. So, that it is nothing, but rotating mass system ok.

And amplitude was resonant amplitude was 5 millimeters, when the speed of rotation was increased considerably beyond the resonant frequency the amplitude approach to a appear to approach a constant value of 0.6 millimeter determine the damping factor of the system. So, this is an interesting whatever observation I have mentioned there actually I will show once again that one and then I explain.

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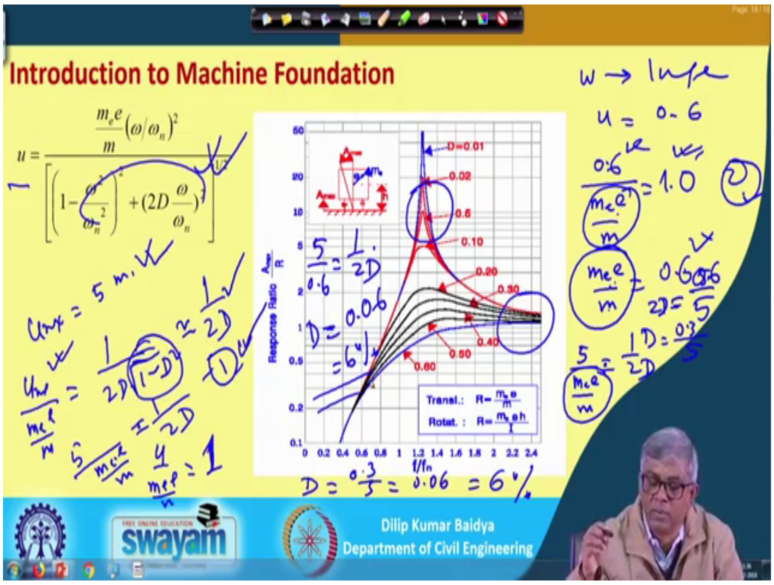

You can see here rotating mass system this is the expression ok. The expression is what? The amplitude here resonant amplitude dumping is unknown resonant amplitude u max, u max actually 5 millimeter is given and your u max actually 5 millimeter it is given and your and u max actually what for this type of u max actually by m e over m will be equal to under root or 2 d under root 1 minus D square. This is the actually your expression for maximum amplitude and these can be retained also when the damping is not verified this can also written as under root 2D ok

So, I have got these and; that means, that is 5 one equation is 5 divided by m e into u over m equal to equal to under root, 2D that is a one equation I am getting ok. And another expression another mention another thing is mentioned that when the frequencies increasing constantly large. So, towards here suppose and it is approaching to a constant value amplitude and that is equal to actually your 0.6.

Here actually amplitude towards when the amplitude omega is large then your amplitude u become 0.6 so; that means, I can say 0.6 divided by m e in to e over m and when the amplitude is very large actually, when amplitude is large I can say u over m e into m that is actually if the ratio become, if I take this thing this side and this magnificent factor becoming 1. So, I can write this as 1 so; that means, from this equation I can write this as 1 so; that means, this is equation number 2.

So, from here what I can get I suppose to know the machine detail that mean what is a eccentric mass, what is eccentricity, what is the marshal vibration nothing else known only two information is given from there I can solve this problem, how I will just show you here. That I if I consider this as a entire thing as one thing then I can see from this equation I can get m e in to e over m will be equal to 0.6 ; that means, this quantity now from this observation this quantity together I get 0.6.

Now, this 0.6 I will put here; that means, I have another equation I have got that is actually 5 over m e into e over m equal to 1 by 2 D, this is another equation I have got these if I put equal 0.6 then you have it is becoming 2D, will be equal to 2D equal to 5 by 0.6 2D equal to 0.6 divided by 5 or D equal to 0.3 divided by 5 and; that means, your D become 0.3 divided by 5, if I calculate that become 0.06 ; that means, 6 percent.

Is it clear this problem? This problem what I have done u max over m e I know that for resonant condition this is the expression and if the damping is low the under root 1 minus 1 square D square is also close to 1. So, I can simply write the equal to this so; that means, 5 divided by m e e by m equal to 2D this is equation 1 and another equation what actually when the amplitude frequency is very large that time your magnificent factor become 1.

So, here actually magnification factor is u over m e over m equal to 1; that means, another expression I have written u is 0.6 or large frequency and then m e over m equal to 1 that is another expression. So, from this expression I get m e e over m equal to 0.6. Now if I put this one in these; that means, here actually 5 by 0.6 is equal to one by 2D ok; that means, 5 by 0 point 0.6 equal to 1 over 2D. So, this equation if you solve then you will get D equal to 0.06 or equal to 6 percent.

So; that means, this I do not know any machine details what is the eccentric mass, what is the speed and what is the total vibration vibrating mass nothing else is known, but by rotating the, by increasing the frequency we have observed the resonance at that point; what is the amplitude is noted? And an increasing the frequency is significantly large and at that point when the amplitude becoming constant that point what is the amplitude is noted.

So, this actually one is 5 millimeter and another is 0.6 millimeter; and 5 millimeters is nothing, but u max and max expression is this from there I get equation this and u actually generally expression or magnification factor u over m e over m equal to this one and for large frequency this quantity become 1. So, I have put this one this is expression 2 and from here I get these equal to 0.6 this one if I substitute to this equate then I will get 5 by 0.6 is equal to 1 by 2D from that D equal to 0.06 actually that is nothing, but 6 percent; that means, damping in this system is 6 percent only. This is another problem; let me go to the next one.

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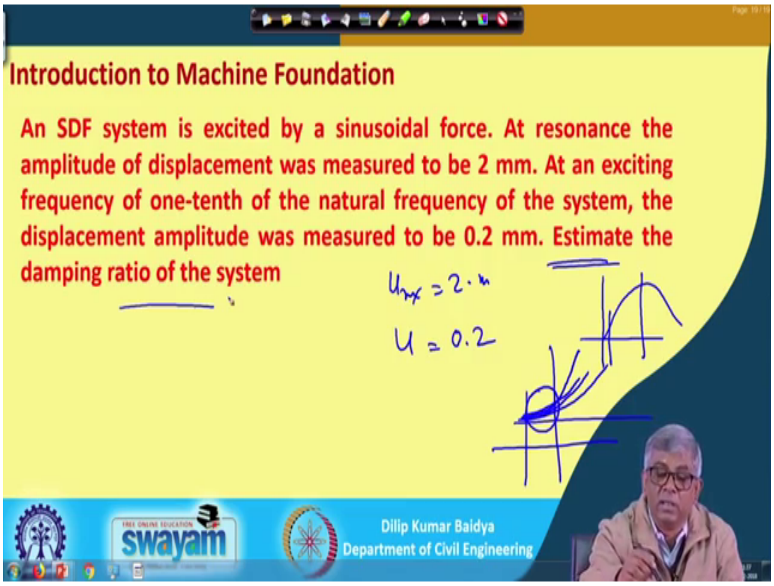

This is another problem actually similar and single degree of the system is excited by a sinusoidal force, and resonance at resonance the amplitude of displacement was measured to be 2 millimeter; that means, u max again u max equal to 2 millimeter at an exciting frequency of one-ten; that means, very small; that means, your response is something like that, resonance is here and at one-tenth frequency; that means, u at very smaller frequency reso it is 0.2.

And if you see that response I have shown before and I will show in the next slide that when the frequency is less your all response some response something like this or at at low frequency that amplitude is almost constant. So, this is the observation you can use by using this problem can be solved the displacement amplitude was 0.2. So, now, nothing else is there what is the magnitude of constant force, what is the vibrating mass nothing is given what is the stiffness, but estimate the damping the. Similar way the problem we have done previously here actually reverse observation let us see in the next slide.

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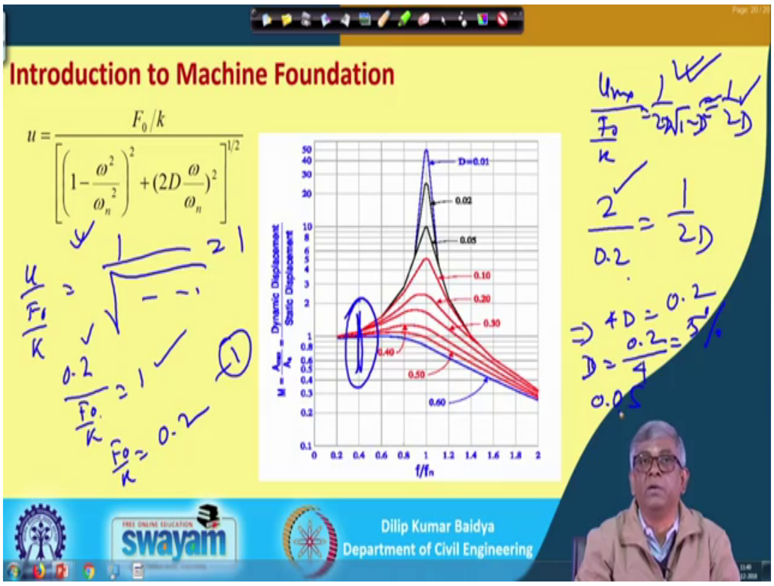

You can see here that when it is a constant force system this is the expression; that means, u over F naught by k equal to under root this quantity and you can see that, when the frequency is very low these become 1 ok. That is observation and u max actually and another is u max over F naught by k equal to 1 over 2D under root 1 minus D square and nothing, but 1 by 2D, this is one observation this is another observation using these two observation. If I use this observation what I will get u is actually 0.2 equal to F naught over k equal to 1 or F naught over k equal to 0.2.

And from here actually what I am getting this is these amplitude was 2 millimeter this was 2 divided by F naught over k is 0.2 equal to 1 by 2D. And from here I can get, I can get 4 D equal to 0.2 or D equal or D equal to 0.2 divided by 4. So, equal to 0.05 or you can say it is 5 percent ok.

So, this is one of the; that means, at frequency ratio, very less frequency ratio when the frequency is very less than this entire quantity become 1, magnificent factor this magnificent factor become 1. So, this is u and this is F naught by. So, this is 1, from here I get one F naught k equal to 0.2 and u max by F naught k equal to 1 by 2D under root 1 minus 2D that is also equal to 1 by 2D. So, this is 2 and this I am getting 0.2 and this is another 1 by 2D solving these I can get damping equal to 5 percent ok.

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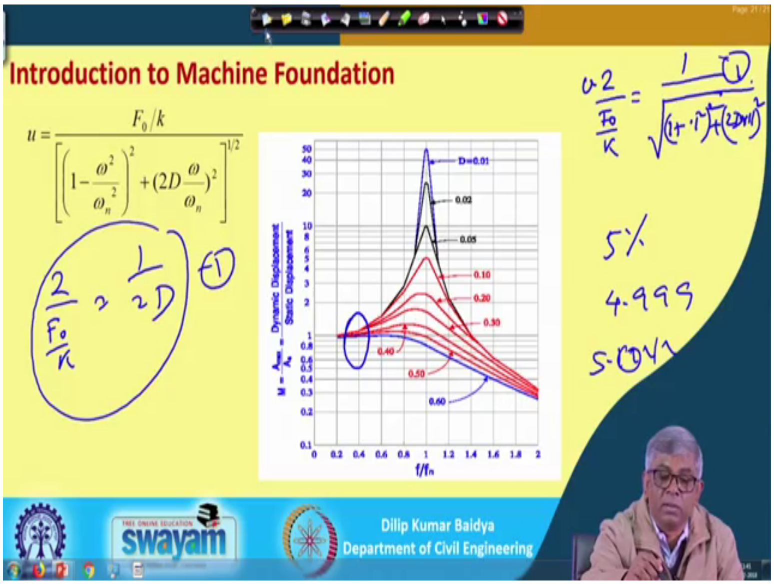

So, otherwise sorry some people can also try to solve this problem by how they can solve it actually one is actually you are getting 2 divided by F naught over k because divided by 1 over under root 1 plus omega over omega and actually one-ten. So, 0.1 square whole square plus 2D a multiplied by 0.1 whole square, one can said this is one equation and another equation 1 can do no this is 0.2.

Another equation is 2 over F naught over k equal to under root 2D. So, solving this two equation one can also find out, but if I use that at lesser frequency the frequency ratio amplitude actually is a magnificent factor this quantity become 1 that is ; that means, if I to if I use this as entire thing as 1 then it is easy to solve quickly and. In fact, if I use this expression or this expression solve finally, instead of 5 percent we may get a 4.999 percent or something or 5.111 percent or 5.01 percent or something. So, very close value we will get.

So, better to use this observation; that means, at very less frequency ratio your magnificent factor is 1 and resonance frequency magnificent factor is by this equation and then equating these two, one can get the solution value for damping I have one more problem, but I will can take this problem later on. So, I will stop here today.

Thank you.