Geotechnical Engineering II / Foundation Engineering

Prof. Dilip Kumar Baidya
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture - 55
Introduction to machine foundation (Contd.)

Good morning, we are now in Introduction to machine foundation almost towards the end of our Foundation Engineering course and I have taken two three lectures. Now, I have to I have planned two three more and today, I will just discuss what we have stopped in the previous class from there I will start.

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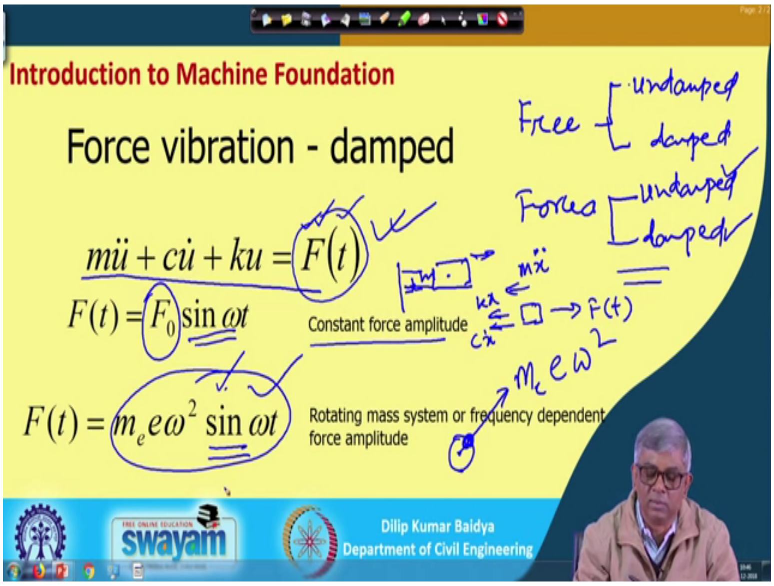

And in this you can see that, we have discussed 4 different types of vibration analysis that is free vibration and force vibration under that it is undamped and this is damped and here also similarly undamped and damped ok.

So, we have done the all the analysis free vibration, damped, undamped we have done force vibration undamped we have done now we are doing damped. And already I have discussed in the previous class also that if you have a forced damped vibration; that means, you have a something system like this and you have a spring and you have a dashboard something like this and this is a mass. And then if I write down the equation, if I you consider a dynamic equilibrium of this mass then there will be given displacement distance.

So, mass multiplied by acceleration will be in this direction, spring force k times x will be this direction, damping force c times x dash x dot that also will be the direction and force is acting here suppose F t. And then if I draw the free body and consider the equilibrium in horizontal direction then I will get this equation. That is the equation of motion for this system.

Now, solution of these will be containing two parts; one is a complimentary part another is particular solution. Complimentary part how to get it, actually you have to put this equal to the force forcing function equal to 0 and then undamped sorry damped free vibration the way you have done the solution, same solution will be there. Additionally, when force equal to this then I will be do the particular solution and these two together will be the total solution. But as I have mentioned that since the because of the damping present in the system that first part will be diminishing with after some time and only it will give you a steady state vibration because of this particular solution.

And that solution I will come to the next slide and F t can be of 2 types as actually with a constant magnitude with the time variation and they have not sin omega t, another will be frequency dependent. This is a constant force amplitude and rotating mass system, frequency dependent means what? Actually this is the suppose ascetic march mass rotating with respect to some point, then in that case because of this frequency that force will be m multiplied e multiplied by omega square, this dynamic force is this much, so that also will be varying with time sin omega t.

So, forcing function can be of these or it can be of these type, so depending upon these 2 types of forcing function the solution also will be a little different. But they are basically same, but once I will substitute these by substitute these one by this one that way actually small change will be there; so, let me go to the next slide.

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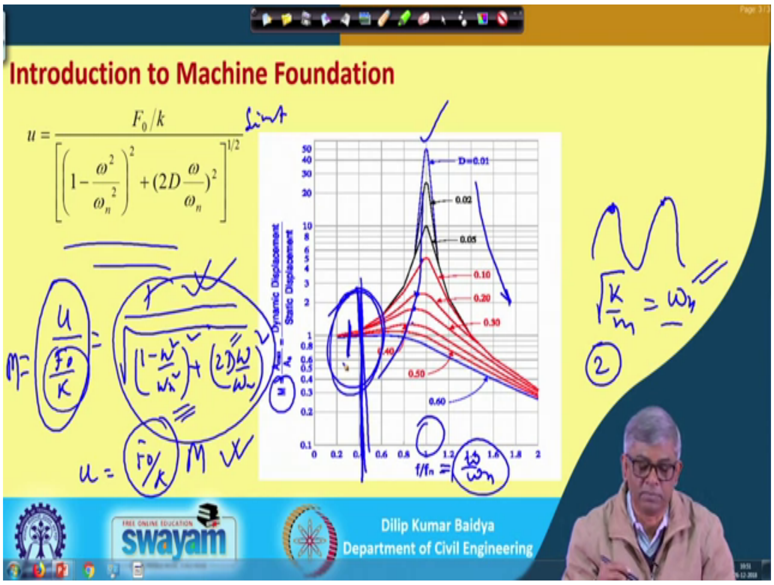

So, that is a solution actually general solution you can see that u actually displacement amplitude of course, sin omega t can be there here, but this is the maximum temperature at a particular frequency this is the maximum amplitude. So, displacement maximum; that means, if you have this then this is the one by these expression it is giving that one ok.

Now, in this actually I can write this one as u over F naught by k equal to under root 1 minus omega square omega n square whole square plus 2 D omega over omega n whole square. I can write this form and this is actually non dimensional I can say something M and this is the shown here M; M actually with the variation of omega over omega n or omega over this is nothing, but omega over omega n also.

So, this variation is that when M variation of M with respect to the frequency ratio and if you observe these that at a lower frequency you can see at a lower frequency with the variation of the damping you can see amplitude near resonant. This is a resonant where actually ratio is 1. The resonant is resonant amplitude is very high and with the increase of the damping the resonant is the resonant amplitude reducing that is one thing.

And at lower frequency you can see whatever, may be the damping the amplitude is almost constant close to these values. So, this is one observation for this is a constant force system when observed by k. So, this from this plot we can see that with the increase of damping, that is the damping increasing this direction with the increase of damping amplitude is decreasing or we can say with a decrease of damping amplitude will increase.

And you can see that, if it is a undamped system the natural frequency will be under root k by m; that means, omega n and omega over omega n equal to 1 is supposed to be peak. But you can see the peak actually, slightly before the that one it is happening you can see, for these resonant we have got this, resonant we have got this resonant may be here. So, resonant is shifting left side with the increase of damping the resonant actually shifting towards left and it is happening before natural frequency ok.

The resonant is occurring at a frequency before natural frequency that is one observation. That is actually not exactly at omega over omega n equal to 1 resonant will occur; resonant will occur slightly before and at lower frequency ratio that your amplitude will be almost m whatever maybe the damping. So, these two important observation can be made from this diagram.

And suppose I want to find out the amplitude at some frequency ratio omega n is known and I know I suppose are my frequency is double the omega n; so, omega over omega n suppose 2. Then I want to find out that then first I will find out this quantity, I will put omega over omega n equal to 2 at damping some value suppose point 1 then I will get this value. These value multiplied by F naught by k will be your amplitude will be u will be equal to F naught over k multiplied by magnification factor.

Magnificent factor is what? This value of this, you need not actually since it's a simple equation you need not use the graph, but one can if I know the ratio of omega over omega n, then I can use this equation. And find out the magnification factor and then magnification factor multiplied by static that static displacement means what? Whatever constant force is there is a dynamic that constant force if you statically applied then what is a static displacement that is nothing, but F naught over k. And that F naught over k actually multiplied by magnificent factor that become the amplitude for the dynamic condition, so, this is the way one can calculate.

Similarly, for any damping value or any frequency ratio one can find out the amplitude using this equation, but what observation you can do suppose I want to know at a very low frequency then actually this I can assume almost close to static displacement itself.

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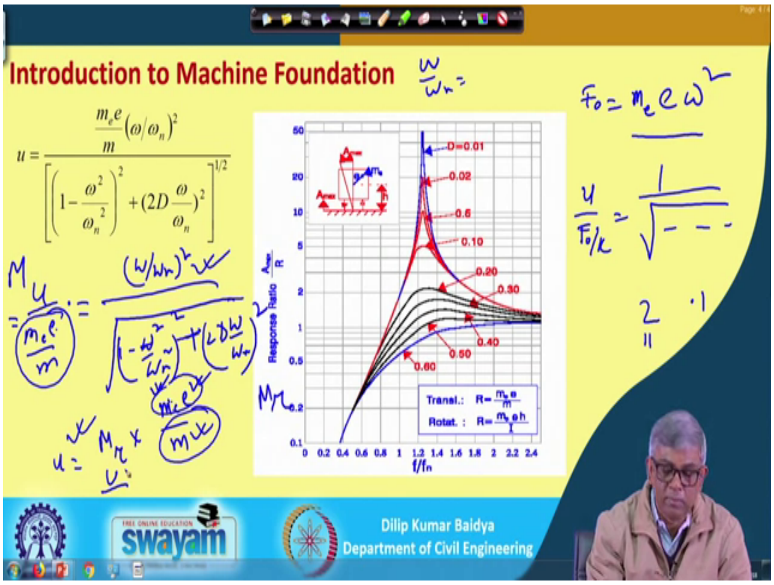

So, let me go to the next slide, when I will put, when I will substitute F naught equal to m into e into omega square, then our previous equation was what was actually u over F naught by k equal to 1 over under root the same this expression.

But here actually in this case it is becoming u over m e into e over m equal to omega over omega n whole square under root the same quantity. Whatever, this 1 minus omega square over omega n square whole square plus 2 D omega over omega n whole square so, this is in show.

The magnificent factor for constant force system and rotating mass system little different you can see this is the thing added. Simply how it came simply I have substituted this F naught equal to these and then I have simplified than that the expression become like this and here instead of m dot by a the F naught over k, this become another quantity; that means, eccentric mass into eccentricity divided by m.

So, this actually so; that means, if I know the frequency ratio suppose 2 and damping suppose point 1, then what I will if I want to find out the amplitude then what I will do? I will put this one omega over ratio is 2 and damping a point 1 in this equation. I will get this is actually I can say M here; this is called M or M r rotating mass system.

So, I will get M r if I get these value then your amplitude will be equal to M r multiplied by m e by m; that means, in the rotating mass system what is the eccentric mass that to has to know, at what eccentricity it is rotating that has to be known and what is the mass of total vibrating mass that has to shown these and these what is the difference? Total vibrating mass means entire foundation basing together.

And this m e means eccentricity mass; eccentric mass means what actually in the machine itself there may be a part rotating part which is rotating, what is the amount? That is actually m e. So, m e into e this two has to be known and then total vibrating mass; that means, putting together that has to be known; if I know these and by calculating M r I can find out displacement.

Another thing is that like that you have got the general equation, but if I simplify; that means, I can maximize this equation; this equation I what value what frequency this is the with the variation of frequency, this is the expression. But if I want to find out at what frequency the amplitude will become maximum you will find that omega over omega n equal to some ratio some frequency ratio and that is because if you substitute then you will see that u max will happen at your u max will be or you can say u over let me clean and then once again.

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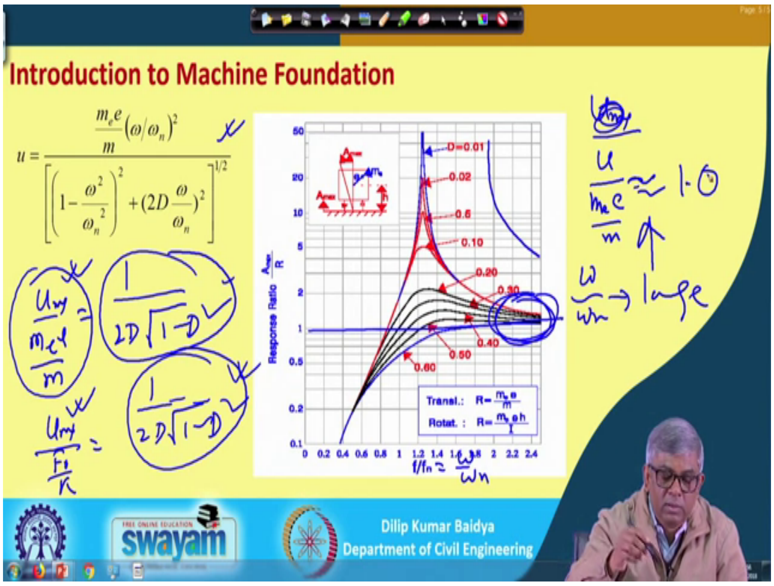

If I do maximize then we will see that u over m e over e or u max will be equal to it will be simply 1 over 2 D under root 1 minus D square. So, for rotating mass system this is the expression similarly, for constant force system u max divided by F naught over k will be equal to 1 over 2 D under root 1 minus D square; that means, at resonant this is u max means displacement amplitude maximum means it is at the resonant that resonant point what is the value depending only damping.

So, you can see both for rotating mass system and constant force system that your resonant is amplitude depends only on the damping value. And how we have done this one that procedure I am not going actually and but what frequency ratio this is becoming I will show in the next slide, both for this and this. And so, from this figure actually again you have to observe a few things you can see here, that again with the increase of frequency ratio you can see this is nothing, but omega over omega n also with the increase of frequency ratio amplitude is increasing and it is reaching to a maximum value which is called a resonant and again it is decreasing.

And again with the increase of damping from this direction you can see amplitude is decreasing and third observation is what? Actually you can see at a very large frequency ratio suppose 2 or 3 (Refer Time: 14:27) or even more we can see that magnificent factor approaching to a value constant value. This magnification factor this one; that means, this is one u max by or not U max u over m e into e over m, that is approaching to a constant value equal to 1; that means, at very high frequency ratio.

Previously for constant force system whatever we have observed at a low frequency ratio; that means, 1 10 or 1 20th of the natural frequency. Then we have seen that initial portion of the response curve irrespective of the damping the value almost constant that is nothing, but equal to m m naught over k. And whereas, the rotating mass system you can see at large frequency ratio at large frequency ratio where frequency ratios 2 or 3 or even more.

There actually that response or magnification factor they are approaching to a constant value which is equal to you can see this is equal to 1. So, I can write that at frequency ratio when approaching to large, then your this amplitude these value equal to 1 that is another observation. So, this two observation from 2 figure can be noted.

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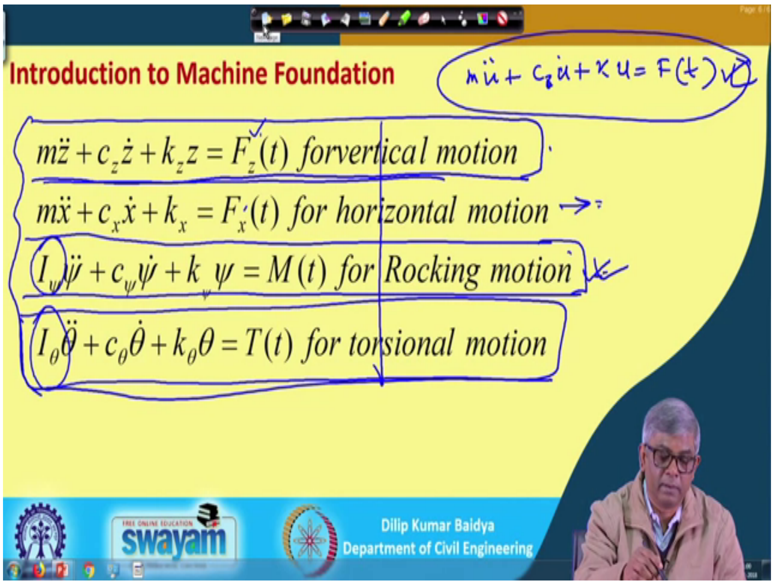

So, with this let me go to the next slide which actually important and you have to remember see so, you can see here, as I have told you that I have written equation for this is the equation we have done actually m u double dot plus c z c into u dot plus k multiplied by u equal to F t I have written like this; this is a general equation of motion.

Now, as I have told before that in the foundation there are 6 degrees of freedom will be there actually 2 rocking, 2 horizontal, 1 vertical and 1 your root torsion. So, this is a vertical equation so, when it will be vertical motion I can write the equation of motion like this, if it is a horizontal motion I can write down the equation of motion like this, if it is a rocking motion I can write the equation of motion like this.

If it is a torsional motion you can write down the equation of motion like this and I have shown before also at this one of course, we have 2 equations can be there for rocking with this direction or with this direction shorter direction and longer direction; so, 2 equation, 2 degree of freedom here.

Similarly, horizontal motion can be in this direction or can be in this direction so, because of that this 2 2 4 5 and 6. So, the respective motion if it is a rocking, if it is a torsion or if it is a horizontal vertical corresponding to that the equation of motion. Now, we can solve all the equation similar way only thing here actually these two cases mass was there.

But here actually second moment of area for rocking motion and mass moment area mass moment actually a moment of inertia for the torsional motion will be there. So, those things I will discuss later on what are the parameters, but if you learn only vertical motion that may be enough at this stage, but if you have a since the vibration can happen in all 3 or 4 directions or 5 6 direction. So, because of that I have just given the equation of motion solution procedure will be exactly the same only you have to replace some parameter nothing else ok.

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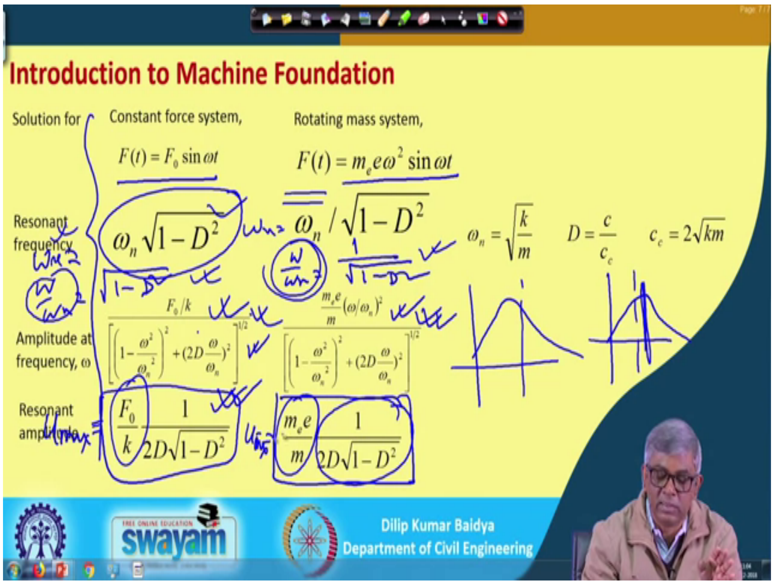

So, now, let me go to and I have told you that 2 solution I have shown for constant force system and rotating mass system. And you can see that the solution important thing you have to remember that when the force is F naught sin omega t and when force equal to m e e omega square sin omega t. In that case resonant frequency will be over will have omega n over resonant will occur at omega will be equal to these or omega over omega n will be equal to 1 under root 1 minus D square.

And whereas, in this case omega over omega n will be 1 by under root 1 minus D square. So, that means, the resonant occur as I have told you that this is actually small smaller than 1 is it not; that means, at operating frequency less than natural frequency will resonant will occur for constant force system, but this one actually greater than 1. So, omega rotating mass frequency will be greater than natural frequency actually ok.

So; that means, we have shown constant force system that your if this is omega over omega n then your natural frequency are something like this ok. And whereas, constant for rotating mass system if this is the one then your natural frequency was something like this. So, resonant was here so; that means, for constant force system your resonant occur before the natural frequency and whereas, for rotating mass system your resonant occur after the natural frequency when you cross the after will cross the natural frequency there is. So, the ratio is 1 over under root 1 minus D square 1 by under root 1 minus D square and here in constant force system it is omega over equal to under root 1 minus D square this is the difference you have to remember.

These natural frequency ratio these frequency ratio to substitute the general equation in these, then you will get at resonant this is the expression you get what I have shown already. So, this is the general solution for constant force system this is for rotating mass system general solution. Now, these frequency ratio if I substitute in these you supposed to get the maximum amplitude and that expression become like this for constant force system and it become these for the rotating mass system; that means, I have already shown here this is actually nothing, but u; u max this is nothing, but u max, this is also u max ok.

So, that means, if I want to find out resonant amplitude directly if I know the damping I will calculate this one and then multiplied by F naught by k I will get resonant amplitude. Similarly, for rotating mass system if I know the damping then I will estimate this one then I will multiply by this one, then I will get the resonant amplitude. So, these table actually one has to remember; that means, for constant force system what is the frequency ratio? Where resonant have occurred or what is the resonant frequency? That means, this omega is this is the omega r will be equal to omega n times 1 minus D square and that means, it will be less than this entire thing will be less than this will be less than omega n.

Similarly, here omega n equal to omega will be equal to omega n divided by 1 minus under root 1 minus D square; that means, it will be greater than omega n. So, these two things to be remembered general solution these and these to be remembered and once if you do substitute this equation and then your amplitude maximum or resonant amplitude will get from this expression these to be remembered.

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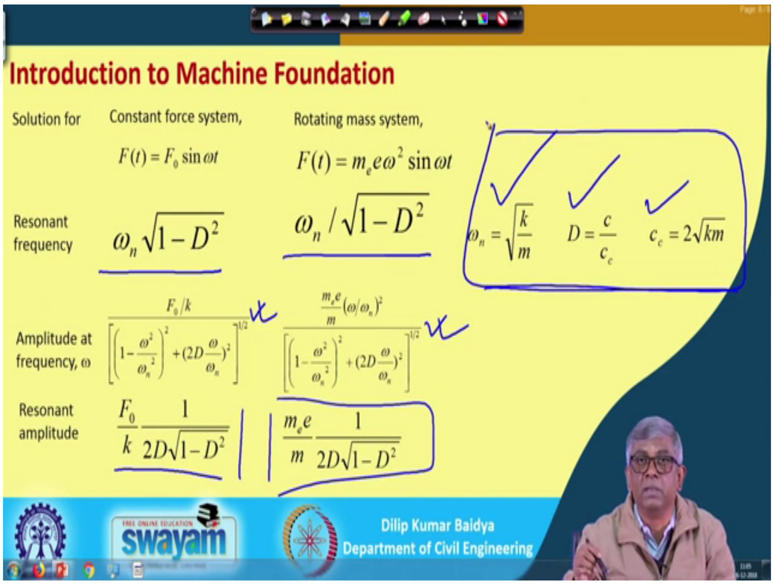

So, this is kept in the tabular form you can see if I clean here, you can see this is the one important thing, this is one important thing, this is already repeatedly I have discussed this is anyway you have to remember. And then finally, this two are resonant amplitude and then what are the other things? Omega n is this and damping is equal to c by c c and c c equal to 2 root km. This almost parameters are also to be remembered this is very very essential for dynamic analysis.

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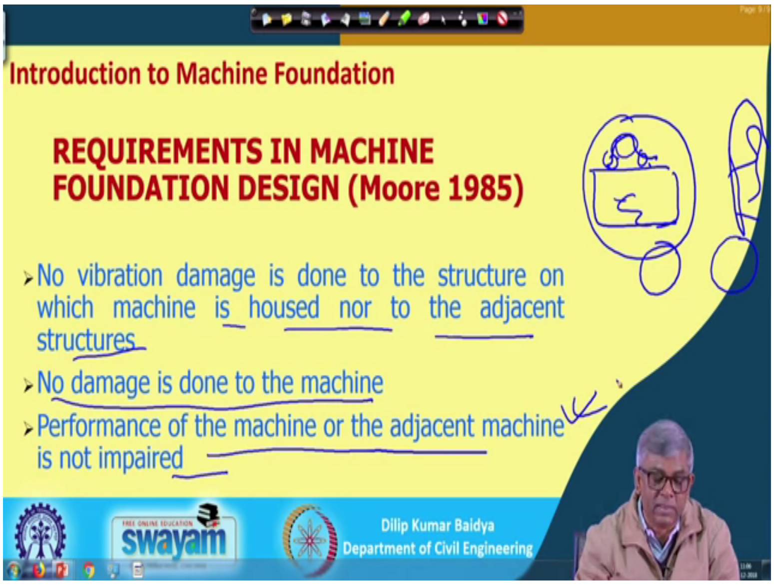

Now, let me quickly go to the requirement for foundation design for vibrating load. So, no vibration is done to the structure on which machine is housed; that means, if I put a machine over this foundation this foundation should not be the damaged or to the adjacent structure also. So, because of the vibration of that the adjacent structure should not also be damaged.

No damage is done to the machine itself, so, if I have if you allow excessive amplitude vibration then because of the shaking of vibration of the machine parts may go wrong. So, that also has to be implemented performance of the mission of the adjacent machine is not impaired. So, if you have excessive machine then sometimes it may get shut down or it may get some maintenance requirement all those things happened because of the high amplitude vibration here nearby machine operation also may be affected to that also has to be prevented.

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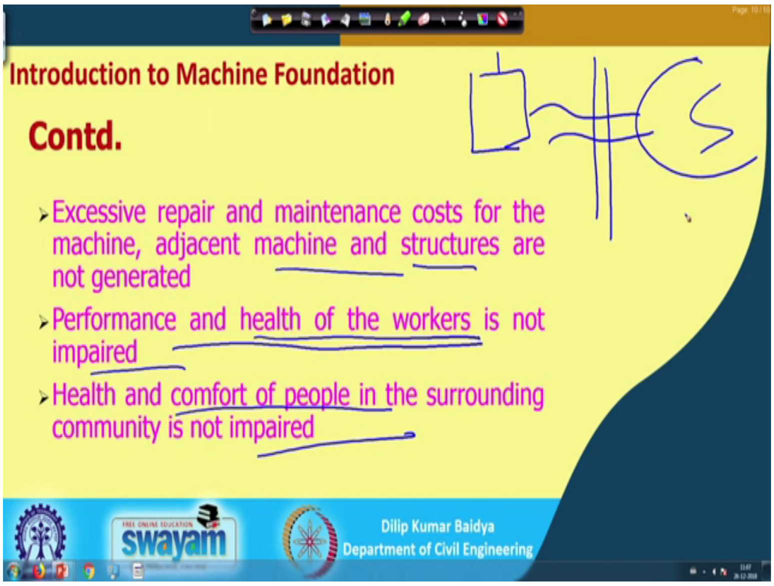

Next one I can see that excessive repair and maintenance. So, if I do not designed properly then you may require excessive maintenance or repair cost that has to be prevented. Performance and health of the worker is not impaired; that means, what if you allow too much of vibration in the system then during vibration, what will happen that people working in the around surrounding places. They may not be that environment, may not be comfortable or it may be a health hazard may occur.

So, because of that you have to follow all those things requirement; that means, what amplitude one person can tolerate you have to keep the amplitude in that level so, that is another requirement. So, performance and health of the worker should not be impaired and health and comfort of surroundings people suppose there is a huge forcing hammer and that creates lot of vibration it goes to surrounding places. So, that also has to be done properly so, that the health and hazard health and comfort of the people in the surrounding area also should not be impaired.

So, these are the actually qualitative that what you have to do while designing? That you have to prevent all those things and how we can prevent? If you keep the amplitude within that acceptable limit then only can all can be prevented.

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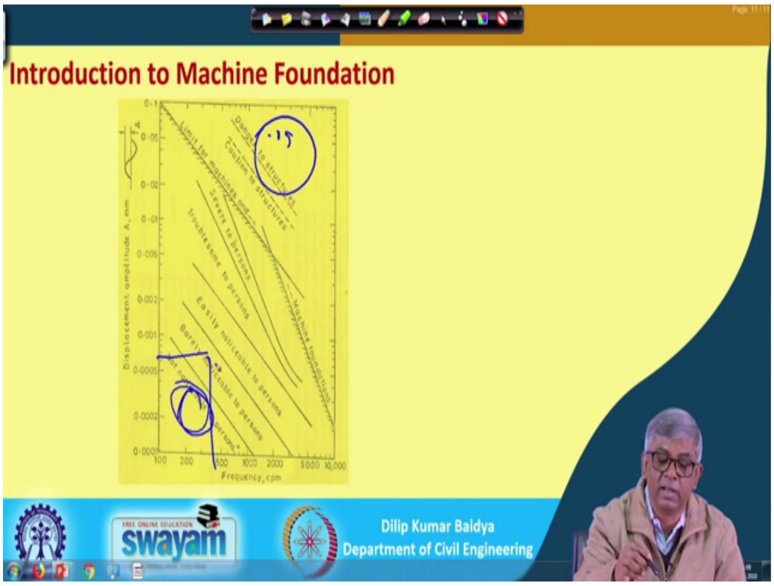

So, and what is the general guideline you can see here in this figure actually if the amplitude is frequency is very high we will be able to tolerate very less amplitude. Whereas, in the low frequency you can tolerate little higher frequency because of that variation of frequency you can see frequency a displacement amplitude there are different range of tolerance you have mentioned.

See these that if I keep the frequency and amplitude in such a way that not noticeable to person; that means, in this actually very safe if I can design within this zone that is very safe then you can see barely noticeable. That means, your frequency and amplitude is in such way kept that sometimes it will be a noticeable that is also safe and easily noticable to person. Now if the frequency in amplitude is such that within these zone then easily noticeable that the people may feel. So, because of that you have to design and there are different ranges are given and you can see beyond this limit if the amplitude become more than 0.5 and frequency become more than 200, then that cannot be acceptable actually that is not acceptable.

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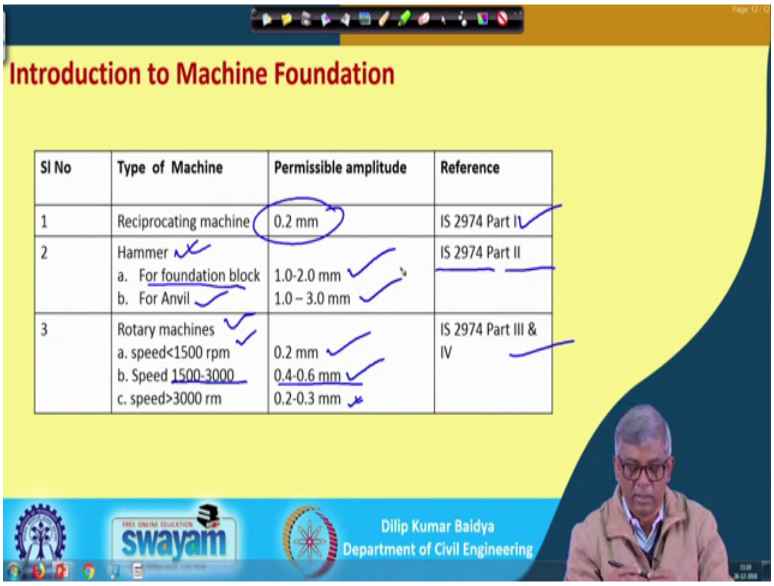

So, this is the limit for where, actually amplitude or frequency can be. So, this is one guideline given by region. And next and you can see that it is given by the code actually for different types of machine and the frequency range and that different amplitude is given the code. Actually, IS 2974 part I is given reciprocating machine permissible amplitude 0.2 millimeter and IS 2974 part II is given for hammer foundation and for foundation block what should be the limiting amplitude?

For anvil what is the limiting frequency? Similarly IS 2974 part III and IV the rotary machine is going because speed less than 1500 rpm. Then 0.2 mm if it is between 15to 3 1500 to 3000, then your this will be 0.4 to 0.6 millimeter and if it is greater than 3000, then 0.2 to 0.3 millimeter amplitude can be partial.

So, these are all codal guidelines so, you have to while designing you have to keep the amplitude within this and if you keep this within this limit most of the problem whatever I have said will be avoided in general.

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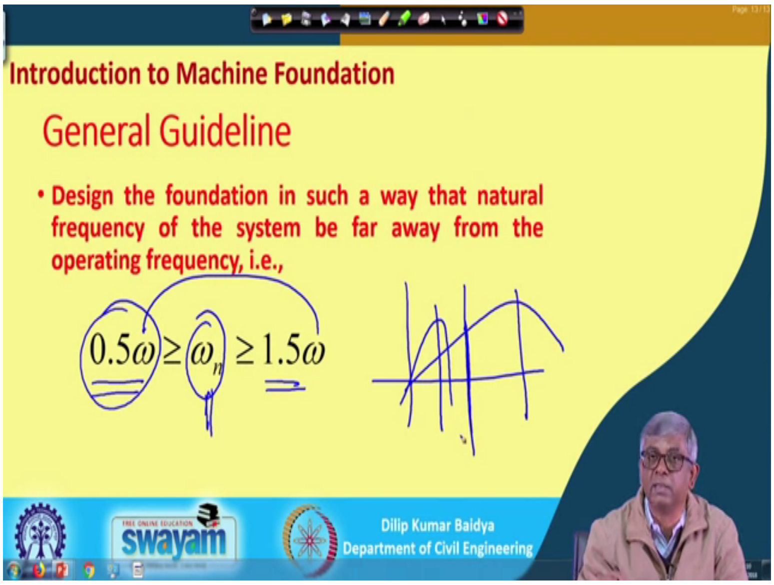

And another typical guideline is that you have to keep omega n actually omega should be like this natural frequency if this is the natural frequency you have to keep 0.5 omega and this is the range actually you have to fix, operating frequencies these to these and then omega n should be so; that means, you have like this. Either, if this is the one either you design like this or you can design like this so, either natural frequency will be here or here if the overall omega is here.

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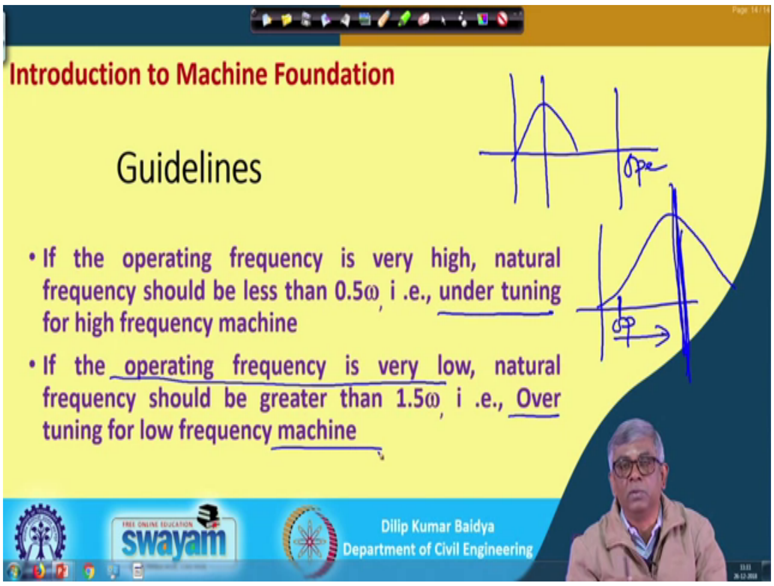

And actually you can see that another two guidelines if the operating frequency is very high the natural frequency. So, operating frequency is very high means somewhere here then you have to design the system something like that. So, natural frequency, so, that is that means, it is under tuning; that means, you have to design foundation in such way natural frequency falls much below the operating frequency. And if the operating frequency is very low suppose and then suppose, if the operating frequency is somewhere here then you have to design the system in such way like this.

That means, natural frequency here that is called over tune; that means, you are designed the foundation natural frequency in such way that it is far away from the operating this is operating; this is operating here; this is operating and this is a natural frequency; so, that is called overtune. So, what is overtune? What is the undertune under? What case you will do undertune? What case you will do overtune? That is actually to these two points it is mentioned ok. So, with this perhaps I will stop here.

Thank you.