Geotechnical Engineering II / Foundation Engineering
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Lecture – 54
Introduction to machine foundation (Contd.)

Let me continue the in the what I have done in the next last lecture, I have mentioned that that vibration solution can be of four different types; one is undamped free vibration, damped free vibration, then undamped force vibration and damped force vibration. So, I have discussed very briefly about undamped and damped free vibration and their solution. How they looks like; the amplitude how varies with time that I have shown. Now, I will try to take that forced vibration and that too undamped force vibration and damped force vibration and we start with the damped undamped force vibration.

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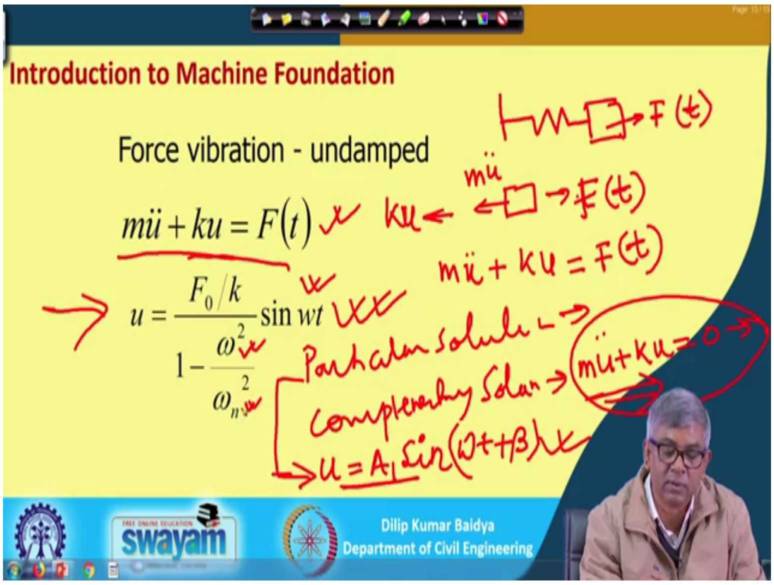

Let me take the first slide and you can see here undamped force vibration. So, undamped force vibration means what? If I consider here and then this is the mass and here there is a force is given F t ok. So, if I draw the free body diagram of these, then we will get what this the mass on that mass there will be m and u double dot will be there and because of the spring, you have if you pull this direction, then there will be k times x k times u is a spring force and this side actually F t.

So, this is the free body diagram. So, if I consider the dynamic equilibrium for this in the horizontal direction, then I am getting m u double dot plus k times u equal to F t ok. So, this is the equation we have got and this is a equation actually this equation solution of this equation will contain two part actually. Actually one will be particular solution one is particular solution; another is complementary solution. Complementary solution is what? It comes from m u dot double dot plus k u equal to 0. If I put this one from there actually we will get a solution dot cell solution already we have done.

So, that will be the one part plus I will have a particular solution. So, particular solution of these actually I just better remove these and then I will do particular solution for this actually for that we can assume u equal to A 1 sin omega t plus beta like that I can assume a initial solution and then finally, we can apply your different boundary condition and then, if I put finally, this through particular solution assuming this and by applying boundary condition that result actually this is the solution.

So, this solution plus solution based on mu double dot plus t equal to 0 together will be the solution of this and it can be shown that in the real system, since there is some amount of damping present. So, this part actually will be absent. So, only response whatever we will get finally, because of this after some time.

So, because of that will try to understand this part actually. So, you can see here with the change of frequency, this is the operating frequency suppose and this is natural frequency, omega over omega n square. With this change of this actually this become smaller and after sometime it will become negative also. So, if you plot the response, then we will see the response. Next page I will show you, the response looks like this ok.

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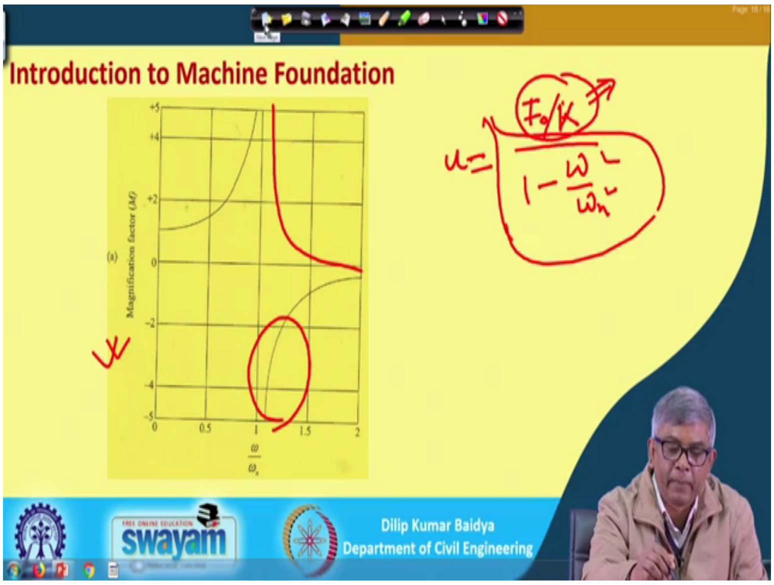

So, the value the magnification factor actually the magnification factor actually what whatever we have got the solution u equal to u equal to F naught over K divided by 1 minus omega over omega n square. So, this part this part is called magnification factor. So that means, with omega over omega n what is the value of magnify; how magnification factor is varying? So, magnification fact this is actually constant force constant force, but divided by stiffness. That means, that is nothing but static displacement whatever initial static displacement multiplied by the magnification factor actually the amplitude at any frequency.

So, to find out that we can see this is the plot and of course, this is negative, but we can take the absolute value. So, if I consider it will be plot will have something like that ok. So, this is the forced undamped vibration that is the solution.

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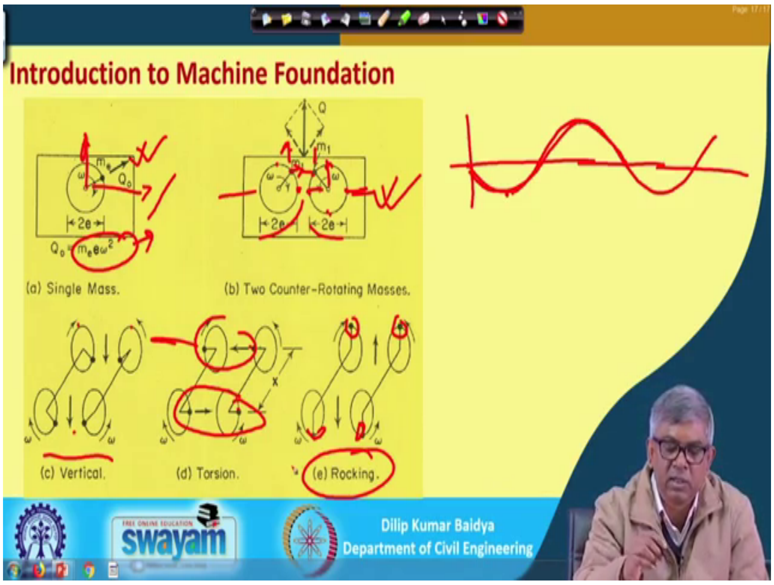

And now, I will discuss the point as I have already discussed before that if there is a eccentric mass am rotating at a speed of omega and then, the force here actually is nothing it is equal to m the eccentric mass if it is m e and this eccentric distance is a e; that means, this diameter of this circular circle is 2 e that means, eccentricity that means, it is rotating with the radius e. So, m eccentric mass m e multiplied by e that means, eccentricity multiplied by the frequency square that becomes the force and that force actually acts radially upward outward so, that mean this direction.

It can have actually two component; horizontal, vertical because of that if that is there then at same time we may get both vertical and horizontal motion. But if I arrange them in a particular way that mean if I use two or rotating part in a particular manner, I can see here and then this force is direction is this. So, I can have two component; one is these direction and other is these direction. Similarly, sorry not here this direction and this direction. Similarly, here actually I can have this direction and this direction and if mass and eccentricity and frequency are same; both mass, then this horizontal face to face will get cancelled and this one actually will be added.

So, like that. So, if I plot now, it will be here suppose; when both are here when both are here actually it will become 0 and then slowly when they are moving these direction, then this amplitude will be increased some value here and then again when it will come here this will be outward and so, it will become 0. So, like that again it will when it will come there it will be increased to a value maximum value and then when it will come again. So, it will become again 0. So, like that with the rotation actually your dynamic force will vary like this ok.

So, this is that means, and that means, every time you can see wherever you go when it is at this point both the horizontal part get cancelled and when it comes actually both the vertical part added right. Here both the vertical part added, when it is here both the horizontal part get cancelled. So that means, no where there is a horizontal component.

So, everywhere will feel the vertical dynamic force and that is the variation actually. Similarly, if I use in combination in two shaft actually that you can see here this is giving vertical. So, both are going vertically and now, here actually we can see this mass and this mass when it is coming in this direction and when this mass and this going in this direction and this is the way that means, it is getting a vibration like this. When these two are this direction, these two are in this direction.

So, it is once it is this way motion and next it is motion like this. So, like that it will be repeated so that means the same mass if I use in a 2 shaft in this manner. Then, I will probably can I will be able to produce actually torsional vibration. Similarly, if I use the mass in another configuration that means, when these two mass is going upward these two mass going downward that means, it is rotating like this. So, once it is when this side is upward this side downward, again when this side upward this side is downward. So, like that its motion is taking place. So, that is called rocking actually.

So that means, when that is a mass it is rotating like this that is rocking that means, when the axis passing through the base it is rotating that is actually rocking and when this mass actually rotating with respect to axis vertical axis that is actually torsion and then, another is actually when it is vibrating like this vertical another is when it is vibrating like this horizontal. So that means, in the real system any that means, we can have four modes of vibration actually; one is vertical, horizontal, rocking and torsion. So, that is the thing by combining the rotating mass in different ways how all types of vibration can be created it is simulated here actually.

So, this is actually single mass rotating. Then it is like that at simultaneously will feel horizontal or vertical motion and if I put together, then we will see that only vertical and then if I combine them in a two shaft both at downward or both are upward. Then, it is pure vertical. When both are in this side, one direction; then this side is opposite direction that means, it is with respect to a vertical axis rotating like this am sorry rotating like this. Then, that is torsion and when this two masses are upward, these two mass are downward that means, with respect to a horizontal axis it is going up and down like this. So, that is actually rocking motion. So that means, all three types of four types of motion are shown here.

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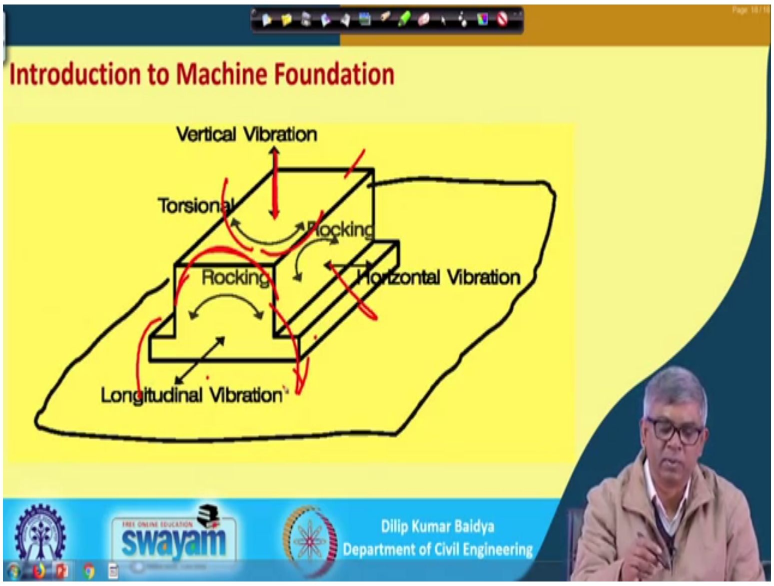

Similarly, if you think of a block like this; this is the block suppose as having some base and has some. So, this is actually you can see now whatever I have shown with respect to this block, I can show you that different modes. You can see here that when the force is acting centrally to this block and repeating like this, then it will be pure vertical motion. Similarly if some force acting on the surface and through this; so, then it will be horizontal motion.

If it acts through the cg of the mass actually; if it is acting on the surface of the block; then, what will happen? At the cg distance from the cg to the surface whatever distance is there a horizontal force multiplied by that distance will create a movement and thus, at the cg.

So, in that case if you apply horizontal force on the surface, then it will not be horizontal motion. When the force is acting through the cg of this block, then only there will be a horizontal motion. So, that is one that vertical is there; then, horizontal if it is through the cg and then with respect to this vertical axis if there is a force like this acting, then there will be rotation.

That means, this block once it is rotating this direction, next time it is rotating these directions and it continues then that is actually torsional mode of block actually rotating like this and then actually see rocking motion that means, there will be horizontal axis here and with respect to these that the block rotates once it is going downwards this side; next time it is going downward this side, then.

So, and it repeats it continues that block moves like this. Then, that is actually rocking motion. Similarly rocking can be of two directions with respect to rotation with shorter axis and rotation with respect to longer axis. So, that way the two types of rocking it is; one is rocking, another is pitching. So, this is called pitching and this is called rocking. So, of course, principally they are same rotation with respect to axis on the base that means, it is like this and when this torsion it is like this that means, with respect to vertical axis it will be rotating like this and vertical means it is pure vertical horizontal means.

So, and the block itself we can have all modes of motion because of the circular rotating mass in the machine, the way it is there if it produce force in such a way that, it is going once this direction, once these direction with respect to vertical, then it will rotation and if it produce vertical force once this side, once this side. Then, it will get rocking motion. So, like that we can also visualize for any block all modes of vibration whatever I have shown previous one.

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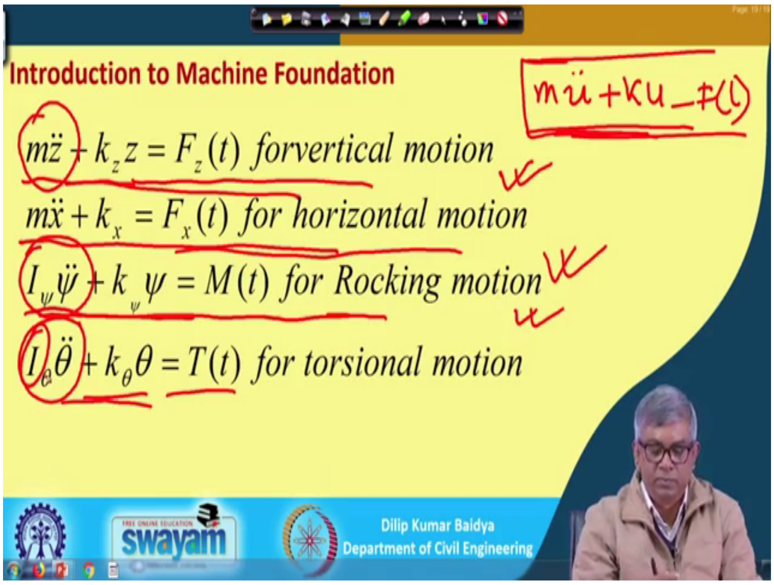

So, because of that whatever I have shown that, that equation of motion for free vibration. So, not free force vibration undamped force vibration I have shown m u double dot plus k times u equal to F t. I have written; this is the equation I have shown. Now, if I give the four modes of vibration of four different notation when it is a vertical motion suppose if I use z; when it is the horizontal motion if I use x and when it is a rocking motion if I use psi. Rocking motion actually what is the amplitude? Amplitude here actually rotation because it is rotating like this.

So, how much it is from the vertical rotating that is actually rotational amplitude. Both rocking and torsion it will have rotational amplitude and rocking means actually it is rotating like this with respect to vertical, how much it is rotating that angle to be is the amplitude and what is a torsional mode; actually with respect to some horizontal distance how much it is going that is actually amplitude of the motion that that also rotational amplitude theta. Psi and theta actually is actually rotational amplitude in rocking mode and in this. So, you can see.

So, similar to these if I write for vertical motion and z actually is a vertical amplitude, then corresponding equation will become like this. Similarly, if I consider the amplitude of horizontal direction is x, then your expression of horizontal motion will be like this.

If I rocking motion, if I consider rotation as psi then your rocking motion will be equation of the rocking motion will be something like that and here, actually another thing to be noted that when it is a horizontal motion or vertical motion, inertia force was there and inertia force was mass into acceleration and when it is a rotational mode actually there actually an inertia force will be different.

Instead of mass actually here actually mass second moment of area multiplied by the rotational acceleration and when actually a torsional mode, it is actually mass moment of inertia multiplied by rotational acceleration and what is this I psi I theta etcetera I will discuss later on.

And so, if we understand this vertical and horizontal motion, other thing can be understood easily; same, similar. Only thing these are the parameters will change what are the changes I will discuss later on. So that means, general equation undamped force vibration equation, I have written like this. Now if I want vertical motion if I want horizontal motion or if you want rocking motion or if you want rotational motion their corresponding equation, I can write like this with noted respecting rotation that means, z is vertical motion, x is the horizontal motion, psi is the rotation with respect to horizontal axis, theta is the rotation with respect to vertical axis.

So, this is the way one can visualize four different equation, but solution method and all will be same. Only thing x will be or u will be replaced by either z or x or psi or theta and mass will be replaced by sometime I psi and I theta depending upon type of rotational mode ok.

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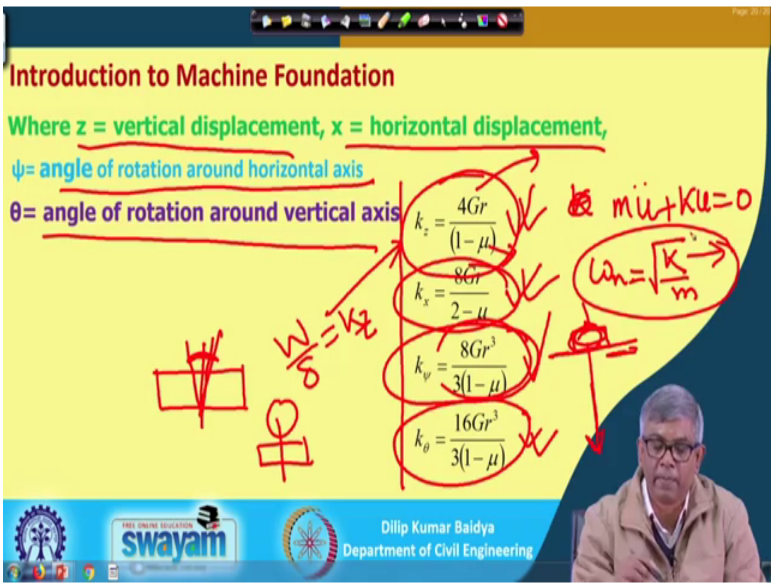

So, now you can see here I have already mentioned that z is the vertical displacement; x is a horizontal displacement, psi angle of rotation around horizontal axis that means, if this is the ma block and rotating with respect to this. So, this block will be rotated rotating like this. So, this angle this angle will be psi. Similarly, if the block is there rotating with respect to these axis.

So, I can I will not be able to show here that means, in each from the initial position how much is rotating that is actually theta. So, theta will be angle of rotation around vertical axis and you can see that that we have mentioned that for display translational mode when I have written equation u sorry m u double dot plus k times u equal to suppose 0, there actually we have m omega n equal to under root k by m. So, where k is the spring coefficient?

So, when I will consider different mode. So, different mode will have different spring coefficient. So, when it is a vertical mode the expression for spring coefficient is like this. When is a when it is a horizontal mode of vibration, then horizontal spring coefficient is like that. When it is a rocking mode rocking spring, coefficient is like that. When it is a torsional mode is a torsional spring, coefficient is like that that. What how it is coming actually? So, I assume a block or circular block on the form on the soil; now the spring stiffness is nothing but because of this loading in this or whatever w acting.

So, it will produce some deflection and by applying elastic theory, I can find out what will be if I w load apply on the homogeneous half space. So, how much will be the deflection I know and then finally, load by deflection is giving you kz and that is actually theoretically you obtain that if I w load if I apply on a surface with a radius r and then whatever deformation will be there and that deformation w divided by that deformation if I do ultimately I get the expression which will be equal to 4 Gr by 1 minus mu; g is the shear modulus of the soil and mu is the Poisson ratio.

Similarly, if I apply horizontal force on the on a block on the ground surface, then there will be horizontal displacement that also theoretically elastically I can find out and that load divided by that horizontal displacement if I do, I will get another expression like k x equal to 8 Gr by 2 means that is horizontal stiffness. Similarly if I a circular block if I give a couple moment of m, then whatever rotation is taking place that rotation that m divided by that rotation if I do, I will get the rotational stiffness k psi which is elastically obtained and it is the expression.

Similarly, if I in a circular block if I give a resting on the soil and if I give a torsion t, torsion of t; then whatever rotation it will be there that torque divided by rotation that if from there actually I will get torsional stiffness.

So, these are all theoretically obtained values and you can see nicely it is given in terms of radius of the radius of the block and the soil properties actually G (Refer Time: 23:16) modulation Poisson ratio. So, k z k x k psi k theta there can be four this will be actually you have to remember actually expression for stiffness for different modes, you have to remember. Because when you want to find our natural frequency first thing we have to use this.

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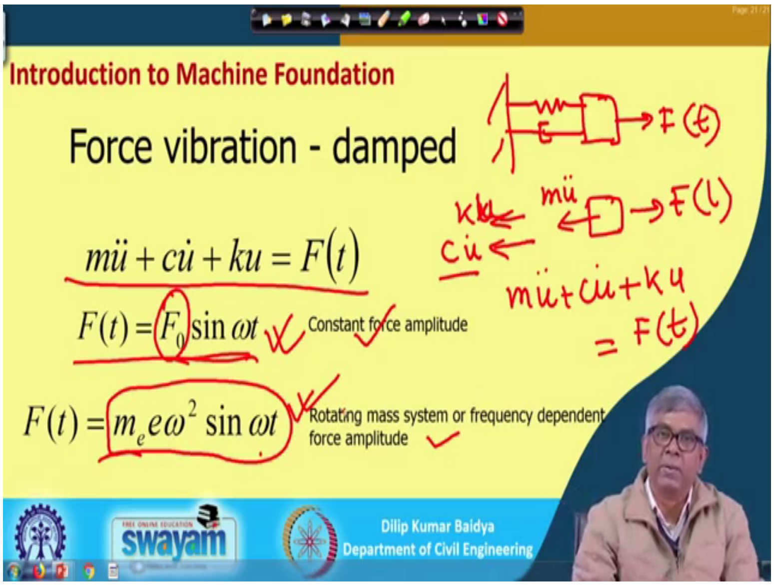

Similarly, now if I forced damped vibration, if I do; then force damp vibration means how it will be it will be something like that. It will be spring and then it will be and this mass and then F t and this is there. So, if I draw the free body of this mass, then on that in this there will be m times u double dot that acceleration will be there. K times x the because of steam spring deformation that force will be there and because of the dashpot; dashpot will be c times u that damping coefficient is actually proportional to velocity.

So, you damping force will be velocity multiplied by damping q. So, these are the things and here actually F t. So, so that means, if I now consider the equilibrium in vertical direction. So, you will see that m u double dot plus cu dot plus k u; it is not x it is u equal to F t equal to F t. So, this is the equation. So, this equation is shown here and you can see F t can be F naught sine omega t or F t can be as I have told that if there is a rotor with a constant force with some frequency, if it repeats; then that will be the F t and in a system there is an eccentric mass and rotates with a frequency omega. At any frequency, what is the F t this is the value? So, the dynamic force can be of this way or it can be of this way.

So, when rotating mass system this is the dynamic force when the constant force is this is the force. So, now, this like undamped force vibration, we have seen that there are two component of solution will have two components; one will be the particular solution that means, taking the equation as it is and another complimentary solution that means, when will F t equal to 0 this equation equate right side equal to 0, then it will give you another solution.

So, this when you will put F t equal to 0, then whatever solution I will get plus keeping this equation as it is I will get another solution that is called particular solution; another one called complementary solution. These two together will be the actual solution of the this system, but as we know that, that complementary part because of the damping present in the system after some time will be will be absent. So because of that, what a particular solution only will produce the vibration in the system.

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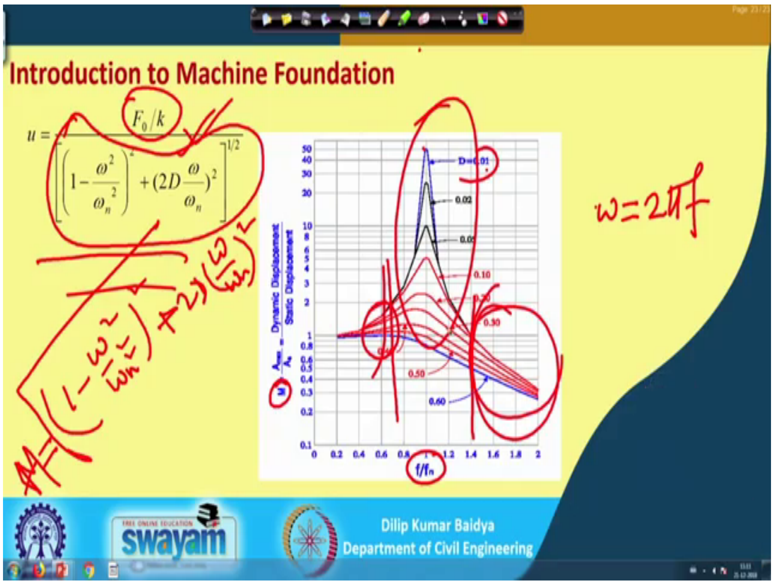

So, taking into that into consideration we will see that sorry we will get finally, the solution something like this. We will get solution like this u will be something like this and if you plot and this is actually delta static and this can be called as magnification factor and this is you can see this is the magnification factor that means, this portion plotted with this, this is plotted with respect to omega over omega n. You can see omega over omega n or omega over omega n and omega equal to 2 pi into F.

So, F pi omega over omega n is nothing but f by fn. So, if I plot this way this expression that means, 1 minus omega over omega n square whole square plus 2 D omega over omega n whole square under root. This is suppose a function and this is actually said m. So, this m versus omega over omega n is plotted for different values of damping you can see and you can see that up to certain frequency ratio, the whatever may be the damping the amplitude is almost you can say constant and if you give if you reach a very high amplitude again whatever may be the damping the amplitude is almost same.

And that means, what is the role in damping actually. Damping has a maximum role close to the resonance ok. If I put if you see damping equal to 0.1 percent this is a finite value and if you put damping equal to 0 in this say this will be infinity actually ok. So that means, damping has a maximum significant role in the resonance zone only. So, other than resonance at high frequency and low frequency; damping does not have much role actually, it has almost close to the static displacement.

Anyway with this, I will stop today. I will see, I will continue the other aspect of this in the next class.

Thank you.