Geotechnical Engineering II / Foundation Engineering

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Lecture – 53
Introduction to machine foundation

Good morning. Today, I will try to start little bit new topic. In fact, last lecture I have given a hint that so far whatever we have discussed it is for static loading and since in many of the competitive exam and also syllabus of many places, it includes Foundation Engineering; also includes a little Introduction of machine foundation. So, because of that I have thought of taking a few lectures on Introduction of machine foundation and or you can say foundation for dynamic loading.

And in this there are there is some basic difference between the static problem and dynamic problem. Sometime many people loosely say that when the force is time dependent that is dynamic, but that much definition may not be enough mainly because what is the exactly basic difference between the static and dynamic is that definitely it is to be time force has to be time dependent and in addition to that has as to it has to satisfy certain condition. Why?

The statement made is not sufficient actually suppose in the geotechnical engineering, we carry out some cyclic loading test and there actually we apply load sometime today, I will apply some load and then next day as I change the loading and again in a particular sequence again, I will change the loading next day.

So, like that that means, with some time dependent force we are applying, but still that is not a dynamic load that is not a dynamic problem. Mainly because it is applied slowly and almost like statically and so, because of that that problem cannot be treated as dynamic. So, to satisfy the problem as a dynamic, what is additional thing comes actually that is actually the force is applied in such a way that inertia force developed within the mass to resist the force. So, that is the actually condition additional condition. So, I will just show you that with my next.

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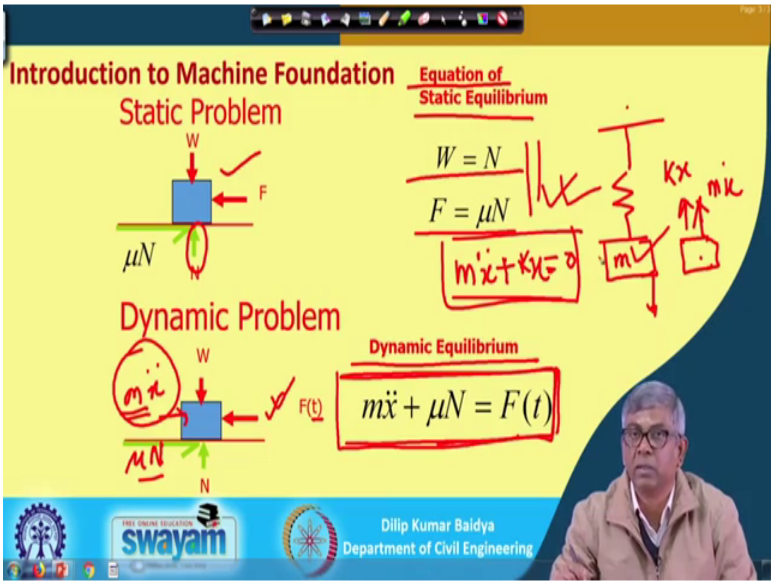

So, this is the slide, you can I will show you here that what is the difference major difference in static and dynamic and you can see here for this is a static case, where actually a block is there on resting on a floor suppose. And when you will try to push this block by applying force, then there will be frictional resistance will develop here and there will be normal force will be reacted by this normal reaction here. And if I then, for since it is a static problem, we can apply the static equilibrium equation; where, actually summation of force equal to 0, summation of moment equal to 0 and here actually since there is no movement.

So, we can say force in vertical direction, force in horizontal direction; if we equate to 0, then you are getting these two equation or relation. Whereas, if the problem is dynamic; that means, if I consider this is dynamic, then in that case you can see here the force is that obviously time dependent and also that it is here and some of the forces are not here actually this will be mu times N will be there and of course, in this direction there will be m x double dot.

If this x actually and you can see these two forces mu N and mx double dot that is actually the force this mu N actually resisting friction and since it is dynamic problem, then this inertia force which is actually axillary mass into acceleration. So, these two forces will be resisting this external force F t and then, if I consider now dynamic will be equilibrium instead of the static equilibrium in horizontal direction, then this will be the equation actually. So, this has to be also solved differently than whatever we do here ok. So, this is that means, the inertia force actually the dynamic system actually inertia force is the most important until all this is develop inertia force that problem cannot be treated as a dynamic problem.

So, that is one thing. Similarly, if we visualize a spring and even mass is resting here and it will fix as if you give an initial displacement here and then, leave it; then also it will create a dynamic system mainly because why it is so? Because if I draw the free body diagram of this and you have given only displacement no force, then you have a we have you have a from the spring there will be resisting force k times suppose x opposite in that this direction. So, when you are pulling this direction. So, the action will be upward direction and since we have pulled this side; so, inertia force also will be in this direction. So, that will be m into x double dot.

So, if this is the system then there is no external force, then I can write in vertical equilibrium. Based on that dynamic condition it will be m x double dot plus k times x equal to 0. So, this is also a dynamic, if you the in a mass which is hanging from a spring if you pulled it little and leave it then it only if the system form a dynamic system. So, that is the an equation of and that motion will follow this equation ok. So, this is the main different and otherwise if it is simply if you keep the mass here mass; mass here just hanging from the spring, then in equilibrium it will be nothing, but your the mass will be equal to k times x.

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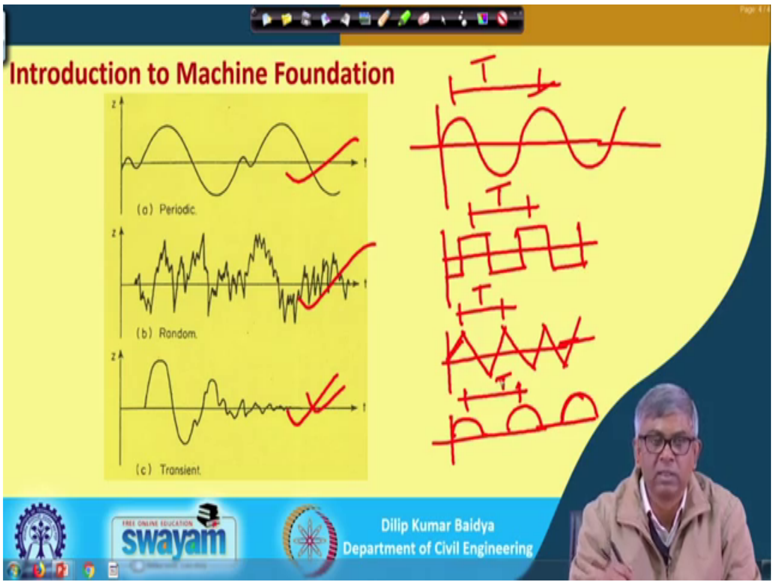

Now, there are different types of dynamic force, you can here next is this is periodic. Periodic means what? Same thing repeating in some particular interval and this is in random and this is actually transient actually that means, you have applied some force and with some time it will be becoming 0. So, this is transient and again periodic force can be of different kind. Actually you can see this can be a periodic and then again, another periodic can be you can say you can say like these.

So, like this is also periodic, then if suppose we can have something like this. If force act like this, this is also periodic or even if it acts like this. So, this is also periodic. So, this periodic that means, this is the time period t after which this is repeating. So, here also this is the time period t and for this is the time period t. After what interval this is repeating. So, this is actually for this case this is t ok. These are all different types of periodic learning.

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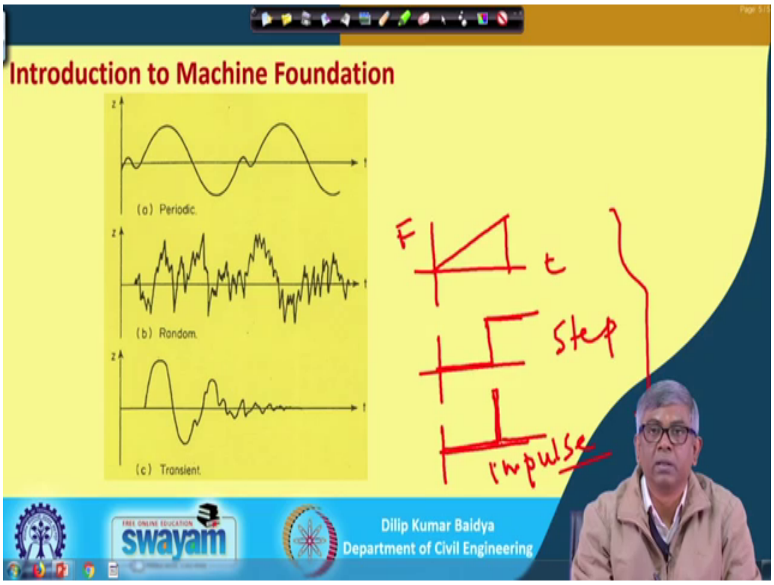

Similarly, this transient also can be of different types you can say this is one, it can be like this; this is one transient. So, this is actually force and this is actually time and similarly, it can be of another step. Suppose it is going like this and then and then it is applying constantly for some time. So, that is another step loading and that there can be another impulse. So, it is going like this for a short very short period this small force is applied and then again coming to 0. So, this is impulse; this is step; then, it is impulse. So, these are all different types of transient loading.

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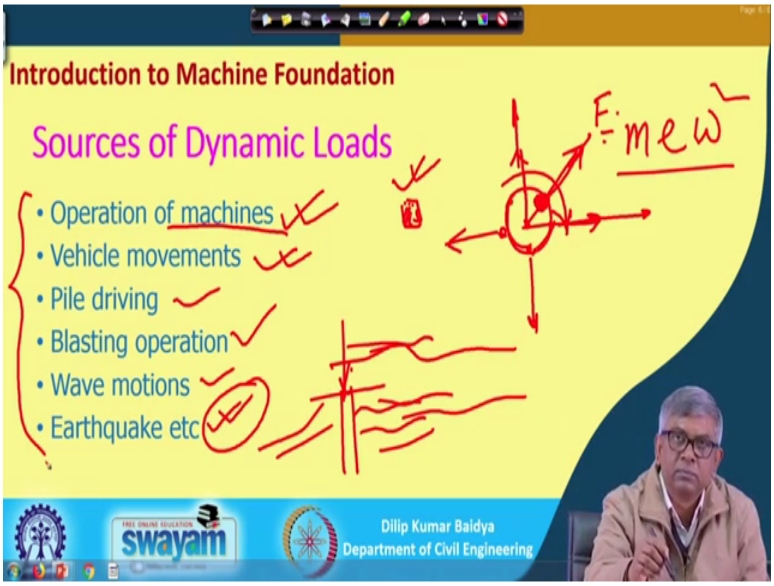

And now you have to let us we have discussed building and foundation all with respect to static loading and what is the source of static loading already known, that is actually yourself weight and different types of live weight those includes actually your static weight. And now, you can say what is the source of dynamic force actually. The one of the most important source of dynamic forces Operation of machines; that means, in the machine actually sometimes there may be some amount of force applied in some interval.

So, in at a particular frequency it is coming that also create dynamic or sometimes in the in the in the machine itself that may be some rotating part, eccentric rotating part if there is a eccentric rotating part suppose this is a mass and it is rotating and in this mass actually there may be some eccentric mass here and it is rotating. So, if it rotates actually, then what will happen? There will be centrifugal force acting upward and which will be some magnitude and then, this force can have actually two component; one is vertical and another is horizontal at any place and when it will be this mass will come to here, it will perfect vertical and when you will come here also, it will be perfect vertical.

When it come here perfect horizontal, when it will come here it will be perfect horizontal like that by rotation actually it will create both horizontal and vertical motion and so, that is what that if the rotating mass is there and then so because of that there will be dynamic force. So, that one is actually a particular fixed amount of force repeating at some interval, then at a particular frequency that also create dynamic force and when there is a rotating machine where which contains some amount of eccentric mass and that because of the rotation of that eccentric mass, you know that centrifugal force will be there and that force will have at different because of different position.

And it will magnitude will be constant, but it will have two components vertical or horizontal and sometimes will be pure horizontal or pure vertical. And I will discuss this one in later stage how we can have pure vertical motion because of the combination of two rotating mass that we will discuss later on. For the time being, we can see that if this because of this rotating part eccentric parts when rotates it will create some force which is m e omega square actually m multiplied by e the radius or extensity into omega square. This is the magnitude of force ok, it is acting this direction. So, F equal to F equal to m e omega square and again it is function of since with change of frequency it is changing. So, that it is dynamic force also changing magnitude.

So, this part I will discuss in detail later on. This is one of the source of sources of one of the sources of dynamic force that means, in operation of machine and another is vehicle movement, when there will be a vehicles and then moves from the undulating grind because of that it will give some impact. It is like a transient dynamic force will create. So, because of this unevenness if someone suddenly hitting; that means, it will create some force and that force as I have shown the transient the initial some magnitude and then it will become 0. And then, pile driving because of that during the driving of pile by impact, then wave generates from there and it will carry some energy and it will move that is that also create vibration.

Blasting operation in the mining, then wave motion and then earthquake that is the most important thing actually for civil engineering, civil engineers is the earthquake loading actually during earthquake actually that lot of different types of wave generates and that waves carry the energy and when it will come to the surface there lot of disturbance occurred and that to be modeled actually how much is the force in a particular earthquake how much force to be taken. So, that there are several different areas actually how to calculate at earthquake loading, we are not discussing. Only we are highlighting, what are the different sources of dynamic load which come to the building or foundation.

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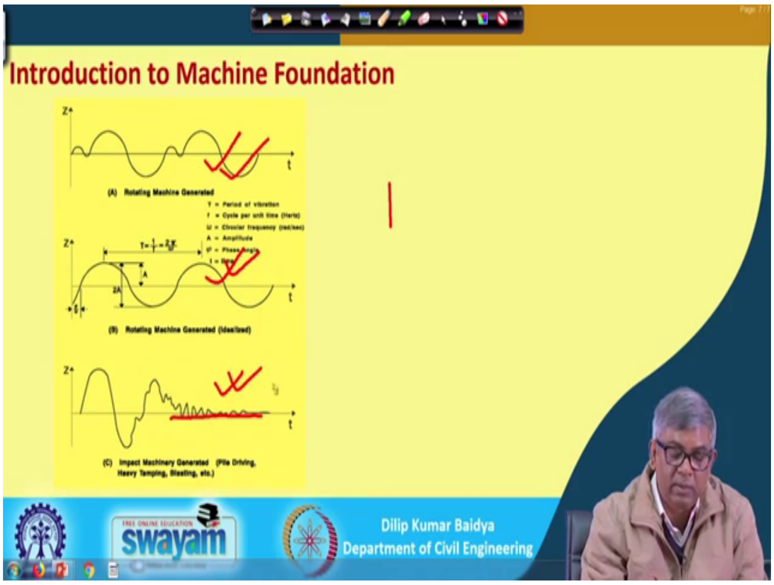

Next part is again I have shown already. So, again repetition actually this is actually by rotating machine, generally the force will be like that with time and that can be idealized like this harmonic ok. So, this will be and if you idealize this way it will be easy to analyze actually. So, because of that we do another actually when impact machine; suppose there are some impact machine means there is a forging hammer. So, on an wheel there will huge mass will fall to save the particular material and so, because of that when it impacts, then it will give dynamic particular magnitude of dynamic force and with time it becomes 0. So, that is another type of dynamic force.

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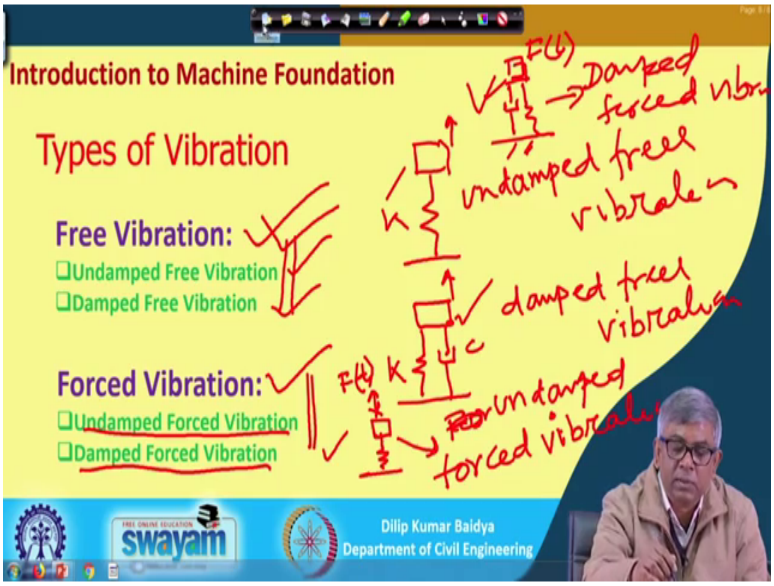

And then, as we know that that there is a dynamic force from different sources and then, to further because of the dynamic force what is happen what will happen actually that there will be vibration and that actually our now aim will be to estimate that amount of vibration ok. So, that vibration again magnitude of vibration again will change with frequency ok. Every system will have a definite natural frequency and when the operating frequency come close to that natural frequency, then it will happen it will have a resonance. During resonance actually there will be amplitude will be significantly high.

So that means, we have to develop the system or a method to estimate those at different places what is the magnitude of amplitude and at the same time we have to estimate the natural frequency of the system and your while designing since we know that resonance point actually is significantly high amplitude is expected. So, we need to separate that separate the natural frequency and operating frequency. That means, you should not keep the operating frequency close to natural frequency mainly because that you have to avoid a resonance.

So, to develop those ideas are those methods actually you have to systematically see the vibration theories and there are if I complete type sorry if the if the every system if I divide vibrating system, if I divide into three two different two different classic groups; one is Free Vibration and another is Forced Vibration. Free vibration means like impact you have applied the force or you are given initial displacement and then, because of that how system is vibrating that is one study that is actually free vibration and another force vibration means actually we are operating machine and because of that there will have some vibration force with dynamic force is generated and that creates actually vibration. So, that is force vibration and force vibration again can be of different kinds that I will discuss later on.

So, again when we will discuss free vibration again it can be of two different types; one is Undamped Free Vibration and Damped Free Vibration. That means, what is damp actual damping actually what? So, when such anything vibrates and because of some damping property present or exists within the material that initially whatever amplitude with time that amplitude will be reduced actually so that the vibration will be ultimately finally, diminished. So, that property is actually damping.

So, we can consider analysis without considering the damping then it is called Undamped free vibration and when you consider the damping then it is called Damped free vibration. So, two different ways the spring vibration can be analyzed. Similarly when we consider damping in the system and the force, then if we consider then it will be so without damping considering damping then undamped forced vibration and when consider in the analysis damping when you damping in considering the analysis, then that will be called damped forced vibration ok. So that means, if I model suppose there is a spring that there is a mass and there is a spring and if it is a dynamic ok.

And then, if I give initial displacement then because of that what will happen that is actually it is undamped free vibration and if I consider now another say mass with is dashpot spring this is k and this is c; this is k. Then if I give initial displacement this will also vibrate, then it is actually damped free vibration ok. Now I assume another situation where the mass is here, spring is here and then, I am giving a dynamic force here F t, initial instead of initial I am giving a force F t which is doing like this in that case without.

So, this is actually force vibration undamped sorry undamped forced vibration and similarly, if I imagine another system suppose mass is here and there is a dashpot and there is a spring and here actually F t; then, that is actually damped force vibration ok. So, like that the four categories; model one undamped free, it is damped free. This is actually undamped force damped force. So, there are three different types of analysis we can do.

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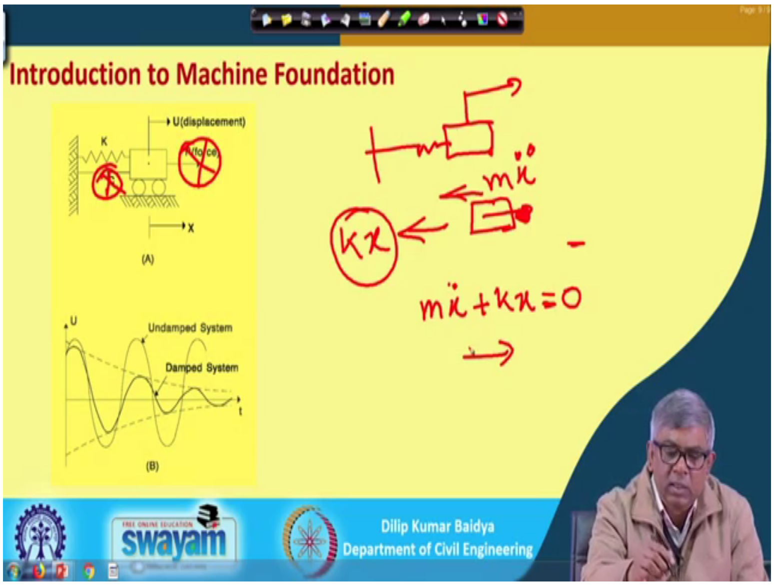

Next you can see here though in the model itself I have shown everything, but this I can separate the sub of force is 0 and dashpot is not there so; that means, your model is something like this and so, in that case and initial displacement is given there and then, if I do a next page I will show that. Then, I will get actually what force I will get this will be mass and in this actually inertia force will be sorry inertia force will be in this direction which will be m x double dot and there will be spring force will be k into x.

If the x displacement is given in this direction; then force in this actually k times x and if I after doing that if I live it, then in within the mass there will be inertia force. Since, I have pulled this side; then, opposite direction inertia force will be there. So, that means, in this mass now two forces are acting; one is mx dot double dot plus k times x and there is no external force acting. So, it will be 0. So, this is the governing equation of motion when I will I will remove this and I remove this p force, only I will give the displacement on the spring supported by a mass; a mass supported by spring, then your governing equation will be this. Next let us go to the next page and then, I will see that your as I have shown instead of x actually suppose I take the displacement where u ok.

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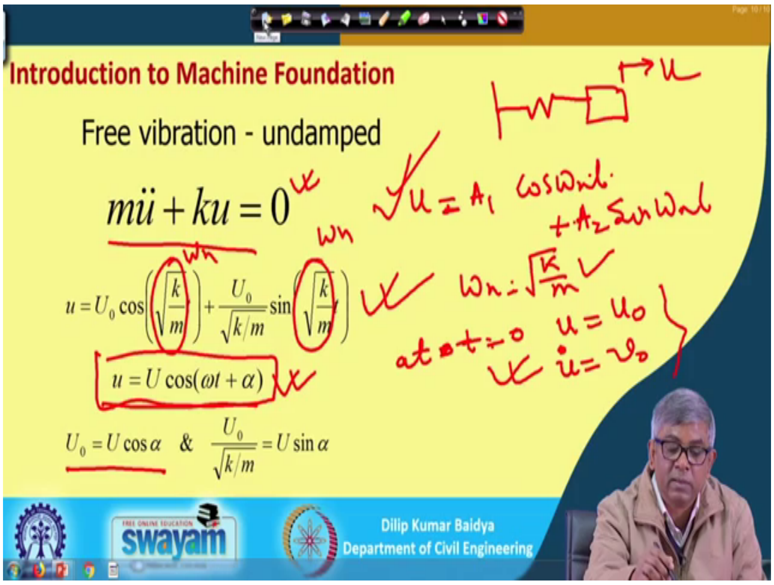

So, here actually the mass this is the mass and this is. So, you are giving these direction that is suppose u; in that case your equation will be mu double dot and k u equal to 0 and this is the equation actually for this equation generally you have a solution, this type of equation we can assume a solution a one A 1 cos omega nt plus A 2 plus A 2 sin omega nt. So, this is the general solution we can take and finally, we can arrive at solution further where, omega n equal to under root under root k by m and your and finally, I can put boundary condition at a at t equal to t equal to 0 u equal to u naught and t equal to 0 u dot when velocity equal to suppose v naught.

If I do this after assuming this is the general solution and taking this as omega n equal to these and putting this boundary condition, I can arrive at a solution like this that u naught cos omega this is actually nothing but omega N this is nothing, but omega n. So, omega nt. you know. So, this can be finally, written in this form, and where u is the amplitude and this is the phase angle and where, u naught actually related to this and sin alpha; this 2 related cos alpha sin alpha u naught and omega n relates and then, I if I substitute in this. Finally, I can get the equation like this and for this equation actually the solution if I plot with time then the whatever I have shown in the previous page, let me go back and sorry ok.

This one also will do. This is the one the ok. This one I can do it. You can see whatever solution we have got that undamped system that free vibration, undamped free vibration this is the solution this is the solution ok.

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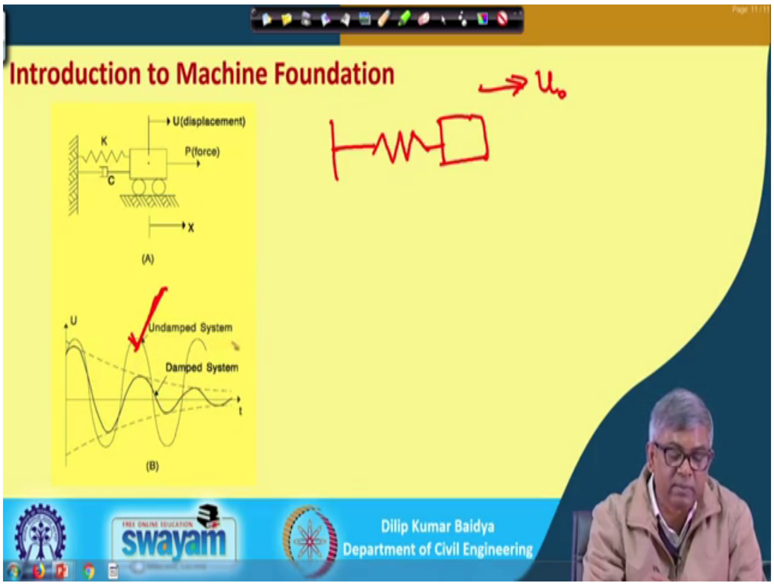

So, you have you can see the two; one is actually it is started with high value and decreasing slowly that I will come later on and one actually keeping constant amplitude that is actually the solution or whatever I have shown previous page that is the solution for this problem; that means, if I take this spring and then, the mass and then I gave the initial displacement u naught and then, I leave it. Then if whatever vibration it will create the vibration amplitude will change like this.

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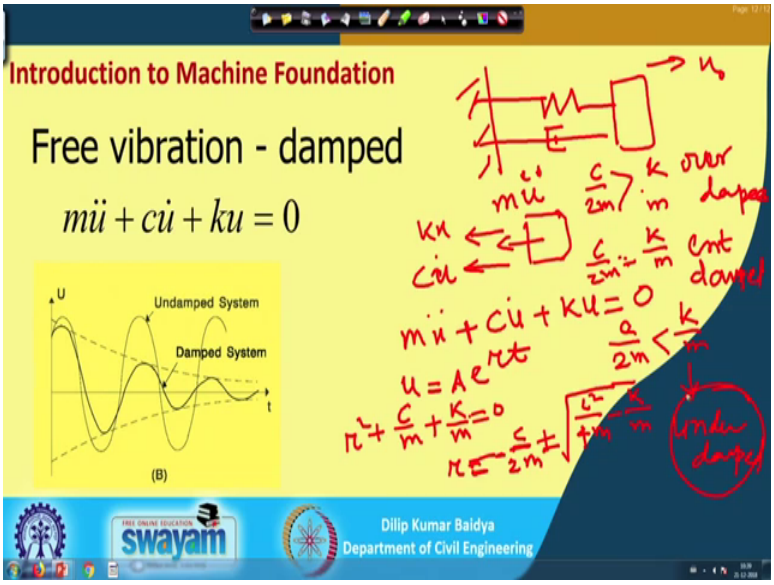

And then, I have already done this and then actually you have another situation what is actually called free damped vibration ok. So, that means this is the one we have this one and then if the mass is like this. Then, we can say we can find out, if I give a displacement u naught initially; then, will have again you can see this will be k times x and damping force will be c times u dot and then, this is the mass in this direction and in this mass again inertia force also will act in this direction. So, mu double dot.

And since, there is no external force. So, you can sum them. So, m u double dot plus c times u dot plus plus k times u that become 0 and if that is the equation if this is the equation; then this equation actually can solve can be solved taking an initial solution as e to the power A A e to the power rt and if you substitute this, then I will get an equation which will be equal to r square plus c by m plus under root k by m equal to 0 and if I solve for r, then I will get root of r will be equal to minus c by c by 2 m plus minus under root under root c square by 4 m minus k by m. I will get the r.

So, now, I can get because of this value combination of these two values I can get three different cases; actually if c over 2 m; c over 2 m greater than k over m that is actually over damped and c by 2 m equal to k by m that is actually critically damped, critically damped and if c over 2 m less than k by m that is under damped. So, three different types of solution will be there and in the practice in these three types of system also exist in the nature and so, but most of the cases actually the system is under damped. So, because of that we can arrive at the under damped solution, the under damped solution is something like this.

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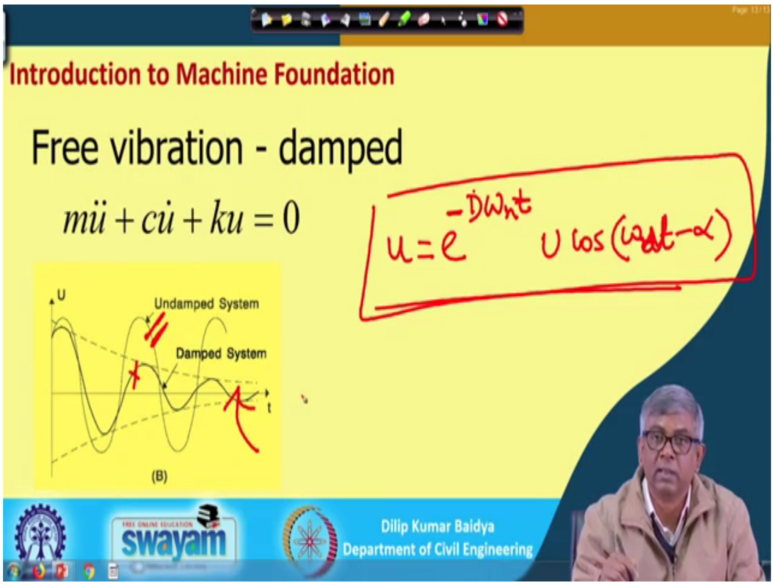

If I do a final solution it like it comes like that u equal to u equal to e to the power minus D omega n t. Then, u cos omega nt. Now, omega dt minus alpha. So, here actually whatever I have shown in these actually you can see that two curves; one is this one is this, another is this. That is this one cross one is slowly decreasing, the this is the plot of this solution is actually this one this is 1 ok. So, this is a the solution of under damped case, there will be again solution for what damped; critically damped also. We are not going all details actually.

So, only thing I will take this solution and finally, how what is the relationship between this peak that I will also will discuss the logarithmic decrement in the later stage. For the time being, I will stop here that when there is a damped free vibration solution is like that and the plot is something like that; that means, it will be with time that amplitude will be decreasing. It will be sometime become 0.

So, with this I will stop here.

Thank you.