

**Geotechnical Engineering II / Foundation Engineering**  
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**Lecture - 36**  
**Stability of earth retaining wall**

Good morning once again I will welcome you to this lecture on Foundation Engineering and we have discussed about the earth pressure theories and stability analysis of earth retaining wall and I have quite elaborately I have discussed the earth pressure calculation for different condition like when the soil retain the cohesion less backfill and then there are different theories are there Rankine's theory, Coulombs theory. And Rankine's theory there is some limitation; that means, it should be a vertical wall, smooth vertical wall and level backfill.

However Rankine's theory as initially as mentioned that, but if it is level backfill inclined backfill or instead of vertical if it is not vertical also how to take care that also I have discussed and finally, using those earth pressure theory actually our primary a objective is actually to analyze the stability of earth returning wall. And for earth returning wall that when you want to do stability analysis we need to know what are the different forces acting on the wall and lateral earth pressure is one of the most significant loading on the returning wall.

So, because of that we have devoted a significant amount of time for finding out the earth pressure for different condition and again initially we have started with cohesion less backfill and we have discussed with different theories and then later on we have also discussed with C5 soil backfill; C5 soil backfill. And how they change and how their pressure diagram how they are what is the total thrust comes to the wall because of the that backfill what is the point of application of the thrust all those things we have discussed and also I have discussed the stability analysis of the wall based on that.

And at the beginning as introduction also I have mentioned different types of retaining wall and some of the things like internally stabilised externally stabilises like that we have divided and we have mostly we have what we have considered they are all externally stabilised ok. So, that gravity retaining wall or cantilever retaining wall or gabion wall all those things actually externally stabilised wall.

And there are actually another type of wall that is called internally stabilised, why internally stabilised because the we use some reinforcement and that reinforcement itself and put it in the soil and that itself helps to stabilising the wall. So, because of that so, I may not be able to take that internally stabilised earth wall in detail, but I will just give you the introduction how we design internally stabilised earth wall or earth retaining wall, reinforce earth retaining wall and then finally, I will try to summarise whatever we have discussed through 2 3 weeks that is stability earth pressure and stability analysis of retaining wall.

So, let me take to the first slide which is actually reinforce earth retaining wall.

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The slide is titled "Stability Analysis of Earth Retaining Wall" and "Reinforced Earth Retaining Wall". It contains a 3D perspective diagram of a retaining wall with multiple horizontal reinforcement layers extending into the soil. To the right of the diagram are handwritten equations:  $T = \gamma z K_a s h$  and  $F = \gamma \tan \delta (2Lw)$ . A small video inset in the bottom right corner shows a man speaking. The slide footer includes logos for "swayam" and "Department of Civil Engineering" along with the name "Dilip Kumar Baidya".

And you can see this figure also I have shown at the beginning and that we can see that, these are the this is actually the wall this is actually wall actually, but this wall unlike gravity retaining wall or cantilever retaining wall they are very light and they are not really wall, they are actually spacing element. And this spacing element actually if I put any load will not be able to hold actually.

So, because of that what is the mechanism here actually this spacing element connected to some reinforcement, you can see at different locations and those reinforcement actually placed within the soil itself and when there will be back soil is fully backfill here, then what will happen because of this soil pressure this spacing element will try to move this direction when it will try to move this and since the reinforcement is

connected to this, then the reinforcement also try to get pulled outward and then because of this and that time that between the soil and this reinforcement and generally we take this type of construction we use granular soil.

So, granular soil will have some friction and the reinforcement will have some friction so between the soil and material and the reinforcing material there will be some frictional force develop and that frictional force if it is enough then this wall will not move it will not fall it will stand. So that means, to prevent falling of this spacing element what you have to do? You have to put this reinforcement sufficiently inside the soil mass ok. So, to put the sufficiently inside the soil mass then how much is the sufficient that has to be calculated actually based on some theory. So, that is the thing I will try to discuss very briefly.

So, what will be the length of the reinforcement inside the soil [vocalized-noise] so and you can see that it is shown here actually there is a this is actually  $h$  this spacing 2 type of spacing, this is one spacing between these and this and there is a another spacing is this actually at one is  $s$  another is  $h$ ,  $s$  is this and  $h$  is this suppose.

Vertical spacing so this is one level of reinforcement, this is second level of reinforcement between these whatever distance that is suppose  $h$  and again if the wall is quite long, then laterally also in some interval you have to put reinforcement. So, that is if this is the one then left this one then this is the one. So, this is the spacing between these two suppose  $s$  then and so what will be the total tension will load tension a tensile load will come to the on the reinforcement there is a mechanism to calculate. So, what we can consider? We can consider that if I consider this reinforcement then half distance here and half distance above will be zone for it.

So, because of that and similarly if I consider this one half from this side, half from this side; that means, for a particular reinforcement from both side half half and from the top and bottom also half half. So, that is why  $\gamma z$ , suppose if I consider at this point what is the lateral pressure?  $\gamma z$  times  $K a$  and if I consider within the depth of  $h$  is constant average because I will just show this one this is the reinforcement level and if I earth pressure that we do like this and I am considering this portion for this rod ok.

Here actually lateral pressure something, here also lateral pressure something. So, this

point actually average of this two. So, because of that I will consider at this point; at this point what is the lateral pressure that is enough, that is why if this is  $z$ ; if this is  $z$  so  $\gamma z$  is the vertical pressure if I multiply by coefficient of earth pressure it will become lateral pressure and then I will take  $s$  spacing and horizontal spacing and  $h$  horizontal spacing. So, that if you multiply then I will get the force and that is the amount of force comes to the rod and that means, this much force because of the backfill it is giving to this rod and which will pull you this side.

Now, I have to put the rod now the reinforcement now inside the soil which is this is the failure plane, suppose if this is the failure plane will be something like this, so failure plane will be something like this. So, you have to put the soil of the reinforcement within the soil mass sufficient length. So, that the developed force will be equal to this, early sequel it should be more than that.

So, how to find out that force? That force again can be calculated if that at that level actually vertical pressure is  $\gamma z$ , if I multiply by  $\tan \delta$  that gives you friction and I will get the friction from both side. So, actually above what the entire length; so,  $L$  will be there and your width length into length multiplied by so this is actually  $\gamma z \tan \delta$  is the pressure. So, this is the reinforcement area suppose  $\gamma z \tan \delta$  is acting here so what will be the total if I multiplied by  $L$  into  $L$  into  $w$  sorry  $L$  multiplied by so  $L$  is this one and  $w$  is this one so that means, surface area I am multiplying.

So,  $\gamma z \tan \delta$  is the friction and  $L$  multiplied by  $w$  is the surface area that become the frictional force and since the each reinforcement will have friction both the top and bottom. So, because of that we are multiplying by 2 so; that means, total frictional force developed actually this and total tensile force because of the backfill coming onto this is this. So, this has to be balanced by this and if I add up a factor of safety suppose 1 or 1.5 or 2 accordingly I will get a other things are known I get the length how much length is required. If I fix a reinforcement of 2 centimetre width or 3 centimetre width so,  $w$  is known so, and  $\tan \delta$ ,  $\gamma z$  everything is known only unknown will be  $L$ . So, we can find out the  $L$ . So, that is what I will go to the next slide.

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The slide, titled "Stability Analysis of Earth Retaining Wall", features two diagrams of wall elements. Diagram (a) shows a wall element of height  $H$  and width  $B$  with a failure plane at an angle  $\delta$  from the horizontal. Diagram (b) shows a similar wall element with a failure plane at an angle  $\delta$  from the vertical. Handwritten notes include a Factor of Safety  $FS = 1.5$  and the equation  $FS = \frac{\gamma z \tan \delta (1/w)}{\gamma z K_a s h}$ . A calculation  $6 \times 0.8 = 4.8$  is written, with a circled "4.8" and a circled "L". A small video inset shows a man speaking.

You can see here so if I know the factor of safety, if I apply some factor of safety suppose this factor of safety is known suppose 1.5 and this I consider a reinforcement at some depth, so  $z$  is known,  $\gamma$  is known,  $\tan \delta$  is known, width of the reinforcement is known and only unknown here  $L$  and here  $\gamma z K_a s h$  everything I have these are all design I will initially assume then only thing is from this calculation I will get a  $L$ .

So, that  $L$  actually how much actually based on calculation we get  $L$  minimum; that means, how much this is the if I assume that this is the failure ways this is the failure ways so that means, within this zone reinforcement will be inactive because soil is moving. So, there is no friction develop there, only thing if I go beyond this that zone only will give you frictional resistance. So, because of that, I have to calculate what is the minimum distance required based on this calculation were getting.

So, your  $L$  Rankines means what if I have a failure failure plane like this then from the geometry this is  $h$  and I know this angle. So, I can find out this length. So, this is actually Rankines length and this is based frictional resistance  $L$  minimum. So, these two together will be actual  $L$  total  $L$ ;  $L$  total and so based on that I can calculate and  $L$  total of course, based on calculation you may get some value, but there is a guideline that you should not give less than 0.8 times  $h$  ok.

So, point  $h$  so if  $h$  is suppose 6 meter and 0.8 actually; that means, 4.8 meter you have

this is 4.8, suppose based on this calculation Rankines length then based on this calculation if you get suppose 4.5 you should not give this you should give 4.8 meter. So, this is the basic principle of reinforce earth calculation; that means, everything same I will consider the Rankines failure plane and then based on that I will assume a vertical spacing and horizontal spacing and based on that you can see at what an interval I put reinforcement and horizontally what interval I will assume and based on that what should be the length required I can calculate from here ok.


So, this is a some basic about reinforce earth analysis ok. So, I will not go beyond that the and this wall again this is actually calculation again what additional you have to do, I have given the length, but whatever force I have calculated so whatever T you have got because of this tension it should not T here also. So, based on that I have to find out or design the thickness also. So, I should know the material strength and based on that T divide by area or I can find out T divided by sigma from there I will get the area and that means, from there actually if I consider unit then I will get the thickness also.

So, that is one thing to be designed, that it should not fail and again, there are many other analysis is there, stability analysis which I will not discuss. So, that I will put it beyond the scope of this so I will not discuss that one. So, this much is enough for your reinforced earth retaining wall. Next actually as I have mentioned that we have spent 2 3 weeks on this topic. So, I will just quickly summarise.

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**Stability Analysis of Earth Retaining Wall**

ACTIVE CASE	PASSIVE CASE
Wall moves out	Wall moves in
Shear stress on failure plane reduces	Shear stress on failure plane increases
Lateral pressure reduces	Lateral pressure increases
Minimum lateral stress achieves when soil fails and full strength is mobilised	Maximum lateral stress achieves when soil fails and full strength is mobilised



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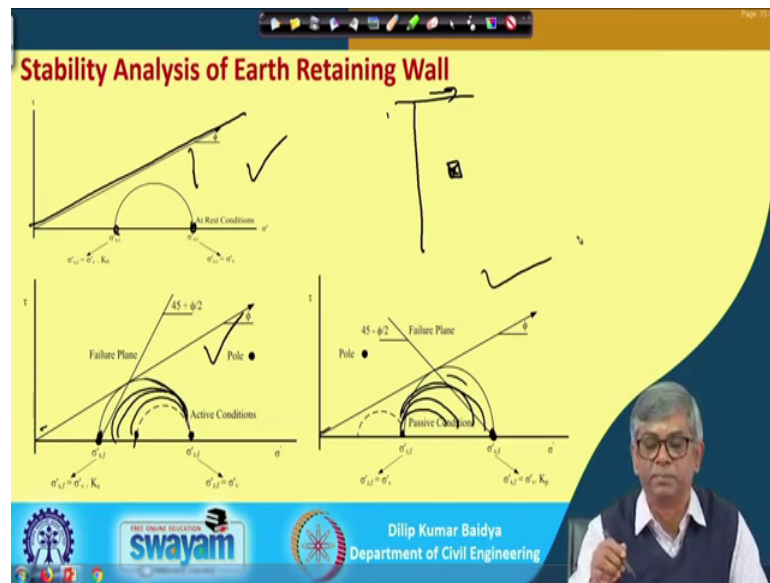
What we have learned through this, actually we have consider one case actually earth pressure at rest; that means, when while there is no movement of wall and when wall moves from backfill; that means, the at rest condition there will be some pressure and when wall move from the away from the backfill pressure will be slowly reduce that is active. When wall moves away from the backfill that is called active and that active pressure will be reducing; reducing.

So, if there is small suppose if there is a wall; if this is the wall and if moves the here, this is active so pressure will be reduced, further it is move this much still it is active, but there will be some movement beyond which if you move then wall will collapse. So, that is the limit actually; that means, just before the merge of failure whatever pressure actually develop that is actually earth pressure active earth pressure.

And similarly when wall moves this direction again suppose this much movement, this much movement, this much movement all are passive, but when it achieved just before the failure that condition actually is called passive pressure, that is what is summarised here. You can see when active case means when wall moves out and when wall moves in shear stress on failure plane reduces that is what whatever we have seen lateral pressure reduces it is compared to here it is less here, less here and this is the minimum, if you make smaller than that then wall will fail.

The minimum lateral stress archives when the soil fails and full strength is mobilised whereas, in passive case wall moves in; that means, towards the backfill shear stress on failure plane increases, lateral pressure also increases, maximum lateral stress archives when soil fails and the full strength mobilised. So, between active and passive what are the differences these are the differences.

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Next actually we have seen that, there are different condition this is the wall and if I consider an element and at rest condition what is the state of stress. So, it is the vertical stress and this is the lateral stress and this is the failure envelope. So, at rest condition your failure envelope will be below the sorry your more circle will be below the envelope whereas, when it is a active case you can see it was initially vertical stress at this is also vertical stress and at rest condition this was the lateral stress and slowly because of the movement of the wall, lateral stress will be reducing and it will rest to the minimum at this point ok.

So, that is actually called lateral stress or passive active earth pressure and when you draw the more circle through this then it will be tangential to the envelope because it is just before the failure. Similarly, when the wall moves this direction, then initially wall vertical stress at rest it was here and slowly it will be increasingly because of the wall movement this direction and when it will be just before failure it will be achieve this maximum value. So, that time if I draw more circle through this, then that also will becoming tangential to the envelope. So, this is at rest, this is active and this is passive.



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**Stability Analysis of Earth Retaining Wall**

$$\beta_{cr} = 45 + \frac{\phi}{2}$$

$$P_a = \frac{1}{2} \gamma H^2 \tan^2 \left( 45 - \frac{\phi}{2} \right)$$

$$P_a = \frac{1}{2} \gamma H^2 k_a$$

$$k_a = \tan^2 \left( 45 - \frac{\phi}{2} \right) = \frac{1 - \sin \phi}{1 + \sin \phi}$$

The slide includes a diagram of a soil failure wedge with a dashed failure surface and a vertical wall. The bottom of the slide features the Swayam logo and the name Dilip Kumar Baidya, Department of Civil Engineering.

And we know that when wall actually the soil movement if I consider different one if everything is giving you active, but at what angle it will actually reach to failure; that gives you actually by geometry you have to find out that critical angle is 45 degree plus 5 by 2 for active and corresponding active thrust is half gamma H square k and active earth pressure coefficient is 1 minus sine phi by 1 plus sine phi.

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**Stability Analysis of Earth Retaining Wall**

$$\beta_{cr} = 45 - \frac{\phi}{2}$$

$$P_p = \frac{1}{2} \gamma H^2 \tan^2 \left( 45 + \frac{\phi}{2} \right) = \frac{1}{2} \gamma H^2 k_p$$

$$k_p = \tan^2 \left( 45 + \frac{\phi}{2} \right) = \frac{1 + \sin \phi}{1 - \sin \phi}$$

The slide includes a diagram of a soil failure wedge with a dashed failure surface and a vertical wall. The bottom of the slide features the Swayam logo and the name Dilip Kumar Baidya, Department of Civil Engineering.

Similarly, you can see when it is a passive; that means, when the wall this is the wall when it moves here, the wall again takes different ways we can consider at what angle

actually it will be reaching to the failure that critical angle geometrically got beta critical actually 45 degree minus phi by 2 corresponding pressure you have got half gamma H square k p and corresponding earth pressure coefficient for passive earth pressure 1 plus sine phi by 1 minus sine phi we have got it. So, this is in detail we have done already. So, I am just summarising.

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The slide, titled "Stability Analysis of Earth Retaining Wall", features a Mohr circle diagram on the left. The vertical axis is labeled 'tau' and the horizontal axis is labeled 'sigma'. A circle is drawn with its center on the sigma axis at a distance of  $(\sigma_1 + \sigma_3)/2$  from the origin. The circle intersects the tau axis at a point where the angle between the radius and the sigma axis is  $\phi$ . The radius is labeled  $R = (\sigma_1 - \sigma_3)/2$ . The horizontal distance from the origin to the center of the circle is  $(\sigma_1 + \sigma_3)/2$ . The horizontal distance from the center to the point where the circle intersects the sigma axis is  $\sigma_3$ . The horizontal distance from the origin to the point where the circle intersects the sigma axis is  $\sigma_1 = \gamma h$ .

Handwritten equations on the right side of the slide are as follows:

$$k_a = \tan^2\left(45 - \frac{\phi}{2}\right) = \frac{1 - \sin \phi}{1 + \sin \phi}$$

$$\sin \theta = \frac{(\sigma_1 - \sigma_3)/2}{(\sigma_1 + \sigma_3)/2} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$$

$$\sigma_3 = \sigma_1 \frac{1 - \sin \theta}{1 + \sin \theta} = \gamma h \frac{1 - \sin \theta}{1 + \sin \theta}$$

$$\sigma_3 = \gamma h \tan^2\left(45 - \frac{\phi}{2}\right) = \gamma h \frac{1 - \sin \phi}{1 + \sin \phi}$$

The slide also includes logos for "swayam" and "Dilip Kumar Baidya, Department of Civil Engineering" at the bottom.



Similarly, if it is based on Rankine's theory, if I draw a more circle for active case, this is sigma 1 and this is sigma 3, sigma 3 is the lateral pressure and if I consider this and envelope will be tangential, now if I consider this triangle from there if I express sine, phi and simplify then I can express sigma 3 in terms of sigma 1 or gamma h and then it is sigma 3 will be gamma h time this.

So, this is nothing, but your k a that is either tan square 4 or I can write gamma h 1 minus sine phi by 1 plus sine phi that also can be written. So,; that means, your k will be equal to tan square 45 degrees minus 5 by 2 or 1 minus sine phi by 1 plus sine phi this way can be express.

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**Stability Analysis of Earth Retaining Wall**

$\sin \phi = \frac{(\sigma_3' - \sigma_1)/2}{(\sigma_3' + \sigma_1)/2} = \frac{\sigma_3' - \sigma_1}{\sigma_3' + \sigma_1}$   
 $k_a = \frac{1}{k_p}$   
 $k_a = \frac{1 - \sin \phi}{1 + \sin \phi}$   
 $k_p = \frac{1 + \sin \phi}{1 - \sin \phi}$   
 $\sigma_3' = \gamma h \tan^2 \left( 45 + \frac{\phi}{2} \right) = \gamma h \frac{1 + \sin \phi}{1 - \sin \phi}$



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Similarly, if it is a passive case you can see here, passive case actually initial sigma was here and these become sigma 3 lateral pressure sigma 3, though it is principally supposed to sigma 1, but I am denoting lateral pressure as sigma 3 because of that and it will be tangential.

Now, again if I take this triangle, if I express sine phi and finally, if I simplify sigma 3 can be expressed as gamma h tan square plus forty 45 degree plus phi by 2 or gamma h; gamma h sorry equal to gamma h 1 1 plus sine phi 1 minus sine phi. So that means, this is actually your k a equal to 1 minus sine phi by 1 plus sine phi and k p equal to 1 plus sine phi by 1 minus sine phi; that means, k a equal to 1 by k p. So, one if you calculate other one need not calculate just inverse of that.

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**Stability Analysis of Earth Retaining Wall**

Granular Backfill

$$k_a = \frac{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}} \cos i$$

$$k_p = \frac{\cos i + \sqrt{\cos^2 i - \cos^2 \phi}}{\cos i - \sqrt{\cos^2 i - \cos^2 \phi}} \cos i$$

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Similarly, if there is as I have the based on Rankine's theory, as I have mentioned that vertical wall and level backfill, but if it is a inclined backfill then corresponding earth pressure coefficients also can be obtained like this  $k_a$ ,  $k_p$  and we can consider the direction of the pressure like this and based on that we can do all calculations which I have shown already during solving the problem.

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**Stability Analysis of Earth Retaining Wall**

$$\sigma_3 = \sigma_1 \frac{1 - \sin \phi}{1 + \sin \phi} - 2c \sqrt{\frac{1 - \sin \phi}{1 + \sin \phi}} = \sigma_1 k_a - 2c \sqrt{k_a}$$

$$\sigma_3 = \sigma_1 \frac{1 + \sin \phi}{1 - \sin \phi} + 2c \sqrt{\frac{1 + \sin \phi}{1 - \sin \phi}} = \sigma_1 k_p + 2c \sqrt{k_p}$$

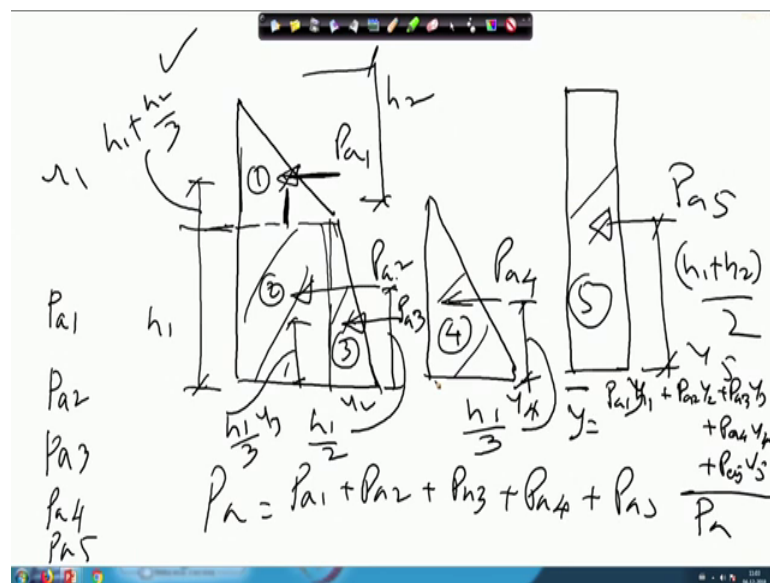
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And similarly now, when it there is C phi soil; when there is a C phi soil and then your this envelope; envelope will not pass through the origin it will be intersecting the y axis

shear axis and it will be meeting x axis somewhere here the that mean if I complete this triangle from there also I can express considering this triangle I can express sigma 3 in terms of sigma 1 and cohesion. That when there is C5 backfill and that is the when it is active case sigma 3 can be explained like that, sigma 1 k minus 2c root k a and when it is a passive case sigma 3 can be expressed as sigma 1 k p plus 2c root k p. And pressure diagram I have shown that when is active case, pressure diagram will be something like this and when it is a passive case your pressure diagram will be something like this ok.

So, these are the things based on that how much depth we can do without support etcetera also I have discussed, this is the depth of tension crack and then double the depth of critical depth will be the depth up to which we can excavate without support. So, these are the things we have discussed in length I hope there will not be any problem.

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And next thing is while solving problem what I have done so suppose there is a pressure diagram, suppose you have better I will take two parts, that is one pressure diagram then, because of the water this is the pressure diagram, because of the surcharge may be this is the pressure diagram like that ok. So, when this type of pressure diagram and quickly I have for the stability analysis I have shown some calculation, generally I have divided in 3 parts 1, 2 and 3, 4 and 5 ok.

So, what you have to do? You have to find out  $P_{a1}$ , first of all you have to find out  $P_{a1}$ , then  $P_{a2}$ , then  $P_{a3}$ ,  $P_{a4}$  and  $P_{a5}$ . So,  $P_{a1}$  is what? Nothing, but this area of this

diagram,  $P a_2$  means this area of this diagram,  $P a_3$  is here of this diagram,  $P a_4$  is this diagram  $P a_5$  is this. So,  $P a$  finally, will be equal to  $P a_1$  plus  $P a_2$  plus  $P a_3$  plus  $P a_4$  plus  $P a_5$  like that ok.

And then finally, we need to find out the point of application of the resultant force, for that actually what we have to do you have to find out the lever arm of each. So, if this is the one for this one from with respect to this one, if this one is acting  $P a_1$  is acting here. So, this lever arm will be equal to if this is  $h_1$  so, this will be  $h_1$  plus this is suppose  $h_2$ ; this is suppose  $h_2$  so,  $h_1$  plus  $h_2$  by 3 because this one will be one-third from the base  $h_1$  plus  $h_2$  by 3.

Similarly, if this is  $P a_2$ ,  $P a_2$  this distance will be nothing, but this distance will be  $h_1$  by 2. Now this is suppose  $P a_3$ . So,  $P a_3$  at what height it will work, this will be actually  $h_1$  by 3 so and this is suppose  $P a_4$  what height it will be? This will be again this is if this is  $h_1$ , so this will be  $h_1$  by 3 and this is suppose  $P a_5$  so then, what is the height this one this is nothing, but suppose  $h_1$  plus  $h_2$  by 2 so total height by 2 because it will be rectangular diagram.

So, finally, resultant we get how you get resultant? We get a point of application we get suppose  $y$  will be equal to  $P a_1 h_1$  plus it is not really  $h_1$  the lever arm suppose  $y_1$  I say,  $y_1$  this is suppose  $y_1$  and this is suppose  $y_2$ , this is suppose  $y_3$ , this is suppose  $y_4$  and this is suppose  $y_5$ ; so  $P a_1 y_1$ ,  $P a_2 y_2$ ,  $P a_3 y_3$ ,  $P a_4 y_4$ , plus  $P a_5 y_5$  divided by  $P a$ . If I do this calculation we get  $\bar{y}$  from the base of the so and then using that. So, quickly where when I have done this calculation how I have done actually the lever arm this actually what is where actually it is acting from the base this is nothing, but this distance plus one third of this height.

Similarly,  $P a_2$  where it is acting it will middle of this rectangle, similarly  $P a_3$  where it is acting it is one third of the side. So, like that we can calculate. So, like this everything I have shown I hope this will be helpful. So, this is the summary stability analysis and earthquake sorry earth pressure calculation and stability analysis of the retaining wall. And I will close this one here.

Thank you.