

**Geotechnical Engineering II / Foundation Engineering**  
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**Lecture – 34**  
**Stability Analysis of Earth Retaining Wall (Contd.)**

Once again, I welcome you to this course and this lecture on Foundation Engineering and that too in a particular topic like Stability Analysis of Earth Retaining Wall. And, in the previous few lectures I have discussed about the earth pressure calculation. Now, actually in the foundation engineering and the soil mechanics generally that is the part where actually we discuss about more in more detail about the earth pressure calculation and all. But, in foundation engineering actually finally, you had to design the retaining wall.

So, there while designing two parts are there one part is structural design that there actually if it is a concrete is used then sometime you will take the help of structural engineer. And, but as a geotechnical engineer what you have to do you have to do the stability analysis stability analysis means you have to understand that what are the different ways wall can fail. And, there are three distinct mode of failure in fact, the fourth one is there which we will not discuss the first mode is actually overturning if the wall is too high suppose and retain the backfill then there is chance of toppling the wall with respect to tow, that is one thing.

So, that means, you have to make sure that there is a sufficient factor of safety against toppling. So, that is one thing to be insured and second thing is next the if the pressure is too high then sometime and if there is no not much fixity on the base and then there maybe because of the high pressure inter wall may slide towards away from the backfill. So, that is base sliding that is called base sliding so; that means, when there is a wall and when there is a pressure there will be sufficient amount of vertical pressure and then below the base of the wall there will be some resistance will be there.

And, then that normal pressure that base resistance will be horizontal friction resistance will be the what? There will be normal multiplied by the  $\tan \phi$  so, that one  $\tan \delta$ . So, that way you can calculate the base resistance and when the wall is pushed horizontal horizontally by active pressure then it will be prevented by that resistance only. So, you

have to see to make sure that base resistance is much higher than the horizontal push by the active pressures that is second one.

And, third one is the that when number of forces acting on the wall for example, there is a gravity weight because of the gravity; that means, self weight of the wall that we act vertically and sometimes there will be some amount of soil weight also will come into the wall that also will be acting vertically on the wall and then whether it is active or passive pressure that will be acting horizontally and if there is incline backfill that will be acting parallel to the backfill slope and then you can take into two components one is vertical and horizontal. Vertical actually component will give you the vertical pressure on the base and horizontal pressure will give you the push to away from the backfill.

Now when there is number of forces acting then all the forces will act somewhere which will be may not be through the centroid of the of the base of the retaining wall. When this happens; that means, when the wall is there when there is a particular structure is there is a compressive force acting through its centroid then there will be compressive pressure throughout the base of the of the wall and there is no issue then how to how you can find out total load divided by the area of the base total compressive force.

But, when there will be a number of forces are acting like horizontal pressure, vertical pressure something else like soil pressure and gravity weight and then if we calculate resultant then we may find that vertical pressure will be is acting somewhere other than the CG then because of the eccentricity; that means, the force is acting away from the CG from distance from the CG and that distance is called eccentricity and when that happens then in addition to the comprehensive load the footing subjected to some amount of moment.

When the footing is having a moment a vertical load in addition to a couple like this then what will happen because of the compressive load there will be compressive pressure and because of this couple one side will be tension and other side will compression. So, that means, compression plus compression one side would be a high compressive pressure and other side will be tension minus compression there will be some amount of little bit of compressive pressure. Sometime if the tension is becoming much higher than the compression then it will be tension also; that means, one side will be without any contact.

So, that; that means, because of this eccentricity what will be the pressure distribution below the base of the of the retaining wall we have to find out and we have to make sure that there is nowhere the tension is occurring at the same time when there is a compressive high compressive pressure and you have to make sure that that compressive pressure will be much below than the bearing capacity of the soil. So, this thing you have to check.

So, for that actually I have taken one problem and I will try to show all those things one by one how to calculate those stability; that means, factor of safety against different condition.

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**Stability Analysis of Earth Retaining Wall**

Investigate the stability against overturning, sliding and bearing pressure of the gravity earth retaining wall shown in the figure. The retaining wall is to support a granular backfill which has unit weight of  $17.5 \text{ kN/m}^3$ , and angle of internal friction of  $32^\circ$ . The coefficient of base friction is 0.5. Allowable soil pressure for the foundation is  $250 \text{ kN/m}^2$ .

0.6 m  
150  
Granular Backfill  
 $\gamma = 17.5 \text{ kN/m}^3$   
 $\phi = 32^\circ$   
5.0 m  
2.7 m

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You can see here this wall actually is actually 5 meter high and you can see there is a gravity retaining wall and this back side is vertical and it has a 2.7 meter width at the base and this wall is perpendicular to the board here I can consider a infinitely long that can be long enough which I consider as long enough and for that I can analyze for a unit length. So, that means, I can take this base I can consider as 2.7 this is 2.7 and I can take unit width of 1 meter.

And, that means, I that that is front view and this is a top view of the wall. Top view it is not really top view top view will see only this top only this one will see 0.6 meter, but the base top view for the base is like this and it is CG will be somewhere here like this and you can see here this angle is not mentioned suppose I will take 15 degrees and it is

retaining a granular backfill with 17.3 it is not 3 it will be 17.5 kilo Newton per meter cube, phi is 32 degrees and it is 5 meter height.

And, gamma c instead of 23, I can take 24 actually kilo Newton per meter cube and there is a tau side 1 meter is there. So, theoretically the this side there will be active pressure and this side because of this way it wall will try to move this way. So, because of this it would be active and because of this active pressure does this side wall is pushing this side, so that means, this is passive. So, the side will be P p and this side will be P a. for the stability analysis purpose.

So, this is actually this P p force will give you extra stability to the wall. So, if I ignore this one and find my find out that it is safe; that means, it is really safe, ok. So, because of that that was some conservative side we can do. So, I will in this problem I will ignore this P p side I will only take the active pressure from the side and whatever relevant things are required I will take and I will try to do the analysis ignoring their passive part.

So, let me do one by one first of all clean this and go to the next slide and suppose, I will create a new page.

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$W_1 = 0.6 \times 5 \times 1 \times 24 = 72$   
 $W_2 = \frac{1}{2} \times 2 \times 1 \times 5 \times 1 \times 24 = 126 \text{ kN} = 74375$   
 $P_v = W_1 + W_2 + P_{pv} = 217.25$   
 $P_v \times 0.5 = 108.625$   
 $P_{ah} = P_a \cos \beta = 71.84 \text{ kN}$   
 $P_{pv} = P_a \sin \beta = 29.75$   
 $F.S. \text{ Sliding} = \frac{P_v \times 0.5}{P_{ah}} = \frac{108.625}{71.84} = 1.51$   
 $F.S. \text{ OB} = \frac{M_r}{M_{ob}} = \frac{W_1 \times (2 + 1 \times 0.3) + W_2 \times 2 + P_{pv} \times 2.7}{P_{ah} \times \frac{1}{3} \times 5} = \frac{401.175}{119.732} = 3.35 \text{ (OK)}$   
 $K_a = \frac{\cos \beta - \sqrt{\cos^2 \beta - \cos^2 \phi}}{\cos \beta + \sqrt{\cos^2 \beta - \cos^2 \phi}} = 0.34$

Suppose this one and the wall is your 5 meter high and it is a inclined and from the Rankine's theory what you have learned, though Rankine's theory is horizontal backfill, but we have learned that when there is a incline we have got x expression for K a equal

to  $\cos \beta$  minus under root  $\cos^2 \beta$  meter minus  $\cos^2 \phi$  divided by  $\cos \beta$  plus under root  $\cos^2 \beta$  minus  $\cos^2 \phi$ . We have got this expression and then we have also shown that the pressure will be finally, resultant pressure will be acting parallel to the backfill slope. So, something like this. So, I can imagine the pressure will be acting like this little bit triangular suppose from here and this is the pressure diagram I can imagine, ok.

And, if I do that, so, at this point actually your pressure will be equal to 5 multiplied by and K a if I put all those if I put the value of  $\cos \beta$  equal to  $\cos \beta$  equal to 15 degrees and  $\phi$  equal to 32 degrees if I put in this expression and then finally, I will get K a equal to 0.34, ok. And, then I can find out here 5 into multiplied by  $\gamma$  is 17.5 and multiplied by 0.34; that means, these become your 29.75 29.75.

And, then I can find out  $P_a$ ; that means, active pressure will be equal to half multiplied by 29.75 multiplied by 29.75 multiplied by your 5 ok. So, that will become your 74 point 74.375 74.375 and suppose about this is the one. So, it is acting at some angle. So, I can imagine this one as vertical and horizontal. So, this one I can say as  $P$  and this one I can say  $P_{av}$ .

So, if I consider that. So,  $P$  will be equal to this angle is actually 15 degrees. So, that will be  $P_a \cos \beta$ . So, this will be  $P$  will be equal to 71.84 71.84 kilo Newton and  $P_{av}$  will be equal to  $P_a \sin \beta$  and that if you calculate that become 19.25, ok.

So, now this we have got and then in addition to that suppose there is a wall dimension I will just draw now and it was something like this. So, this is suppose  $W_1$  and this is suppose  $W_2$ . So, I can calculate  $W_1$  will be equal to 0.6 multiplied by 5 multiplied by 1 multiplied by 24 concrete where. So, these become your  $W_1$  become your 72 kilo Newton and  $W_2$  will be equal to half multiplied by this is 0.6; that means, this one is 2.1. So, it will be half multiplied by 2.1 multiplied by 5 multiplied by 1 multiplied by 24 that gives you  $W$  to become 126 kilo Newton, ok.

So, now I have; that means, how many forces are there  $W_1$   $W_2$   $P_{av}$  and  $P$  so; that means, I can find out  $P_{vertical}$  total  $P_v$   $P_v$  will be equal to  $W_1$  plus  $W_2$  plus  $P_{av}$ . So, that gives you  $P_v$  equal to 217.25 217.25 and; that means, so, factor of safety against sliding suppose it will be actually resistance will be  $P_v$  multiplied by frictional actually is given 0.5 frictional resistance at the base will be called is given as 0.5.

So,  $P_v$  multiplied by 0.5 that is actually resistance and what is the horizontal force actually it will try to push the wall for sliding that is  $P$  horizontally 71.84. So, that will be equal to 71.84. So, if I calculate these that gives you a value equal to 1.51 and it fact it is in the Indian code what should be the factor of safety required for factor of safety against sliding it is given. You know that do not have to remember generally it is 1.5 and it is greater than 1.5. So, that means, again sliding it is ok,.

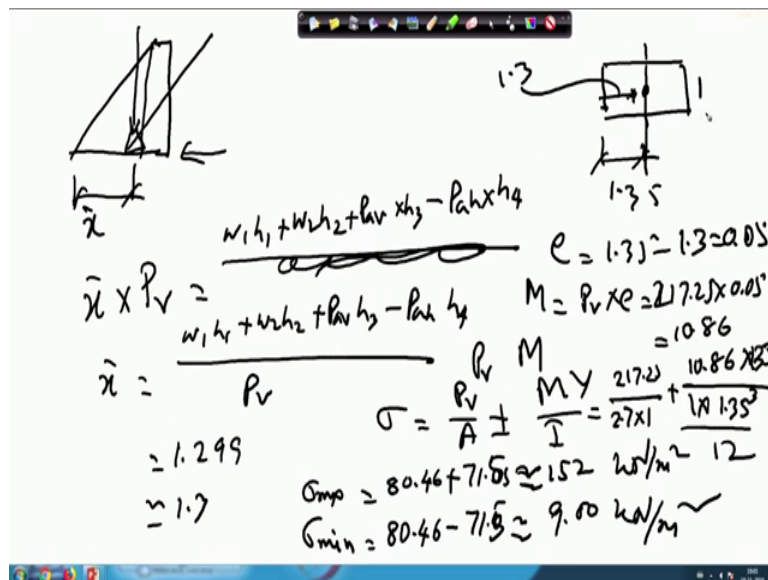
And, now factor of safety against over turning, factor of safety against overturning. So, overturning what will happen; you can see these forces acting here this forces acting here and and  $P_v$  also acting along  $P_{av}$  that is  $P_v$   $P_{av}$  that is acting this. So, this three forces acting. So, if overturning will be when the wall will overturn the overturn will be with respect to that when the pressure is from the side if at all over turn the overturn with overturn with respect to this point here this point here; so, that means, it will be overturning like this.

So, that means, when since all the vertical forces are acting this direction, so, it will be resisting force. So, that means, resisting moment  $M_r$  divided by overturning a ot. So,  $M_r$  will be how much; that means,  $W_1$  multiplied by what is the distance from here actually? It will be point 2.1 plus 0.3. So,  $W_1$  at a at much this much distance is acting plus  $W_2$  in multiplied by how much distance two third of 2.1, two third of 2.1 plus  $P_{av}$  multiplied by how much distance? 2.1 multiplied by at 2.7 multiplied by I am sorry that that will be at 2.7 that will be at 2.7 divided by overturning moment is what? Overturning moment is  $P$  multiplied by how much at a distance it will be one third of 5.

So, if I put all the values and then simplify and calculate then you will see that if I put all those values here and you can get the then you will get ultimately it will be 401.175 divided by 119.733. So, and that will come actually 3.35. 3.35 ok. So, this is actually factor (Refer Time: 17:44). So, it is quite good value. So, it is also looks like and whether it is or not what is the factor of safety desired that will be given and one can decide that what value actually required.

Now, whether or not if it is not ok; obviously, you have to change the dimension you have to redesign oh sorry and then next page actually I will go suppose you have to find out now the factor of safety against a base pressure and we can see now that we can find out that where actually CG is acting.

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$$\bar{x} \times P_v = \frac{w_1 h_1 + w_2 h_2 + P_v h_3 - P_a h_4}{P_v}$$

$$\bar{x} = 1.299 \approx 1.3$$

$$e = 1.35 - 1.3 = 0.05$$

$$M = P_v \times e = 217.25 \times 0.05 = 10.86$$

$$\sigma = \frac{P_v}{A} \pm \frac{M y}{I} = \frac{217.25}{27 \times 1} \pm \frac{10.86 \times 1.35}{18 \times 1.35^3}$$

$$\sigma_{max} = 80.46 + 71.53 \approx 152 \text{ kN/m}^2$$

$$\sigma_{min} = 80.46 - 71.53 \approx 9.00 \text{ kN/m}^2$$

Suppose, this is the base of the this is this is the one and different forces are acting at different point. So, I can find out that vertical pressure resultant vertical pressure acting somewhere somewhere here and then or in some angle actually acting pressure are maybe some angle and then I can consider vertical and horizontal and suppose these distance is suppose x bar.

So, x bar multiplied by; suppose, P v will be equal to that restoring moment minus overturning moment, so, that means, W 1 h 1 W 1 h 1 plus W 2 h 2 plus P av plus P av multiplied by h 3 minus P multiplied by h 4, suppose divided by sorry, these are this should be equal this to be equated and then I will get x bar will be equal to W 1 h 1 plus W 2 h 2 plus P av h 3 minus P a h h 4 divided by P v.

And, if I put W 1 h 1 I have shown in the previous page calculation W 1 h 2, h 3, h 4 also I have mentioned all those things there. So, I have put all those values and then I will get this value equal to 1.299 or so. So, for approximately suppose 1.3 now, if I consider now the base of the footing suppose this one and this is 2.7 base of the footing is total 2.7, this is one point 2.7 means this is 1.35 and; that means, force is acting somewhere here from this is 1.3; that means, e will be equal to 1.35 minus 1.3; that means, 0.05.

That means your m will become P v multiplied by e and P v actually we have calculated how much P v was whatever was there and that if you P v was 2 1 217.25. 217.25 multiplied 0.05. If I do that then it comes around 10.86 M become 10.86.

And, now I have discussed also bearing capacity time that when there is eccentric moment is active then I can consider the force eccentric force is acting that force can be concerned at the center in addition to a moment. So, when  $P_v$  and  $P_v$  and  $M$  are acting together then your sigma become  $P_v$  by a plus minus  $MY$  by  $I$  this is the formula. So, this formula if I use then you can see then it will be  $217.25$  divided by  $A$  is how much a is nothing, but  $2.7$  multiplied by  $1$  this is  $1$  and plus  $M$  become  $10.86$  multiplied by  $Y$  will be how much? This is this is maximum distance  $1.35$ .  $1.35$  and your this is rotated with respect to these axes. So, your  $I$ , so,  $I$  become the  $1$  multiplied by  $1.35$  cube divided by  $12$ .

So, then I can find out sigma max and I can find out sigma minimum if I calculate this one then it become  $80.46$  plus  $80.46$  plus  $71.05$  or  $70.5$ . So, that gives you  $151$  point at approximately  $152$  and sigma minimum will be  $80.46$  minus  $71.5$  and that gives you a approximately  $8.96$ ; that means, around  $9$  kilo Newton per meter cube ok.

So, that means, this side your pressure is  $152$  and this side is only  $9$  and you can see that from this distribution we could make sure that eccentricity actually small very small and eccentricity should not be greater than the  $b$  by  $6$  and  $b$  is this  $b$  by  $6$  whatever value calculate and you can see less than that that is one thing.

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The image shows handwritten engineering calculations and diagrams. At the top, there are three diagrams: a triangle representing a stress distribution, a rectangular area with a trapezoidal load distribution, and a rectangular cross-section with dimensions  $1.35$  and  $1.35$ . Below the diagrams, the following calculations are written:

$$\bar{x} \times P_v = \frac{w_1 h_1 + w_2 h_2 + h_v x h_3 - P_a x h_4}{w_1 h_1 + w_2 h_2 + h_v x h_3 - P_a h_4}$$

$$e = 1.35 - 1.3 = 0.05$$

$$M = P_v \times e = 217.25 \times 0.05 = 10.86$$

$$F.S.B.C. = \frac{\bar{x}}{S_{xx}} = \frac{1.299}{1.5} = 1.299 \approx 1.3$$

$$\sigma = \frac{P_v}{A} \pm \frac{M Y}{I_x} = \frac{217.25}{27 \times 1} \pm \frac{10.86 \times 1.35}{27 \times 1.35^3}$$

$$\sigma_{max} = 80.46 + 71.05 \approx 152$$

$$\sigma_{min} = 80.46 - 71.05 \approx 9.41$$



And, you can see that we want to make sure that there is no tension below the footing you can see here also this is also positive this is also called positive. So, in between also it will be positive.

So, if I draw the pressure diagram below the retaining wall the pressure diagram will be something like this. So, this will be maximum here and this will be minimum. So, this one will be 9 and this will be 152. So, that means, throughout the base of the footing it is positive value.

So, this is actually sufficient now the; that means, we have got the positive pressure throughout the depth. Now, if you want to find out factor of safety against bearing capacity suppose bearing capacity is given soil; bearing capacity is given suppose 500, ok.

So, factor of safety against BC bearing capacity suppose 500 divided by 150 152. So, whatever value it comes that will be quite close to suppose 3 2.523 actually generally desired so; that means, it will be ok. If it is not obviously, you have to change the dimension of the footing either base or height all those things to be changed and accordingly show that after analyzing these we get the expected value which is desired, ok.

So, like this we can we can solve the problem.

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**Stability Analysis of Earth Retaining Wall**

Investigate the stability against overturning, sliding and bearing pressure of the gravity earth retaining wall shown in the figure. The retaining wall is to support a granular backfill which has unit weight of 17.5 kN/m<sup>3</sup>, and angle of internal friction of 32°. The coefficient of base friction is 0.5. Allowable soil pressure for the foundation is 250 kN/m<sup>2</sup>.

$\sigma = \frac{P}{A} \pm \frac{My}{I}$

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Now, this is one type of problem I have taken, but this problem actually this problem actually I have taken like the slope. Instead of slope it can be only horizontal, now this is not there if it is horizontal here then your directly your  $P$  active was here there is no need of making component and in that case we will have only 1 2 and this is 3 only 1, 2, 3. This is one type of problem I have taken a little complicated; that means, I have taken slope backfill. So, because of that this pressure was like this and then I have taken component that are  $P_{av}$  and  $P$  and then I have to consider four forces and then I have done the stability analysis.

Sometime, instead of the vertical back the wall can be cross section of the wall can be something like this and maybe something like this. If it is so, then what I have to do? I have to while doing the stability analysis from the from this point I will draw a vertical line from here, these vertical lines. So, I will consider these as the wall, this as the wall and I will calculate  $P_a$  acting here and; that means, I will divide this cell point that 1 2 this is 3 and this soil weight also will be coming in the on the wall assuming that I have to I have to consider this weight of the soil also as a fourth part.

And, then rest of the analysis will be similar I have to find out factor of safety first of all vertical forces I will calculate and then vertical force multiplied by the friction, I will get the resistance that divided by the horizontal pressure that will give you a factor of safety against overturning sliding and then for finding out the factor of safety against overturning what I had to do all the forces 1, 2, 3, 4 all are actually this side. So, these are all restoring I had to find out CG of all those  $P_1 h_1$ ,  $P_2 h_2$ ,  $P_3 h_3$  like that I will do and overturning moment will be this  $P_a$  multiplied by these height and from there I will get the factor of safety against overturning.

Similarly, for finding out the bearing capacity I have to find out the resultant point of application if it is not on the CG then I had to find out the eccentricity then find out movement then apply the equation  $\sigma = P/a \pm MY/I$  by this equation you have to find out  $\sigma_{max}$  and  $\sigma_{min}$ . So, that is another type of problem.

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**Stability Analysis of Earth Retaining Wall**

Investigate the stability against overturning, sliding and bearing pressure of the gravity earth retaining wall shown in the figure. The retaining wall is to support a granular backfill which has unit weight of  $17.5 \text{ kN/m}^3$ , and angle of internal friction of  $32^\circ$ . The coefficient of base friction is  $0.5$ . Allowable soil pressure for the foundation is  $250 \text{ kN/m}^2$ .

Granular Backfill  
 $\gamma = 17.3 \text{ kN/m}^3$   
 $\phi = 32^\circ$

$\mu = 0.5$   
 $q = 250 \text{ kN/m}^2$

Dimensions:  $0.6 \text{ m}$ ,  $5.0 \text{ m}$ ,  $2.7 \text{ m}$ ,  $1.0 \text{ m}$

Handwritten diagram:  $5$ ,  $h_{\text{mod}} = h + h \tan^2(45 - \phi/2)$

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Then another type of problem also possible that is actually suppose the wall is similar; suppose, like these like these like these and in addition to that it is slope backfill. Here actually what I have to do I have to draw it line and I had to go up to the slope then suppose the wall height was something like suppose 5 meter and now this slope angle was something, then I have to consider this height also. Now, height  $h$  modified I have to calculate which will be equal  $h$  plus this height  $h$  plus this height. So, this height how I will find out I know this, I know this angle, so, from there I can find out this.

So, if this is the type of problem then again I will do similar way 1, this is 2, this is 3 and this will be 3 this will be 4 and he can ignore either this point or you can consider another part 5 also and like that 1, 2, 3, 4, 5 component will take and then the rest of the calculation is similar.  $P$  will be acting somewhere at a distance  $h$  by 3 and then find out resistance then resistance divide by a horizontal pressure will be factor of safety against sliding. Then you can find out restoring movement, find out overturning movement, ratio of that will be factor of safety against overturning.

Then, find out the CG the point of application of the resultant force; if it is not in the middle find out the eccentricity. Eccentricity multiplied by vertical force will be your moment once you get the moment it will be  $\sigma_{\text{max}}$   $\sigma_{\text{min}}$  will be equal to  $P$   $v$  divided by area multi plus minus  $MY$  by  $I$  and while calculating I actually always you

have to calculate these as base and this has a height, because it is rotating with respect to this access.

So, like that whatever may be the type of problem you have to the problem I have solved if additionally this type of slope is their backs back slope is there then what I will do from the hills wine I will draw a vertical line and after the ground surface I will be grow and then I will see the level backfill then height will not change. If the slope backfill height of the wall will be modified, it will be more than the actual height that to be calculated and the considering that height of the wall you have to calculate earth pressure and then all the things otherwise a rest of are similar.

So, I will take another problem also in the next class maybe and then I will show some more that whatever if it is how to modified height and all those to be calculated and stability analysis to be done that I will show in the next class.

Thank you.