Geotechnical Engineering II / Foundation Engineering
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Lecture - 31
Earth Pressure Theories (Contd.)

Good morning. Once again I welcome you all to this lecture Foundation Engineering and we are discussing on Earth Pressure Theories. And in fact, our main objective is to do stability analysis of the earth retaining wall and I have mentioned you before that the main component in earth stability analysis of the retaining wall is the earth pressure calculation.

There are other loading, but earth pressure is one of the most important component and first of all you need to know how to calculate the earth pressure and where they act. And for that I have already discussed ways method, Rankine's theory columns theory and now and most of the cases via what I have discussed here so far it is for we cohesion less soil. And; that means, backfill material is sand mostly and but sometimes pure sand may not be the situation most of the cases.

So, you may have to use sometimes c phi soil also, and if we use c phi soil then how to calculate earth pressure behind the wall under different conditions like active and passive case. So, we will see that first and then subsequently we will see some other applications.

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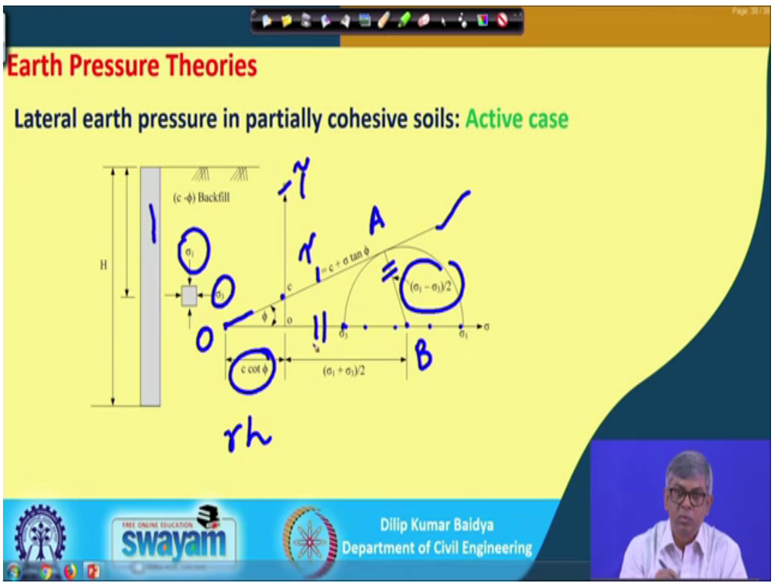

So, let me see that this is a case of active earth pressure and we already we have we know that this is the and this is wall and in fact, let me show, this is actually behind the wall and this is the backfill and c phi soil. As usual I can find out sigma 1 by normal procedures sigma 1 is nothing, but gamma multiplied by h.

And then I had to find out sigma 3 which is nothing, but here lateral pressure or earth pressure and so, if the soil is c phi then if you plot in a tau verses sigma or s or tau you can s also you can think of tau actually tau verses sigma or shear strength versus sigma. If you plot and then generally you will have the envelope it will be will be intersect at tau axis and if we extend it will meet somewhere here.

Now that mean this is the failure envelope, this is the failure envelope and if full active case develop in this case then corresponding to that there will be more envelop; Mohr circle and that this may be the Mohr circle for that condition. So; that means, this is originally sigma 1 and with the movement of the wall away from the backfill sigma 1 was decreasing suppose, somewhere here or then somewhere here like that finally, at exactly at fully active condition sigma 3 sigma 1 reduces to these values suppose this is sigma 3.

So, this sigma 3 is nothing, but the lateral earth pressure in the wall in active condition. So, this circle will be tangential to this. Now, if I use this failure envelope on this Mohr circle and then from these actually if we write this tau this also has to be written as tau. So, if I use these geometrically and you can see that this is the center of the circle and then if I join that here at where it is tangent. So, that become the radius and then this radius will be actually this is sigma 1 minus sigma 3 by 2 definitely and again we can find out this distance as sigma 1 plus sigma 3 by 2 this distance and of course, c is this one if a consider these as a c, this portion as c this is phi. So, this distance will be c cot phi.

Now using this geometry and we can consider this we this triangle, suppose this is O A and B this triangle I have take I if I take and if I express sine phi then sin phi by you will be these divided by this. So, that is the thing I will try to do in the next slide.

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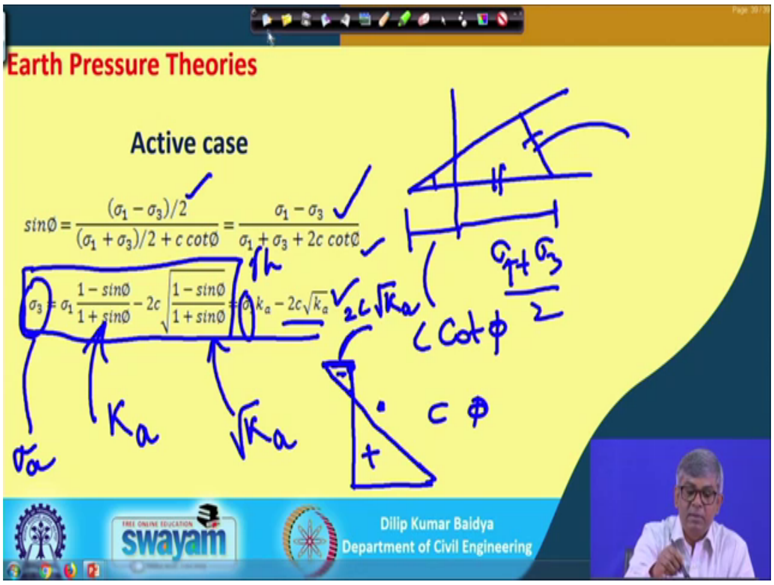

You can see here that we have expressed sin phi and that is radius by this up word news actually here and if you see that this is the radius and this is the; that means, if I draw the diagram it was something like that and it was something like this. So, this is radius and this is this distance was sigma 1 plus sigma 1 plus sigma 3 by 2 and this was c cot phi.

So, that is why; so, these our sin phi is this is phi sin phi is these divided by this. So, if I do that then sigma 1 minus sigma 3 by 2 sigma 1 plus sigma 3 by 2 plus c cot phi and if you simplify further it become like this. So, sigma 1 minus sigma 3 by sigma 1 plus sigma 3 plus c 2 cot phi and now if I substitute then and, if we express sigma 3 because our I know sigma 1 because sigma 1 is nothing, but gamma times h. And I need to find out sigma 3 and; that means, I want to express sigma 3 in terms of sigma 1 c and other parameters like k a k p whatever maybe.

So, if I simplify this one and express sigma 3 then it will become in this form. So, in this form sigma 3 will be sigma 1 1 minus sin phi by 1 plus phi minus 2 c under root 1 minus sin phi by 1 plus sin phi. And you can see this is nothing, but we know k a already we have done and this is actually nothing, but root k a.

So, I can write that sigma 3 will be equal to sigma 1 k a minus 2 c root k; that means, so at any depth suppose this is the wall and it contains c phi soil here, then if I want to find out the pressure at any depth, then what I have to do? I have to find out gamma times a sigma 1 multiplied by k a that is normal one and it has to be reduced because of this cohesion part; that how it will reduce? 2 c times under root k a.

So, we can and if it is suppose if I want to find out a surface; that means, when your normal stress; that means, sigma 1 is 0 then you can see still there is a value minus 2 c root k a so; that means, at the surface there will be minus value there will be a minus value which will be called to minus 2 c root k a. So, this will be there at the surface and when you go deeper and deeper this minus will be reduced because when h is some small value so it will be plus and this is minus which will be reduced.

And add some depth at some depth it will become 0 and then finally, when it becomes further increase the depth then it will become positive value like this. So; that means, for this case when soil the wall retained back, wall retained c phi backfill then in that case your earth pressure diagram will be like this. This portion will be negative and this portion be positive so; that means, of course, this will give you thrust in this direction and this will give you tension. So, that I will discuss later on what is the effect of it.

So, for the time being what I want to mention here actually to make clear that if the backfill retains c phi soil how to find out lateral pressure; that means, this is sigma 3 is nothing but I can say here; I can say here sigma active sigma 3 here sigma active. And, your this will be in terms of sigma 1 k a; that means, sigma 1 is nothing, but gamma times h. Of course, if it is a water table is there gamma summers time h minus 2 c root k. So, this is the way one can find out the lateral pressure for c phi file.

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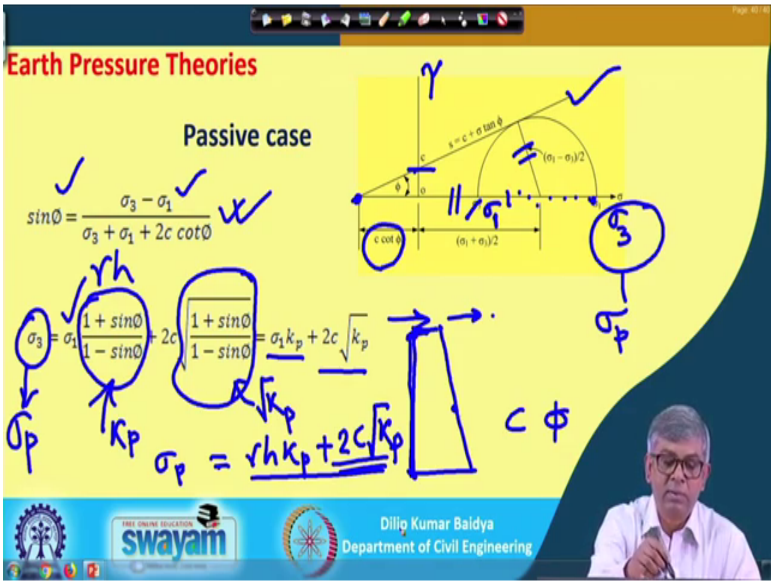

Similarly, if I take a passive case it will be similar actually in fact it will be reverse of active k that if the wall is somewhere here and I returning c phi soil the wall moves in this direction when wall move this direction then initially there will be there that we have sigma. So, that should be actually in terms of our understanding this must be sigma 1 because that is known non vertical stress and this will be slowly, when wall moves this direction while moves this direction this value increase. And finally, it reach to a maximum value before failure so that suppose this is the one that must be suppose sigma 3, because that is that is the one though magnitude wise this is sigma 1 that is larger.

But I am here sigma 3; that means, which is unknown initially sigma 1 as applied it is there and then with the movement of the wall pressure is increasing it is becoming sigma 3 and that sigma 3 actually is the nothing, but sigma passive sigma passive the earlier cases sigma active and single passive. So now, this will be a Mohr circle and if I draw this Mohr circle tangential to the envelope of the failure envelope the soil.

And I know that this is a failure envelope and this is generally when c phi soil the failure envelope will be intersecting this is suppose tau axis and it will intersect at a c where this must be the value of c and then if I extend this line this is the this is the failure envelope we extend it will intersect the x axis somewhere here. That means, sigma axis some over here and that distance if it is phi then I can find out from the geometry c cot phi.

So, again similar to that if I do it here then you can see here that sigma 3 here actually bigger sigma 3 for these cases week. So, bigger value so, sigma 3 minus sigma 1 similar to that I if I take this triangle this triangle and then sin phi will be these divided by these and if I do that and simplify then this two will get this equation. And once you get this equation, once you get this equation; once you get this equation then go further simplify to find out sigma 3 because which is unknown to us.

So, sigma 3 here I have expressed and sigma 3 here actually nothing, but sigma p passive pressure and this will be when you expressed, what in terms of what I should express? I should express in terms of sigma 1 which is known which is equal to gamma times h and if it is water table is there it is gamma summers time h and this become 1 plus sin phi by 1 minus sin phi plus 2 c multiplied by under root 1 plus sin phi by 1 minus sin phi.

Now, we can see this, this is nothing, but k p and this one nothing, but under root k p. So, this one again for the simplification I can do and I can write this sigma 3 equal to sigma 1 k p plus 2 c rate root k p or I can write gamma k p plus 2 c root k p. So, when there is a c phi soil the pressure diagram pressure expression sigma p sigma p will be a lateral earth pressure or lateral passive pressure is I can be express like this.

So that means, if I height is 0; that means, that at the surface that are these are the surface of the top surface of the wall I want to find out the pressure you can see this part become 0, but still there is a positive parts. So; that means, when c phi soil passive case it will start with a positive value and then if I h increases then this will be further increase the positive value; that means, your pressure diagram will be like this.

Whereas, we have seen that when it is the active case at the surface there is a negative value and then it will increase further and it with a positive value, but for passive case it is throughout positive with at the top surface some value and at the bottom surface some other value which higher magnitude. So, this is actually for earth pressure diagram sorry procedure for calculating active pressure and passive pressure for your c phi soil when the backfill retain c phi soil. So, two important expression you have to remember, that is actually I just go to the next slide and I will show you.

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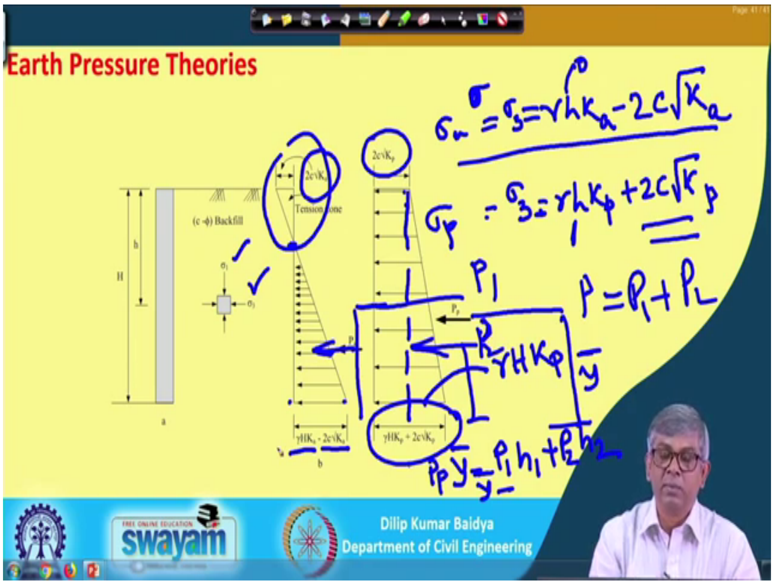

You can see that as I have shown before that if this is the wall and at height actually this is sigma 1 and this is sigma 3 and then if I write sigma 1 or I can say sigma 3 as sigma 3 when it is active it become gamma h k a minus 2 c root K a and when it is passive; that means, sigma 3 will be called to gamma h k p plus 2 c root K p. If I keep side by side this is a difference I can see you can see here and this is nothing, but this is nothing, but sigma active and this is nothing, but sigma passive.

So now, with the variation of h depth if I want to draw the pressure distribution earth pressure distribution over depth then for the active case you can see as I mentioned that when h become 0 then I will get this whole portion will be, the whole part of the 0 and only value will be minus 2 c root K. So, that is actually that the surface we will get a value negative value 2 c root K a and when h increases then this minus plus.

So, it will be negative value will be decreasing and at some time when negative value and positive value are equal then it will become 0 and when further increase average it will give you a value at the bottom of the wall something like that gamma h root K a sorry gamma h k a minus 2 c root k a.

So, this is actually earth pressure distribution, active earth pressure distribution for c phi soil behind the wall. Similarly if there is a passive case then you can see here that when you h put h equal to 0 then entire thing will be 0; that means, your value will be at the surface will be 2 c root K p and when you increase h then from here this will be constant throughout the depth. Suppose and then assuming this part is missing, this part is missing this is a constant part only with the increase of h these value will be increasing linear do this part will be there.

So; that means, you have this part this part 2 c root K p and at this part this part will be gamma h K p and these two together will have the earth pressure at the bottom. So, now, if this is the situation you know the pressure distribution, then if you want to find out the thrust then what I will do? I will find out the area of this diagram. So, this area I can divide into two parts, for this rectangular part I will do suppose this P 1 and triangular part I can say P 2 and then p will be equal to P 1 plus P 2. So, total thrust will be called to P 1 plus P 2 so, like that I can calculate.

Similarly you can find out the point of application of that earth pressure how to find out for P 1 I can find out what is the hone for P 2 also I can find out what is the h 1 this is our P 2. So, this is h 1. So, finally, I can take and P p, suppose this is acting and this is hide actually suppose y bar then what I can do P p multiplied by y bar will be equal to P 1 into h 1 plus p 2 into h 2. So, this way so, everything is known P 1 h 1 p 2 h 2 P p everything is known only unknown is y. So, I can find out from this equation.

So, that way actually one can find out that total thrust then part of application of the, similarly for this case generally negative part ignore we find out the area of this diagram and this will active at one-third hide from the base of the triangle. So, that is the way on can find out.

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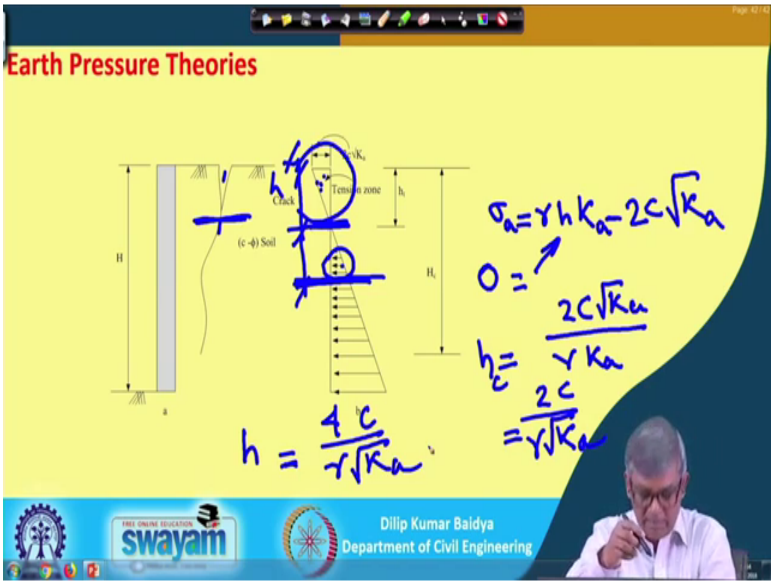

Next one is as I mentioned that when the retaining wall retains a backfill with c phi then we have seen that at the surface we are getting negative pressure, but negative pressure file cannot take. As a result this is nothing more tension and because of that tension there will crack will form and you can see this is the thing I can visualize that because of c phi soil it craft crack will for and it will go some up to some depth, up to what depth this crack will go. That means, were actually pressure will become 0 you can find out and; that means, you when we excavate generally we need to give support, but when it is c phi soil and because of these if you discovered it this will pressure is getting released.

So, because of that there is no need to put support actually, up to some deep depth actually up to certain depth we will can excavate the c phi soil without any support. So, if you want to find out up to what depth you can excavate without support that is the thing I can this analysis can be utilized. So, you can see here that first of all I can find out at what depth you have tension become 0. So, I know that your formula is, our formula is sigma active will be equal to gamma h ka minus 2 c root K a this is the expression c phi say that sigma become 0. So, if I equate this two 0 then from here I will get a value of h from here actually if I equate to 0 then I will get h equal to 2 c this is the 1 h equal 2 c root K a divided by gamma K a so; that means, it will be 2 c by gamma root K a.

So; that means, at what depth; that means, I have set I assume that at a depth h your pressure or tension becomes 0; that means, this is the 1. So, this is the 1 h. So, and I have expressed the general expression for sigma lateral pressure this and I equate this one to 0 because I have assume that h depth that becomes 0 if I equate to this expression 0 then from simplify I get h equal to 2 c y gamma root K a. That means, up to this there actually tension crack will go, but since this is a pressure release because of earth excavation; that means, equal to this double the amount if I cut also without support it can stand. So, that is why. So, depth of unsupported cart will be just twice of that.

So, this is actually just depth of tension crack. So, if I release this one so; that means, these minus and plus this resultant become 0. So, because of that though we could find out that depth at which the pressure is becoming 0 is the 2 c by root K a, but depth up to which we can cut without support will be twice that depth; why it is so? Because this minus diagram on this plus diagram if I go h here. So, this diagram on this diagram will be identical and then this will be with the increase of h from here it is negatively increasing and with the increase of h it is positively decrease it this way. So, this diagram and this diagram will be equal.

So, because of that so, they resultant so this is minus and this is plus. So, because of that result and pressure with 0 and; that means, up to that if you cut; that means, wall is not giving any vacuole is not giving any pressure on the wall so; that means, up to this depth we can excavate without any support. That means, h c h c is a critical depth and h cut or excavation without support will be that be twice if I do; that means, will be four c by gamma root K a ok. So, this one I will show you in detail in the next slide again and you can see here.

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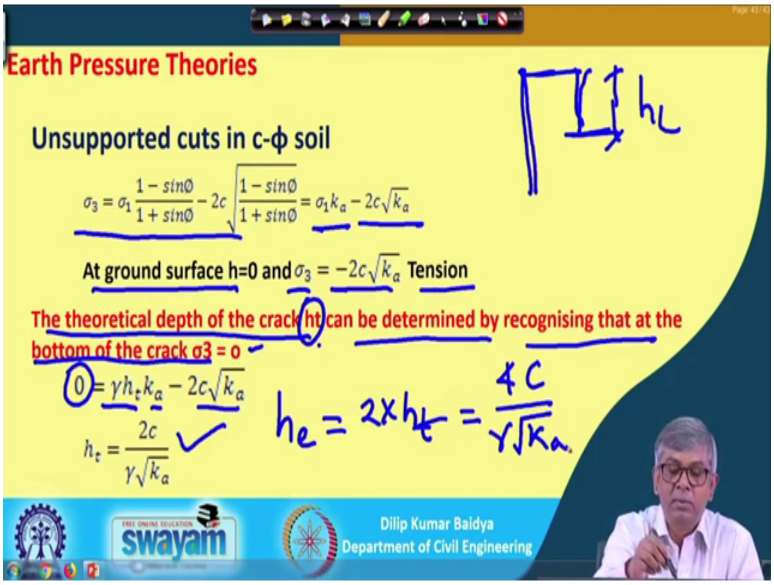

So, that s the thing I have done here you can see that our general expression for sigma 3 equal to sigma 1 gamma k a minus 2 c root k a this is a formula, at ground surface h equal to 0 and sigma 3 become minus 2 that is tension that already I mentioned and the theoretical depth of the crack ht theoretical depth at which that tension crack will reach that I suppose give a different name ht can be determined by recognizing that at the bottom of the crack sigma 3 will be 0. That means, that this is the wall and this is the ground surface and crack something like this it comes.

So, this is the depth of tension crack. So, h t if I share. So, that ht; that means, at this depth happens sigma 3 will be 0. So, this 0 to be equal to gamma ht k a minus 2 c root k a I do, then from here I can get ht equal to 2 c by gamma root k a. So, depth of excavation should be depth of excavation that when h excavation will be without support will be twice of ht so; that means, equal to equal to 4 c by gamma root k a. So, this is the way one can find out. So, these are useful for some earth pressure calculation also later on will show the use of it.

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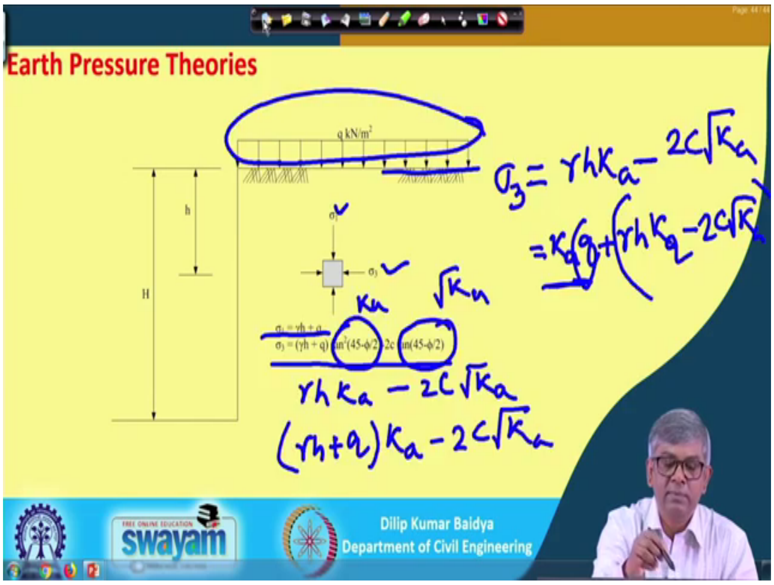

Next part is that in addition to these sometime there maybe some surcharge s there may be surcharge s pressure this is the ground surface you can see this is ground surface and on that ground surface additionally if there is a pressure. How we can visualize this one during excavation or some construction work there will be some loading truck or there will be excavation equipment or something else is there. In that case you can visualize there is some amount of distributed load acting on that surface, if it is there then how to consider that?

So, that actually so, this is sigma 1 because of this loading and this is sigma 3 is the lateral pressure, sigma 1 actually we can visualize as sigma 1 as gamma h plus q and then sigma 3 can be visualized as sigma 3 is a gamma h instead of gamma h k instead of gamma h k a this is nothing, but ka and this is nothing, but root k a this is k a and this is root k a. So, gamma h k a minus 2 c root k a 2 c root k a, this is the one we can visualize, but when there is a surcharge I can visualize as gamma h plus q plus q k a minus 2 c root k a.

So; that means, I can what way it is different? That means, one additional term so, sigma 3 was sigma 3 was sigma 3 was these gamma h k a minus minus 2 c root k a. Now if I simplify this one it is becoming k a multiplied by q plus gamma h k a minus 2 c root k a so; that means, this part was there already, now because of this q this is additional part is coming. That is actually throughout the depth it is independent of depth. So, if I draw the pressure diagram so; that means, if I draw the diagram for c phi soil and then additionally if I give the q times k a another constant throughout the depth than that is the total picture diagram for the surcharge case. So, this can be seen in the next slide let me see.

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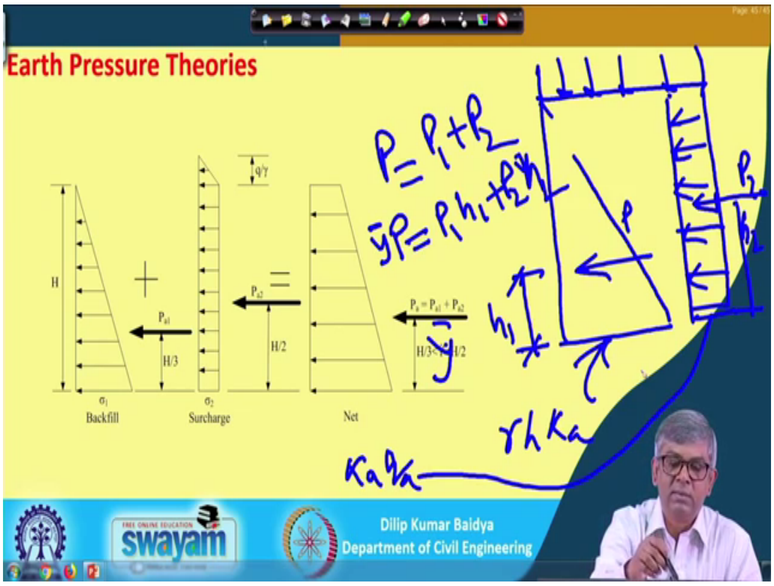

You can see that, you can see suppose that was the surcharge was there I will draw that one again; suppose this is the wall and this is there is a surcharge there is a surcharge and suppose this is a suppose cohesion less soil I will take first. So, cohesion less soil if it is a cohesion less soil then pressure diagram will be something like this gamma h ka. So, it will be gamma h ka is it not and in addition to that I can find out the effect of surcharge, what I will do effective surcharge? I will do this diagram, this will be equal to this value will be equal to this value will be equal to throughout everywhere it will be k a times q a.

So, now, if I want to find out the thrust then what will happen? So, this triangular diagram this will be somewhere P 1 acting somewhere here and this is suppose P 2 acting somewhere here P 1 and this is P 1 and this is P 2. So, P will be equal to P 1 plus P 2 and resultant actually I can find out y bar multiplied by P equal to P 1 h 1 plus P 2 h 2 h 2. So, P 1 h 1 means this is h 1 and this is suppose h 2. So, and y bar actually suppose is the resultant thrust somewhere here this is the one resultant thrust actually somewhere actually this is the resultant thrust. So, that become y bar suppose if I consider.

So, that way actually we can find out the thrust; so, what is the thing actually now to note. If it is a surcharge then only this is the additional q times k a and this is the one I have considered cohesion less soil, if instead of cohesion less soil if it is c phi then corresponding diagram I would have door drawn in addition I could have drawn this. So that means, so first of all without surcharge you draw the pressure diagram and then add another diagram which will be throughout the depth pressure intensity cost and which is equal to k a times q which is be equal to like this and then sum it then you can find out the resultant pressure and resultant thrust and resultant all those things ok.

With this actually almost earth pressure theories I have whatever relevant I have just completed. And now I will try to show the calculation; that means, earth pressure calculation through some application and then stability analysis ok.

Thank you.