Geotechnical Engineering II / Foundation Engineering
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Lecture – 30
Earth Pressure Theories (Contd.)

Let me continue with the Earth Pressure Theories once again. And so far, I have discussed about different cases of earth pressure like at rest, active condition, passive condition and how to find out active earth pressure, passive earth pressure by Wedge method and by Rankine's theory. And both the cases actually we have considered the smooth vertical wall and level backfill and but actually the backfill will never be mostly will not be level it can be inclined; particularly in the hilly areas when the soil will be retained by the wall always it will be inclined.

So, to find out pressure earth pressure for those cases that will have minor modification will be required and perhaps we can extend the Rankine's theory and by that we can find out.

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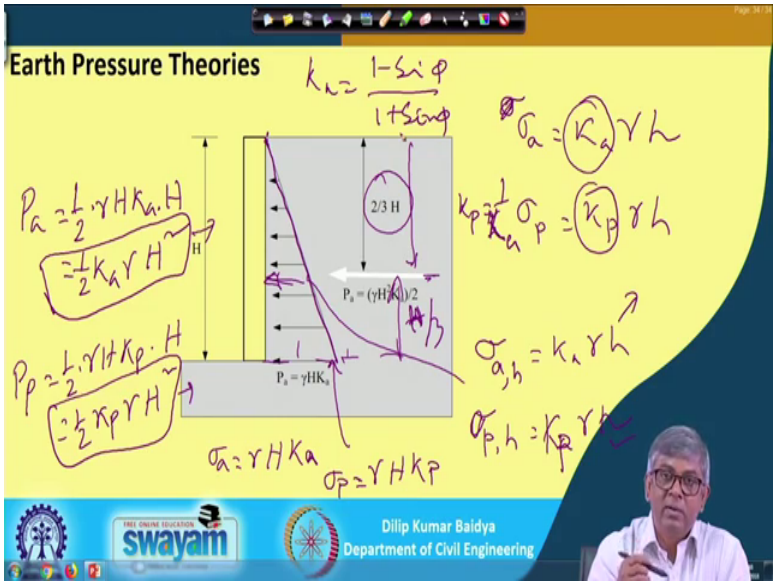

Let us see one by one that before going to that I have mentioned that earth pressure theories that is K a that means, we have mentioned for active case sigma or I can say sigma a suppose, active K a times gamma h. And if it is sigma passive we can write K p gamma h. So, gamma h will be common only change K a and K p.

So, if this is a wall height H and then I will have pressure at any depth if I want to find out pressure at depth H, so, sigma a at h then it will be K a time gamma times h. And similarly, if I want to find out sigma p at h that K p times gamma; K p times gamma h.

So, here actually we can see now if I put h equal to 0 then it becomes 0 and h become any values of 5 or 10 or 20. So, or in between if I value the value then it will be linear. So that means, your pressure diagram, earth pressure diagram, lateral earth pressure diagram supposed to be a triangular diagram like this. And then if this is a triangular diagram, so at any depth that means, at the bottom most that means, if the wall height is H then if it is a sigma a then your pressure will be gamma H K a. And if it is a passive case then sigma p will be equal to gamma H K p. So, whatever whether it is active or passive accordingly you have to change this.

Now, if I know this distribution like this and if diagram become triangular then I can find out what is the total thrust on the wall. How to find your thrust area of the triangle? Then that means, so I can find out P a equal to a half and this will be gamma H K a multiplied by again H. So, it will become half K a gamma H square. Similarly, if I want to find out P p then it will be half gamma H K p multiplied by H, so it will be half K p gamma H square. So, this is the passive thrust, this is active thrust.

So, calculation is same vertical calculation we know vertical pressure calculation we know, if I consider a depth H then you need to weight of the soil up to that to be considered and then finally, you know the diagram and then you can find out the area of the triangle they become. So, this is actually triangle since the triangular diagram then this is whether it is K a or K p, where it will act actually? It will act you know the through the centeriod of the triangle.

So, if I from here actually it will be at one-third height from the base or two-third height from the vertex so that means, this will be either two-third H or here one-third H, H by 3. Whether its active or passive the point of application of the thrust will be the same, that means H by 3 height above base or 2 by 3 H from the vertex. So, this is the one thing to be known.

Second thing is either it is whether K a or K p only you have to change the notation and accordingly you have to use either K a or K p, and if you know the value of phi then we can find out K a equal to 1 minus sin phi by 1 plus sin phi and K p will be equal to 1 by

sin phi 1 by K a; 1 by K a. So that means, whether it is active thrust or passive thrust only you need to know phi, find out K a, find out K p and then by using these you can find out thrust and point of this value of thrust will magnitude of the thrust will be different for active and passive case, but point of application of the thrust will be at the same location that means, H by 3 above the base of the footing. This is the one thing to be remembered.

Similarly, if this is the dry granular (Refer Time: 06:31) soil then this is the procedure, but if it is layered or some other the water table changes is there then it will be little different that has to be considered as everywhere it is in terms of effective stress. So, for that you have to consider effective stress only.

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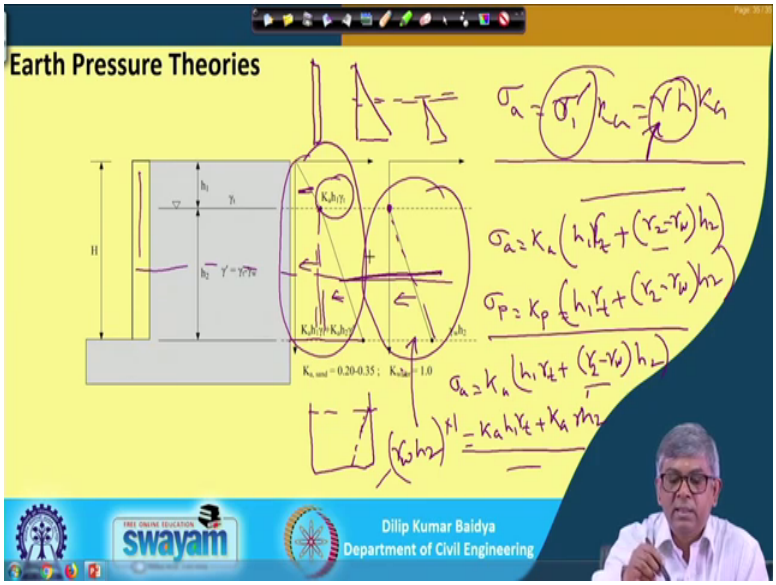

For example, so this is the wall and this is the wall and water table is here though it is same soil granular soil, but water table was here. Now, previously it was dry, but here it is water table is there. So that means, I can find out K a K p at the same way it will not change it will not depend on earth pressure water table condition, it depends on the soil type only, but your earth pressure diagram will change total thrust also it will change because of this water table location.

How it is actually? Anytime, whatever you do whether it is sigma active equal to it is nothing, but sigma 1 dash K a that means, whatever I am telling initially gamma h K a, this is gamma h actually when it is submerged then we have to take (Refer Time: 07:48)

unit weight that is the thing actually. So that means, water table when it is there then accordingly you have to calculate the effective stress, vertical effective stress accordingly.

So, for example, up to this it will be K a h 1 gamma t, but when you will go beyond this then what will happen suppose at a height h 2 from here then it will be your sigma active will be equal to K a times your h 1 gamma t plus. Now, from here actually it will be gamma 2 minus gamma w into h 2 that become effective stress at this point. Gamma 2 minus gamma w into h 2 is the at this point and multiplied by gamma this.

Similarly, if I want to find out sigma passive, so, sigma p will be again same thing only this will change multiplied by h 1 gamma t plus gamma 2 minus gamma w h 2. So, that become (Refer Time: 09:07). So, now, if that means, if I want to find out pressure at this point what will be there? Sigma active will become K a times h 1 gamma t plus gamma 2 minus gamma w multiplied by this height actually suppose this is suppose h 2 from here to here h 2, so again it will be h 2.

So, this is the way actually we can do or I can write K a h 1 gamma t plus K a this is gamma sub merge h 2 gamma submerge h 2. So, similarly if it is K p it will be accordingly. So that means, you have to keep in mind that if there is water table you have to take effective vertical stress then multiply by K a to get the lateral earth pressure active earth pressure or lateral passive pressure. So, this is the way we have done for active pressure because of the soil where this is the pressure.

But here you have to remember one more point that since water table is there and because the water also will give pressure, suppose if you in a tank there is some water then water also will give pressure on the wall at the base, so water pressure also increase like this. So, because of that from here actually since it surface actually it will be 0 and when we will go from here actually h 2 depth. So, at this point your water pressure will be gamma w multiplied by h 2 and here actually supposed to be multiplied by active or passive earth pressure for water actually active pressure or passive pressure coefficient is 1. So, it will be full multiplied by 1 actually.

So, when it will be a normal cohesion less soil, then your if this is the wall suppose and then your active or passive pressure diagram will be something like this. But when it is a submerged that will be two layer or water table is there then you will have these in

addition to that if there is water table from here then because of that water pressure that triangular diagram to be considered that also will be the exerting pressure on the wall. So, the area of this triangle plus area of this triangle will be the total thrust of the on the wall, that is one thing.

Second thing you have to find out the point of application of the thrust. So that means, I have to find out now I know point of application for this point of application for this. So, I can divide into a number of parts now, point of application this, point of application of these, then point of application is this so that means, different components. Now, I know the cg of different areas from convenient areas like rectangle, square, triangle these are the areas we know trapezium also know, but it is difficult to remember. So, your diagram is something like this let me erase and then I will once again I will say.

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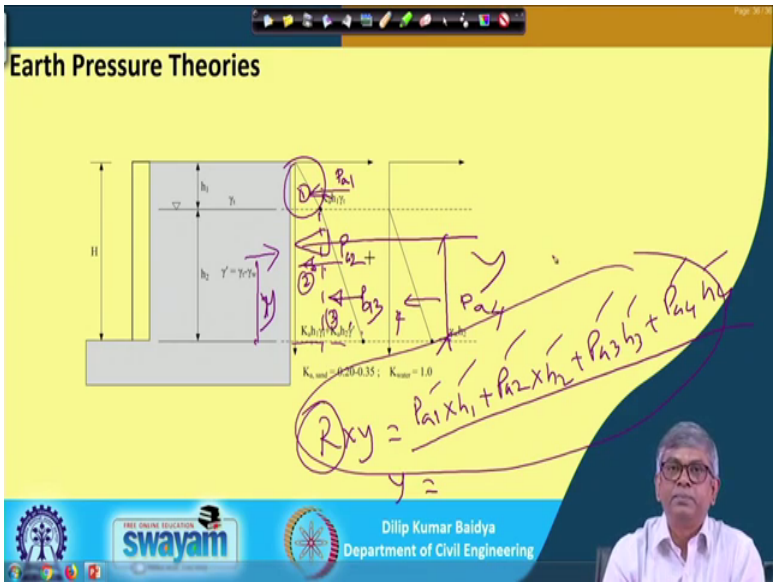

So, if this is the diagram actually, if this is the diagram we can see I can divide into this is part 1, I can divide in to part 2, I can this is 3 and this is 4. Now, I can consider at this one as P a 1, I can consider these as P a 2, I can consider these as a P a 3, I can consider these as a P a 4.

So, point of application of for all cases I know, suppose I want to find out the point of application from the base, so I know it is at what height. I find out area of these multiplied by these height I will multiply then I know this area and at what I did is acting

I know. So, this area we have multiplied by this height I will do. Similarly, for this area at what height it is acting I can know I know.

So, I will find out the area of these multiplied by these height plus, I know the area of these and at what height it is acting I know, so I will multiply by these then all I will be summing up that is these are all giving moment in direction and I will consider that suppose P a acting this way at a particular height suppose y.

So, that R multiplied by y will be equal to P a 1 multiplied by h 1 plus P a 2 multiplied by h 2 plus P a 3 multiplied by h 3 plus P a 4 multiplied by h 4. So, like that if I do and all are known. So, P a 1 h 1, P a 2 h 2, P a 3 h 3, P a 4 h 4 all are known, only unknown and R also known. R is what? Summation of all diagram, area of this diagram. So, all the unknown is y.

So that means, finally, if the diagram is like these irregular then I have to find out the resultant point of application. And what is the total thrust? Area of these diagram plus area of these diagrams is total thrust. And what is the point of application of the thrust? You have to find out this way that means, you have to come divide into number of parts their cg location you have to find out and then by summing the entire area I will get that total thrust and then taking moment like this all areas with respect to base and with respect to resultant of the with respect to base equate them then we will get the value of y. That means, finally, you may find out the thrust is acting somewhere here that will be your y ok.

So, this is the way so that means, when it is the like water table is there because of that earth pressure is changing. Similarly, there may be two layers, one layer is phi 1 another layer is phi 2, then your gamma 1 h 1, gamma 2 h 2 like that you have to do. So, because of that the diagram maybe irregular, but ultimately when it becomes irregular then you have to divide with number of parts of known areas and their cg and then find out resultant by this way ok. So, let me proceed to the next thing.

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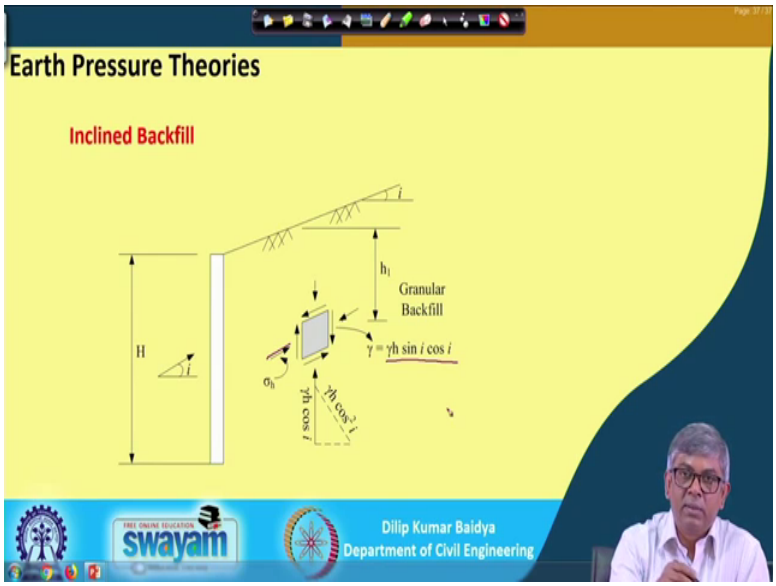

So, that is one will show some application actually by solving problem. So, before that the so far what we are considering that level or horizontal backfill horizontal backfill, but if the backfill is something like this inclined if the inclination angle i, then we can imagine the failure is like this. And in that case they are not this is actually gamma h sin i cos i and this on that plane this is the force stress this is the force the force. So, by considering this geometry of these, one can extend the Rankine's theory to find out the value of K a K p for this inclined backfill.

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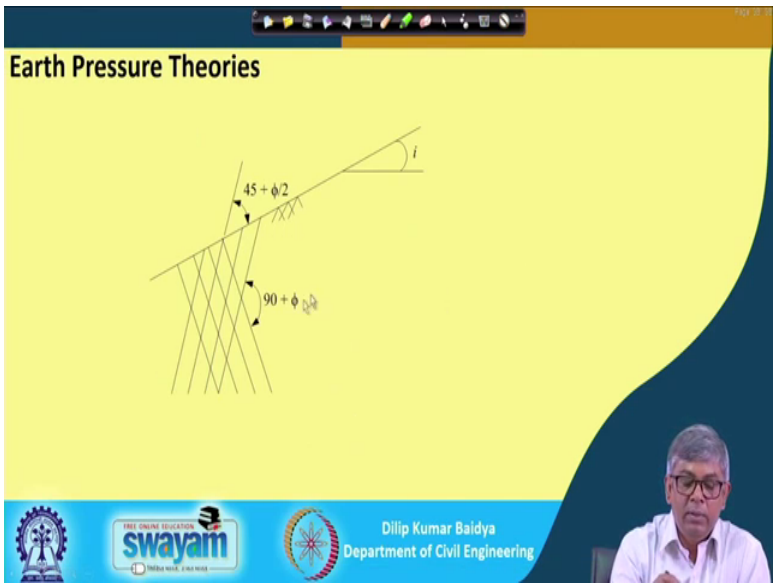

So, for that we can see, we can failure plane will be accordingly, so this is the one, so, with this with sorry. With respect to these, with respect to this is with respect to this will be 45 degree plus phi by 2 or 45 degree minus phi by 2 so that is the shown here.

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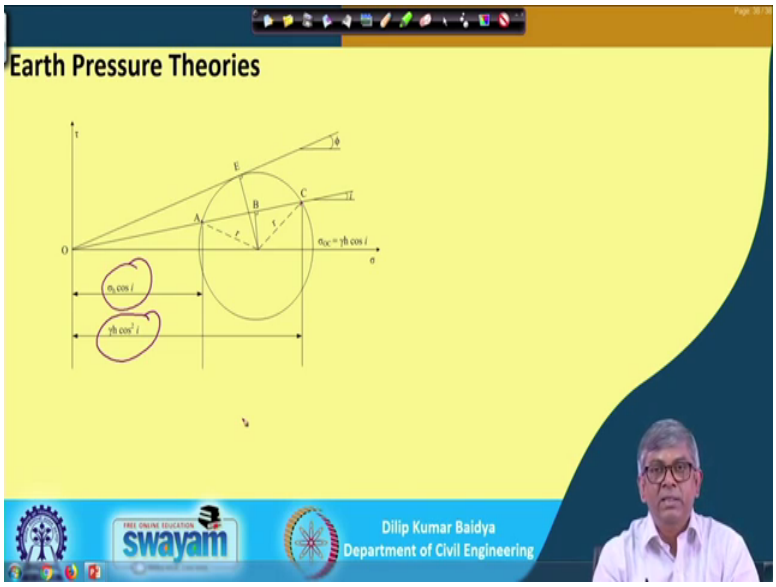

And next is and you can see now if I draw know the failure envelope and the Mohr circles and if I draw a line with angle i then it is intersecting to point actually and their values actually you can find our which is shown in the diagram sigma a cos i gamma a cos square i. And accordingly using this I can establish what is the value of sigma a K a and K p. Let me go to the next slide.

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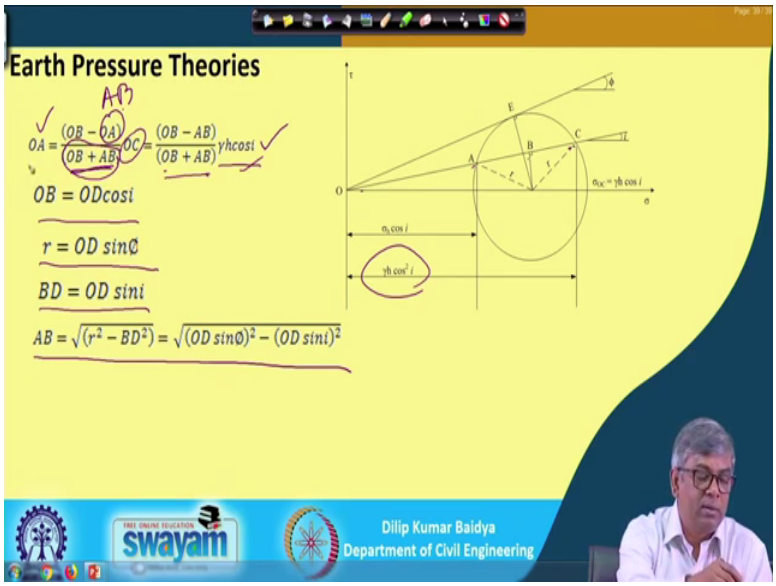

If you see this again once again I will take this one and you can see here that OA equal to OB; OB minus; OB minus AB actually, this is actually AB. So, it is wrong. OB minus AB actually, OB minus AB, so it becomes OA and then OB minus OA and then below actually I have multiplied I have written OB plus AB, OB plus AB and this is nothing but OC. See these distance and these distance will be same, this distance will be same this distance will be same. OB plus AB I have taken and that is in OC, so I have divided here and multiplied here, so ultimately this OA equal to these I can write.

So, if I write these, so OB AB by OB AB and gamma h cos. So, gamma h cos i. OC is, OC actually is your gamma h cos square i. So, cos i we have written and OB equal to from the geometry we can find out r equal to these, BD equal to these, than AB equal to this. So, these are all I am getting geometrically and this is of course from the soil mechanics you might have learned in detail but may not be required, but I am just for reviews revising actually I am taking this one. So, let me go to the next slide.

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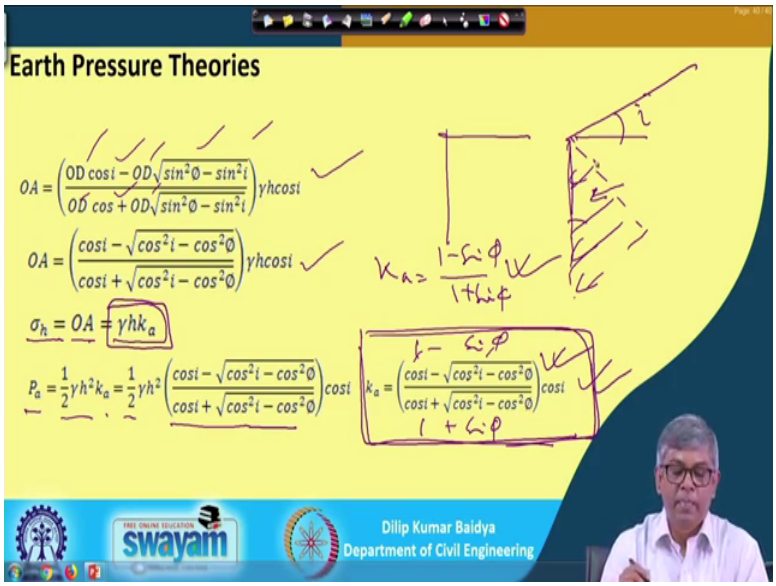

So, after knowing all those things and if I substitute then I will get OA equal to ultimately we can see if I substitute all those things OA equal to whatever was there, OA minus AB and if I substitute then you are getting like this gamma h square cos i and then if I simplify you can see all are OD actually or in terms of OD, so, OD gets cancelled.

So, ultimately you can see cos i minus under root cos square minus cos square phi by under cos i plus under root cos square i minus cos square i gamma h cos i. So, and sigma h equal to OA, so sigma h is nothing but OA in the figure itself you will see and that is (Refer Time: 19:59) by gamma h ka ok, so this is gamma h k a. And so, P a will be half gamma h square k a so that means, half gamma h square cos i under root cos square. So, this is actually your, so that means, your k a equal to nothing but this, your expression for k a now when there is a level backfill. Your k a was 1 minus sin phi by 1 plus sin phi, now I have taken the backfill with angle i and then your expression for k is this.

If you put i equal to 0; if you put i equal to 0 you can see these become 1, 1 minus cos square phi means under root 1 minus sin square phi and cos square. So, it will be it will become sin and cos i become 1, so 1 minus sin phi it become and this is again 1, it becomes sin phi. So that means, we have got a expression now geometrically that coefficient of earth pressure for inclined backfill which is expressed on this and since as I have mentioned that it is generalized expression that means, if I put i equal to 0 then my inclination of backfill angle equal to 0 we supposed to get the normal active earth

pressure coefficient. So, if you put i equal to 0 in this expression you will see that k a will become like this ok.

So that means, if there is a level backfill then we can do like this that means, your acupressure diagram will be acting like this and but your coefficient of earth pressure will be little modified like this. And your diagram maybe taken can be like this.

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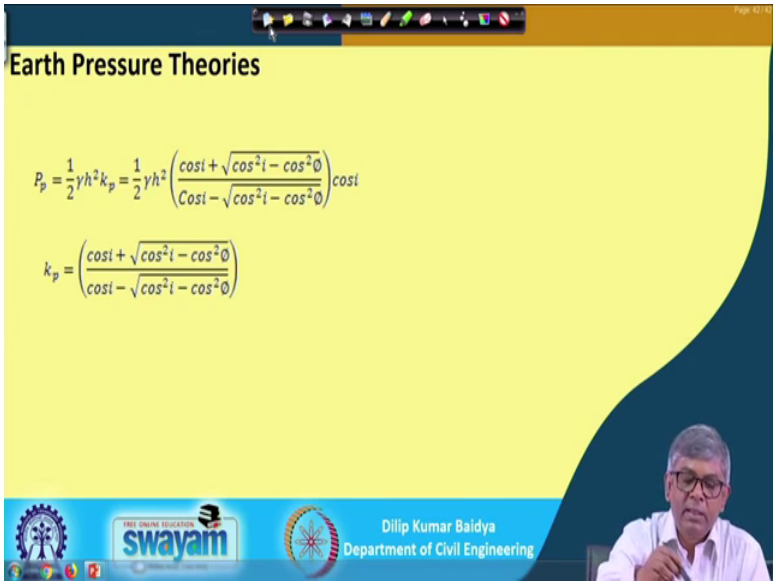

Next, actually you can see similarly you can similar way if I do then you can find out P p also equal to half gamma h square k p, half gamma h square cos i again this is actually nothing but. So, there actually it was minus here written plus here, now it becomes plus here minus here. So, as you have seen before k p become cos i plus cos square i minus cos phi and cos i minus under root cos square over the cos i. So, this is again k p, so cos i was there actually, so, that can be taken.

So, this is actually when it is a backfill is inclined, then your earth pressure coefficient for passive case and active case already we have seen in the previous slide. Now, so far actually in the Rankine's theory and whatever we have seen that, it is wall is frictionless and there is a vertical and horizontal backfill and then somehow, we put extend the inclined backfill.

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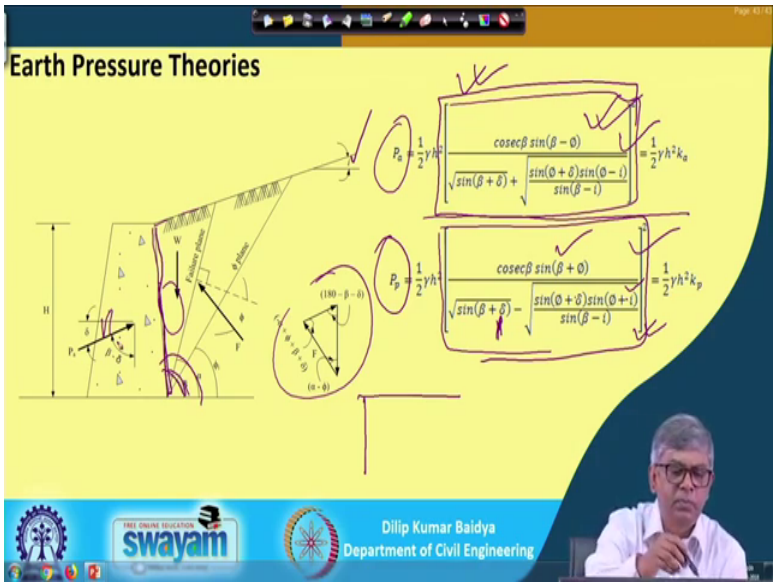

Now, there is a Coulomb's theory. In it in this actually this is very much universal that means, the some of the assumption can be eliminated that means, actually we can see this is as per Coulomb's theory that it can be any angle this instead of back vertical wall. It can also inclined backfill, back also in inclined sorry, wall back also can be inclined you can see this is very common also in many cases.

So far in the Rankine's theory those are vertical, but this theory Coulomb's theory can consider actually inclined back. And Rankine's theory initially horizontal, but now in the from the beginning itself Coulomb's theory can take any angle of backfill. And again, it was frictionless now it can take some friction and you can see the delta, decline angle of angle friction between the wall and the soil that delta is also there.

So, based on that again we can imagine the different failure angle and then according to that consistently Wedge you can find out the your force polygon and then for the force polygon you can find out what is k a, what is k p. And then again maximize minimize from there you can find out earth pressure at active condition earth pressure for passive condition. And you can find out that finally, that P a as per Coulomb's theory is the value is given like this half gamma h square and this is the entire thing which is nothing but earth pressure coefficient for active condition as per Coulomb's theory and this one again nothing, but active earth pressure coefficient for passive condition as per Coulomb's theory.

So, this is this expression is little lengthy and you can see it has so many things it is it has i, in it this is i. It has a angle of beta that means, inclination of back you can see. It has a declination angle this the sorry this angle of friction angle of the wall friction, delta also is there. So, 3 additional thing. There only sin phi soil properties phi only was there. In addition to this we have here delta i and beta.

Now, if you put beta equal to 90 degrees that means, vertical and i equal to 0 here and your delta also equal to 0, then we can find out that this also can be modified at the same as previous case that means, by Rankine's theory 1 minus sin phi by 1 plus sin phi same thing will be arrive at. So, this is the one is quite accurate actually, one can get very good estimate of earth pressure because of this actual condition we are taking, but because of this lengthy expression people generally not used it, most of the time people use Rankine's theory with minor modification.

Sometime whatever it is getting by Rankine's theory little modified 10 percent, 20 percent more or less can be taken and then based on that they can design, but otherwise one can take this Coulomb's theory as it is and one can find out earth pressure which is support supposed to be better one. But because of this complicated expression of earth pressure coefficient most of the time this will be this is generally avoided, but otherwise this is this can be also used.

So, this expression both the expression is actually quite lengthy, otherwise there is no issue is a very much generalized because we are taking back and backfill angle we are taking wall back angle, and we are taking wall friction angle. So, all three are included in this. So, this is actually a Coulomb's theory. So, sometimes what is the difference between the Rankine's theory and Coulomb's theory? Many times this question is there. So, that can be that one has to remember or one has to keep in mind and same time either this formula of course one cannot remember, but you know that if you put those values it will be reduces to the Rankine's formula value Rankine's formula also.

So, like this now, so far we have considered that Rankine's theory or Wedge theory or Coulomb's theory whatever we have taken cohesion less backfill, but many a times soil backfill may have both cohesion and friction. In that case, how to find out the earth pressure and then what is the how to find out the thrust behind the wall that will be again

we need to look it look into that and that maybe I will see in the subsequent few modules.

And then after doing that I have to finally, after getting the pressure and then I had to do the further stability analysis how wall can fail. When the wall determines the soil then wall can have different types of failure, one type of failure is toppling or overturning and there can be sliding like this it can slide. This is the wall and because of the excessive pressure it may slide like this and there can be over turnings sliding, and then it can have bearing capacity failure also actually.

That means, because of these wall weight, pressure wall pressure, then point of application of the force is passing in such a way that your pressure below the wall will not be uniform, because resistance will be there. And most of the time and if it is designed in such way that the sometime the wall may have very little pressure one side and very much very high pressure other side.

So, when the pressure become very high one side it has to be seen that high pressure should not exceed the bearing capacity of the soil. So, that bearing failure or bearing that is what, that means what adding failure? Shear failure or sliding failure and bearing capacity failure that means, under the loading condition whatever the pressure below the footing that should not exceed the bearing capacity of the soil. You should have some factor of safety.

And there are another type of failure that is global failure, that means entire wall will fail with global failure that there is another check. So, we will try to do global failure we may not able to do, otherwise that sliding failure, overturning failure and bearing capacity failure, these 3 things factor of safety calculation how to do considering the stability of the wall will also discuss maybe in the few subsequent module.

Thank you.