

Geotechnical Engineering II / Foundation Engineering
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Lecture – 03
Quick Review of Soil Mechanics (Contd.)

Good morning. Once again let me welcome you on this Foundation Engineering lecture. And, today once again I will continue with the same topic, that is quick review of soil mechanics. And, I have discussed already 2 aspects; one is origin of the soil and another is classification engineering classification.

And, today I will again try to cover this weight volume relationship and effective stress concept, which is actually very much essential in every calculation. In fact, and sometime when we will actual solve the foundation problem, we may not go in detail about assuming that we have the enough background and quickly calculate that.

So, since you have to use frequent this things. So, I am just reviewing quickly and let me go to the first slide.

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Weight Volume relationship, Effective stress,

(a) Dry soil

(b) Saturated soil

(c) Partially saturated soil

Void ratio, $e = \frac{V_v}{V_s}$

Porosity, $n = \frac{V_v}{V} = \frac{V_v}{V_v + V_s} = \frac{e}{1+e}$

Degree of saturation, $S_r = \frac{V_w}{V_v}$

$G_s = \frac{W_s}{V_s \gamma_w} = \frac{M_s}{V_s \rho_w}$ $\rho_w = 1.0 \frac{gm}{cm^3} = 1.0 \frac{Mg}{m^3}$

Specific gravity, G_s , of most soil range between 2.6 and 2.75

$\gamma_w = \rho_w g$ $g = \text{acceleration due to gravity} = 9.81 \text{ m/s}^2$ $w = \frac{W_w}{W_s}$ or $W_w = wW_s$

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That is weight volume relationship actually you can see that when the soil mass is idealized dry soil mass if you idealized, then what we will get actually you can idealize the all solid part together. And, suppose like this you can this is the total volume of the

solid, then when the soil. And then if I idealize the solid part and put together this is the volume of solid suppose and this is the air, where there is a dry soil there will be 2 face is only air and solid.

And, again when this air is fully a filled of by water only then it will become which is called saturated soil, and then it will be again still it will be 2 face diagram and it will be solid and water this water volume will be replaced by air volume and solid will be as it is, and this is a saturated soil. And, this water whatever whether is a water or air we generally call it as a void. So, this is the within the soil mass whatever the voids are there it can be either filled up by air or by water.

So, this is the one. So, both are actually 2 face diagram when it is completely dry or completely were saturated both cases in 2 faces. Either water solid or air solid, but in most cases soil will be the 3 faces, partly will be filled up by air, partly by water, the entire void space will be 2 parts now. Partly the whatever air voids are there if you put together this may be the volume of air, and whatever water pressure with the soil mass that you put together this may be the volume of water. And solid even answers.

So, this is the 3 face diagram. And, this is if I give this all notations then we can I have number of things based on these we have defined number of parameters like; void ratio, which is equal to volume of void by volume of solid, then porosity again volume of void by total volume. And, interestingly you can find out if this is the definition then what will be the value of void ratio minimum value and maximum value, if we imagine the soil is completely filled up by solid there is no voids ok.

Then void become 0 and then your solid become full then it become 0 and you can see you can imagine other side also, that the soil mass is very very negligible and become close to 0, then in that case these become 0. And these become entire volume then that become very large value; that means, theoretically void ratio can be between 0 and infinity.

But most of the time it will have a finite value; that means, it can go beyond one also void ratio 11.5 that also possible. Similarly, porosity if I see this diagram then this value ratio of course, they have definite relationship between porosity and void ratio, but from this definition again we can see that porosity can vary only between 0 and 1. So, you can see from their definition.

Next is that G_s is nothing, but total solid present divided by V_s and these are the different definition we use, ρ_w is 1 gram per centimeter meter cube or 1 mega gram per meter cube. These are actually units some time we make mistakes. So, that should be very careful and see γ_w is ρ_w into g and generally g is the acceleration due to gravity and which is 9.81.

And, this 9.81 when we use accurate actual wanted to use the accurate calculation, then will be 9.81 can be used. Otherwise in civil engineering calculation all loads and forces are so, huge.

So this, we generally for simplicity in calculation we some time use instead of 9.81 10 kilo Newton 10 actually; and as a result of that the calculation will be simplified. And, this specific gravity whatever is definition is given based on that one can estimate in the laboratory. However, that most of the soil particles whatever we have investigated over the time for particle specific gravity. The value generally does not go below 0.2 0.6 some time it go, but most of the time most soil will range between 2.6 to 2.75.

Sometime it in some problem the specific gravity is not given most of the time you are allowed to assume a suitable value between 2.6 to 2.75 most of the time 2.67 2.7 we can assume. And, whatever calculation we get it will be quite accurate.

So, these are the things actually you as a civil engineer one has to remember, that that unit weight of water is 1000 kg per meter cube and your specific gravity of solid soil; soil solid will be between 2.6 to 2.75. These are the things to be remembered because sometime it will be assume this is known actually it should remember these are the things.

Next thing is some time water content is another important term that is weight of water by weight of solid this is actually defined as water content, and water content is very very important, because some time with the addition of water soil behavior change greatly. So, you need to know what water content present water is present what amount of water present in the soil. And so, we always find out from the sample water content of the soil and how we find out? We take initially moist weight and then put it in a oven and after 24 hours and we take again dry weight and from there loss of weight will be nothing, but weight of water.

And, then we can weigh also solid difference of finally, weight also actually is the weight of solid. So, difference between 2 weights is the weight of water and final weight is the weight of solid. So, this ratio will give you water content. So, these are the things basic things actually everywhere every calculation will be required. So, because of that I am just reviewing quickly.

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Weight Volume relationship, Effective stress,

bulk unit weight, $\gamma_{bulk} = \frac{(G + Se)\gamma_w}{1+e}$ $\gamma_{sat} = \frac{G_s + e}{1+e} \gamma_w$ $\gamma_d = \frac{G_s \gamma_w}{1+e}$

Buoyant unit weight = saturated unit weight - unit weight of water = $\frac{(G_s + e)}{1+e} \gamma_w - \gamma_w = \frac{G_s - 1}{1+e} \gamma_w$

$\gamma_{bulk} = \frac{W}{V} (1+w)$ or $\gamma_d = \frac{\gamma_{bulk}}{(1+w)}$

Handwritten notes:
 $Se = WG$
 $\frac{G(1+w)}{1+e} \cdot \gamma_w$

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Next is weight volume relationship again there are number of unit weights, you can see I will just first bring all together and sorry. You can see this bulk unit of weight bulk unit weight of soil this is one formula G plus a plus 1 and there is a relationship S into e equal to W multiplied by G .

So, there actually instead of S C if I put WG , then it become G 1 plus W divided by 1 plus e γ_w . So, this is one formula and at also sometime this can be used. And, again this is the one. So, bulk unit weight either this, this equation or this equation, when degree of saturation and void ratio is given you can use this, when void ratio and water content is given then we can use this equation.

Similarly, now if I take this equation now degree of saturation is known suppose] s is 10 or degree of saturation 10 percent, 15 percent, 20 percent, or even 100 percent or even 0. So, based on that I can modify this equation for different other bulk unit ratio other unit weight like; suppose degree of saturation is 100 percent; that means, soil is completely

saturated then s become one and I put this s value 1, then this equation we get which is the unit weight of saturate soil saturated soil.

And, now if I put in this s equal to 0; that means completely dry then in that case s become 0 and this part become 0. So, this equation will be modified to this form that mean dry unit weight of the soil will be $G s \gamma_w$ by $1 + e$. So, there are so bulk unit weight, saturated unit weight, and dry unit weight. And, next actually some time buoyant unit weight; that means, water when it is under water this will have buoyant unit weight, which will be acting actually.

So, the buoyant unit weight of the soil will be saturated unit weight minus unit weight of water. And, if you do that the saturated unit weight of this and unit weight of water this and then if I subtract, then I will arrive at this equation ; that means, G minus 1 by $1 + e$ into γ_w .

This is also some time unit weight of or sometime is not given, but you know the G value and if you know the e value, then below water table unit weight you can calculate from this equation. Similarly, if it is unit weight is not given, but degree of saturation or water content is given and void ratio is given, then either using this equation or this equation you can find out the bulk unit weight.

Similarly, if it is a dry unit weight suppose I want to find out dry unit weight and unit if you know the void ratio, then you can find out by using this equation. So, these are the things; that means, you need to know some time void ratio is one of the important per say parameter, then degree of saturation or water content. So, if you know those things γ_w always you can assume as 9.81 or 10, and then rest of the calculation can be done. And, there is a relationship between bulk unit weight and the dry unit weight.

So, this is another relationship γ_{bulk} will be equal to γ_d by γ_d will be equal to γ_{bulk} by γ_w . So, these also can be established by using the all parameter whatever we have introduced and γ_d if you know the γ_{bulk} and if you can find out the water content, then you can find out the γ_d also. That means, partially saturated solution those are the things I have told this is the completely this is ideal; that means, either saturated or dry.

But, bulk unit weight is the in between stage most of the soil in the construction soil will have partially saturated. That means water content will be there less than saturated. In that case you can find out the bulk unit weight by either this and this, but how to find out the dry unit weight, if you know the water content then by this. And, this is very very important in many road construction because when you construct the road the construction material. That means, soil is recommended based on the dry unit weight, that we have to achieve the this much dry unit weight. And, a based so, because of that we have to select number of sides and from those sides again, you have to you have to carry out this proctor test and then find out you have to find out what is the maximum dry unit weight can be achieved?

And best way that we can select the site. And, when will come to the site side and during construction you know that what unit weight we have to achieve. Then what we have to do time to time during construction, you have to sample it get the bulk unit weight by some mechanism and then determine the water content, and then we will get dry unit weight. When you see that dry unit weight achieved and dry unit weight recommended they are close enough then we can certify the construction is done as per as per specification.

So, that is the thing this gamma d is very very useful most of the time in embankment construction.

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Weight Volume relationship, Effective stress,

$$P = \sum N' + uA$$

$$\frac{P}{A} = \frac{\sum N'}{A} + u$$

$$\sigma = \sigma' + u$$

$$\sigma' = \sigma - u$$

$$u = \gamma_w \times h$$

$$\sigma = \gamma \times h \times \gamma_s = \frac{w}{\sigma}$$

effective stress

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And, next is the effective stress weight volume relationship one part and then effective stress they are closely related to each other.

And, you can see that that effective stress when you have discussed in soil mechanics or you have studied from some book and this is the final formula we generally use, that is $\sigma_{\text{effective}}$ will be equal to total stress minus u . And, how it is done actually from this formulation actually if I draw a line here, and then it will pass through number of contact points. And, each contact point the force will be in the different direction, but you can take the component in the vertical direction. And, if I write equation of equilibrium in the vertical direction then you get this equation. And, as per if you divide by A in all sides, then you will get this equation and as per definition σ_{total} stress equal to actually P applied over the cross sectional area.

So, this is σ and by definition actually summation of all normal component divided by the area is our effective this is our definition, which may not be correct the actually you are dividing by the total area ok. So, that is not actually contact points will be much less than the actual area.

But, this is as per our definition the effective stress is defined at that whether it different contact points, whatever normal components are there summation of all normal components divided by the total area is defined as effective stress. So, if I do this then this become so, this equation modified to this equation and then equation to modified to this whether $\sigma_{\text{effective}}$ equal to $\sigma_{\text{total}} - u$ $\sigma_{\text{effective}}$ is nothing, but effective stress effective stress. Sometime is some book is write by $\bar{\sigma}$.

So, both are same and you can use any off them. And so this becomes like this, but we are interested to find out some $\sigma_{\text{effective}}$ that will effective stress. So, effective stress if I express different separately then it become $\sigma_{\text{total}} - u$. That means, if there is a ground here like this and if I want to find out and water table suppose also here.

Then, I if I want to find out the effective stress shear then what I have to do I have to find out the total load acting over this divided by area and if I consider a unit area here. If, I consider unit cross sectional area and this length of the column is l , then total volume will be unit area multiplied by l this become the volume. And, if I multiply by unit weight that become the w actually and then w the total weight divided by area you can get to 1.

So; that means, we can nothing, but so, this is this sigma this sigma is nothing, but actually this weight by area. So, that is actually nothing, but gamma times since it is unit weight unit length. So, all the gamma times l gamma times l is enough to find out the total stress at this point.

So, similarly if you want to find out a water pressure at this point; so I can consider again unit column of water here. So, at this point total weight of water will be again unit weight 1, multiplied by l, multiplied by gamma w and this divided by 1 by 1; so this nothing but gamma w so, u equal nothing, but gamma w into l. That means, total stress will be equal to gamma of water I am sorry gamma of soil multiplied by the depth at which you have to find out the total stress

Similarly, if you want to find out the water pressure at the particular depth then unit weight of water multiplied by the depth at which I want to find out the water pressure will be equal to the pore water pressure that. Now, we have got 2 component this become sigma this become u, then I can find out what is the effective stress acting here that is sigma y as u. So, this is the way actually we need to calculate. Now so, this is the effective stress concept.

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Weight Volume relationship, Effective stress,

Effective vertical stress due to Self Weight of Soil, $\sigma_v = \gamma_{sat} z$

Pore water pressure, $u = \gamma_w z$

$\sigma_v' = \sigma_v - u = (\gamma_{sat} - \gamma_w) z = \gamma' z$

$\gamma' = \gamma_b - \gamma_w$

$\gamma_b = \frac{G}{1 + e} \cdot \gamma_w$

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And, now as I have already explained you can see once again I have repeated repeating here. Effective vertical stress due to Self Weight of Soil, as I have drawn suppose if this is the soil and I want to find out at this point; that means, because of a self-weight of soil

whatever pressure is coming here, that is actually sigma b or sigma that will be equal to not necessarily it will be gamma saturated always. If, it is water table is here only then only gamma saturated into Z, otherwise if it is a partially saturated then it will be gamma bulk into Z.

So, this is the sigma v and pore water pressure will be again if I consider water table is here and then this is the depth is Z, then your pore water pressure equal to gamma w into Z. And, assume that water table is here then this equation also valid for sigma v. Now, sigma dash will be ultimately sigma v minus u and that is actually gamma sat minus gamma w into Z; that means, gamma submerge multiplied by Z.

So, gamma submerge how to find out if you know that is actually some time G minus 1 by 1 plus e multiplied by gamma w you can use it, or if I can find out gamma bulk and minus gamma w divided by direct calculation of these 2 parameter we can do this or if you know the specific gravity and void ratio we can also find this way. So, this is the calculations porosity or for effective stress.

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Weight Volume relationship, Effective stress,

A layer of saturated clay layer 5 m thick is overlain by 5 m deep sand layer. The water table being 2 m below the ground surface. The saturated unit weights of the clay and sand are 19 and 20 kN/m³, respectively; above the water table the unit weight of the sand is 17 kN/m³. Determine the effective stress at various depths. If the sand above the water table gets saturated with capillary water, how are the above stresses affected?

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Now, let us see a problem.

A layer of saturated clay saturated clay layer sorry layer is to 5 meter thick is overlain this sorry; this layer may not be required. A saturated clay layer 5 meter thick is overlain

by 5 meter deep sand layer and water table being 2 meter pressure if I draw this, if I draw this it comes like this suppose this is 5 meter that is sand and this is 5 meter suppose clay.

So, this is suppose clay and this is suppose sand, and water table is somewhere here ; that means, this is 2 meter, this is 3 meter, and this is actually your 5 meter. And, you can see that the unit weight unit weight of clay is 19 and sand is 20 and water sand unit weight above water table is actually given 17.

So, now if I want to find out the effective stress at different location; that means, I want to find out at this location, I want to find out at location water table location, I want to find out the in between layer, I want to find out at the bottom of the layer. So, at this point sigma total sigma at the surface will be 0 and total sigma at this point will be 2 multiplied by 17, 2 multiplied by 17 and at this point your your sigma will be 2 multiplied by 17 plus 3 multiplied by 20. And, at this point your sigma will be 2 multiplied by 17 plus 3 multiplied by 20 plus 5 multiplied by 19. So, if I do all those thing then it become 0 here, in this become 34 here and this become 94 this is 20 actually this become 94 and this become 189.

So, this is actually sigma. Similarly, I can find out u at the surface it is 0, at the water table also 0, at 3 meter it will be 3 into 10, 3 multiplied by 10 and at this level 8 multiplied by 10. And, then if I want to find out sigma dash, then what I will be doing I will be doing 0 minus 0 at this surface actually effective stress is 0, and then at water table level that is 2 meter depth. So, it is 0 at 2 meter depth it will be 34 and at 5 meter depth it will be 94 minus 30, that will be 64 and at 10 meter depth it will be 189 minus 80. So, it will be 109.

So, this is actually sigma dash nothing, but sigma minus u. And, this is sigma calculation this column this column is sigma this column is u and this column is sigma dash. So, this is the way one can calculate. Now, this is another thing is mentioned here that if the sand above the water table get saturated with capillary water, how are the above stresses affected?.

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Weight Volume relationship, Effective stress,

A layer of saturated clay layer 5 m thick is overlain by 5 m deep sand layer. The water table being 2 m below the ground surface. The saturated unit weights of the clay and sand are 19 and 20 kN/m³, respectively; above the water table the unit weight of the sand is 17 kN/m³. Determine the effective stress at various depths. If the sand above the water table gets saturated with capillary water, how are the above stresses affected?

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So, saturated means now water table is here water table is here, but it is saturated by capillary rise. In that case actually when there is a capillary rise unit weight is here 20 here also you have to consider now 20, but pore water pressure variation when you do pore water pressure u variation earlier was u variation of earlier case was like this; that means, it was 0 and it was 80, but present condition your u diagram will be your u diagram will be so, sorry. So, your u diagram will be something like this; that means, above water table when water is getting up that will be negative. This will minus and this will plus this will be same 80, but there will be minus 2 multiplied by 10.

So, when there is it is not saturated this was the pore pressure diagram and when it is saturated by capillary rise this is the pore pressure diagram, this is the difference in pore water pressure will be observed. And, in addition to this there will be little change in effective stress also because total stress will change now. So, how much change will be there the this step we have considered total unit weight by earlier we have consider 17 into multiplied by 2, but it will be taken now 20 multiplied by, it will be taken it will be total will be 20 multiplied by 2 or actually I can take 20 multiplied by 5, 20 multiplied by 5 plus 19 multiplied by 5.

So, it will be 100 95. So, there is a change in total stress it was 189 now it become 195. And, if I want to find out effective stress at that this point now before it was 80 189 minus 189 minus 80 it was 109.

Now, it become 195 minus 80. So, it will become 115. So, this there is a little change in effective stress so, because of this. So, these are actually the minor changes when the water soil above the water table get saturated. So, there will be change in pore water pressure diagram, and again there is one more change effective stress theoretically at the surface will be total stress is 0 0 here at this point total stress is 0 0 minus minus u . So, it will become 20 at the surface theoretically ok.

So, this is the only change we can find out when soil is get saturated. So, this is the thing very frequently will be used in effective stress calculation in many calculation particularly a compressibility problem, you need to know or stress and problem also you need to know what is the effective stress at any depth? Because, shear strength is function of effective stress, compressibility also function of effective stress and every frequently we calculate this.

So, because of that I have a once again elaborately discuss this.

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Weight Volume relationship, Effective stress,

Variation in Effective stress With the Shift in Ground Water Table

For water table below the ground surface, a rise in the water table causes a decrease in the effective stress, and a fall in the water table produces an increase in the effective stress

For water level above the ground surface, a fluctuation in the exposed water level does not alter the effective stress

The effect of a shift of in the ground surface will cause a change in the effective stress of magnitude equal to the change in the overburden pressure

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Next part is a variation is effective already I have highlighted variation in effective stress with the shift in ground water table; you can for water table below the ground surface, a rise in water table cause decrease in effective stress, and a fall in water table produces an increase in the effective stress.

So; that means, if your if your water table somewhere here and then if water table goes up suppose because of the seasonal variation water table goes up, then what will happen your total stress will be increase little, but pore water pressure will be significantly increase because of this, this γ_w into this much height. And so, that much water table that much pore water pressure if you subtract, then automatically effective stress will be decreasing; that means, if water table moves in this direction, then effective stress will be decrease and if the water table moves this direction, then your what effective stress will be increasing.

That means water table as if it is if it goes deeper and deeper, then you will expect more effective stress in the soil. Similarly, for water level above the ground surface a fluctuation the exposed water level does not alter the effective stress; that means what if your ground surface is somewhere here and water is somewhere here. Now, if water table comes down this side or goes up this side, it will not make any difference if I want to consider a point here to find out effective stress.

So, if you find out of effective stress here then because of this change of water table above this direction or this direction where water table was originally above water table and still fluctuation is above water table above ground water level sorry ground surface, then there will be there will not be any change in effective stress. And, the last point whatever I have shown during the competition the effect of shift of in the ground surface will cause a change in effective stress of magnitude equal to the change in the overburden pressure; that means, what I have shown then when because of this this water table was water table was here, and ground surface was there, and this much depth was saturated by capillary rise.

And, because of that I have considered unit weight from 17 to 20 so; that means, that how much change will be there, because of the effective stress of magnitude equal to the change in the overburden pressure. There actually it become actually a 1092 115, then how much was changed it was 9 sorry 6 1 0 9 2 1 0 1 1 5, that 6 increased and 6 increased because of what the unit weight change from 17 to 20 so, that 3 multiplied by 2 6.

So, that is the thing is mentioned here effective effect of shift in the ground surface will cause a change in the effective stress of magnitude equal to the change in the overburden

pressure ok. So, this is the thing 3 important point regarding effective stress just highlighted here.

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Weight Volume relationship, Effective stress,

If instead of unit weight of soil, void ratio, specific gravity and water contents are given,
 For Example, void ratio of sand $e_s = 0.7$, void ratio of clay, $e_c = 0.95$, specific gravity of sand 2.7 and specific gravity of clay 2.65, the problem can be solved as shown

$$\gamma_d = \frac{G_s \gamma_w}{1+e}$$

$$\gamma_s = \frac{G + S e \gamma_w}{1+e}$$

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And, next actually if the previous problem whatever I have solved there actually unit weight all given, but instead of giving of what unit weight if they if they give you the void ratio and your specific gravity, then we can find out dry unit weight, then saturated unit weight, then your dry saturated bulk unit weight and based on that we can again calculate similarly what is the total pressure, what is the pore water pressure, what is the effective pressure? Only one step more you have to do what based on this is supposed gamma bulk, you have to use suppose gamma you have to read some gamma d, then G s gamma w by 1 plus e.

So, gamma sand above water table you have to find out you use this equation because e is given G is given gamma w can be assumed. Similarly, below water table you want to find out. So, you can find out gamma bulk will be equal to G plus S C by 1 plus e into gamma w. By this equation you can find out gamma sorry this is gamma saturated; that means, below water table if the sand or clay unit weight we can find out, because the unit void ratio of the soil of the clay and sand is given. Degree of saturation when below water table it will be 100 percent and G can be assumed or it is given so, you can find out to. After computing this and the procedure is same; that means, find out the total

pressure, find out the pore pressure, and by subtracting them we will get the effective stress.

So, that is all I think we can stop here this part that is weight volume relationship and effective stress concept. And, this will be very frequently will be used in many foundation engineering problem ok. So, this is the reason why I have elaborated once again.

Thank you.