Geotechnical Engineering II / Foundation Engineering
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Lecture – 29
Earth Pressure Theories (Contd.)

Once again let me continue with Earth Pressure Theories. In the last module I have discussed different types of earth pressure qualitatively; that means, what is earth pressure at rest, what is active pressure, what is passive pressure? Three aspect I have discussed. How wall moves, how major to major principles to minor principle it become and what is the failure angle during the angle of the failure plane, during active case during passive case all those things I have discussed.

Now, as I have mentioned that we need to quantify the magnitude of the earth pressure because in that will be required in the design. So, how earth pressure varies behind the wall and what is the magnitude that to be estimated. So, for that purpose now will take one by one; you can see.

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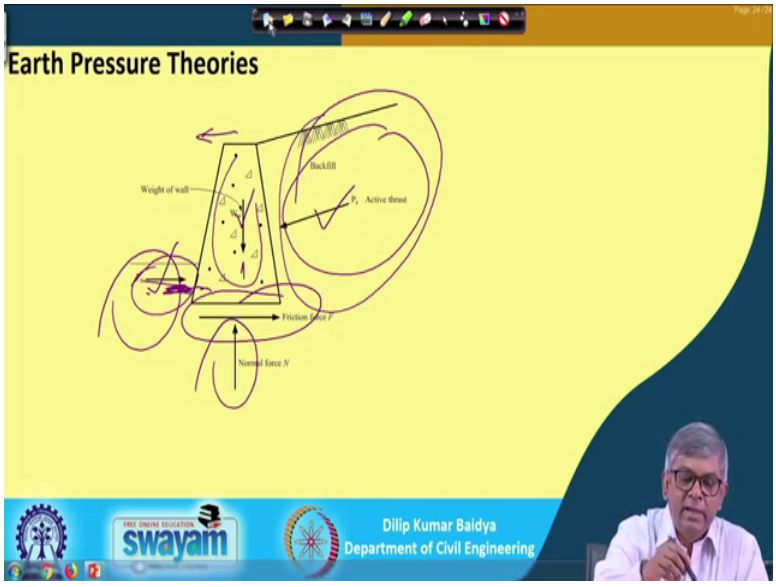

First of all if I take a typical retaining wall it looks like this; this is a typical gravity retaining wall. On the gravity retaining wall that we will have number of forces will be there and you can see the its own weight will act like this and obviously, it will have a normal reaction and soil retained here. So, there will be some active pressure, there will

be some soil here and because of this weight there will be obvious if there is no other things are kept here then the tendency of movement of the wall will be towards this direction. So, that means wall will move in this direction.

So, if wall moves this direction; that means, there will be active pressure and of towards the toe side you can see it is embedded. So, when wall move this direction; that means, this soil actually; that means, this wall actually it is pushing this side, towards the backfill. So, that portion will become passive. So, in the same wall itself somewhere there will be active, somewhere there will be passive; how? Because this is the height, because of the huge height this entire weight will try to push the wall this side and when it try to move wall this side; that means, this side active and when the wall is moving this side towards this soil then; that means, this is passive.

So; that means, there will be weight of the soil there will be on the toe side there will be passive pressure, the backfill that we active pressure and there will be normal reaction and of course, since it is moving so at the base there will be frictional resistance. So, all those forces will be there and why we need to find out everything and perhaps this weight by if you know the geometry and if you know the concrete unit you can find out. But and this the reaction or friction related to this only and so all the unknown is this and this. So, to find out this that means you have to learn how to find out, we know the condition of active and passive, but what is the magnitude that has to be estimated.

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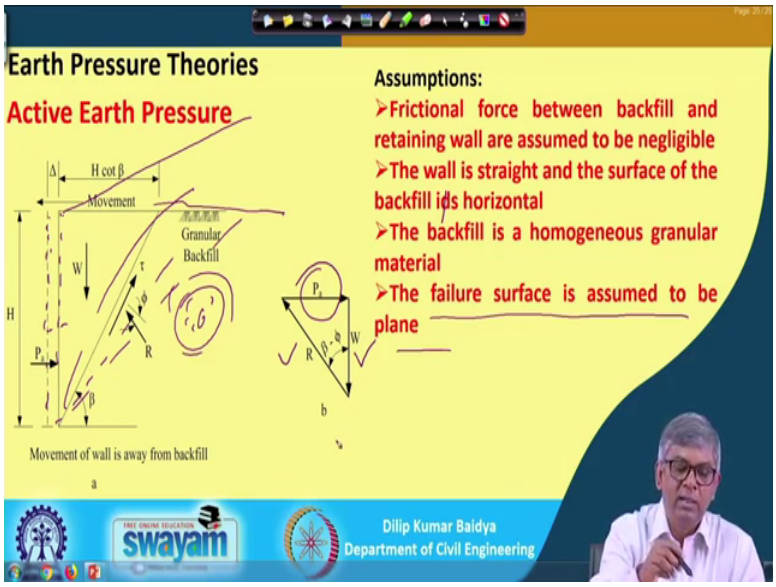

So, for that let us take just geometrically and that will be this is supposed wall, initially I have taken this the sorry this is the wall actually and this wall I am considering for active case and because of this active, since active case is the wall movement away from the backfill so it is going that way.

So, the suppose new position of the wall is this delta is the movement of the wall and if it moves and then I can find out the some soil ways it will move and because of these there are number of forces acting is W here and along this failure plane there will be tau and there will be reaction.

So, which inclination of phi and this side of course, active P active. So, all those forces if I draw in the form of a force polygon then it will come like this, the reaction will be here W will be here and if I complete this one the P a will be this one. And while doing this what are assumption we made? Frictional force between the backfill and retaining wall are assumed to be negligible, the wall is straight and the surface of the backfill is d is not required is horizontal.

That means, you can see this is the horizontal backfill you have taken it could have been like this also the backfill is a homogeneous granular metal, this is actually granular material. If it is other material then this will not happen, the failure surface is assumed to be plane; that means, instead of these failure surface could have been like this also it is not. So, whatever based on these assumption we could visualize the failure ways and corresponding to this failure values I can visualize the force polygon like this.

And if this is the force polygon then we can relate P a with respect to W and other things and if this is a particular condition and then this ways actually having a particular angle beta I have considered if it angle changes, but different values of beta I can get different values of force polygon. And for that I will get different values of P a; that means, I have to find out now what value of beta the P a become maximum then that will become active earth pressure.

So, for the time being I have considered beta as variable and based on that I have assumed the force polygon and then I have assumed the failure ways and then based on that I have considered the force polygon. And if this is the force polygon the little bit of analysis I can do, that analysis will be something like this.

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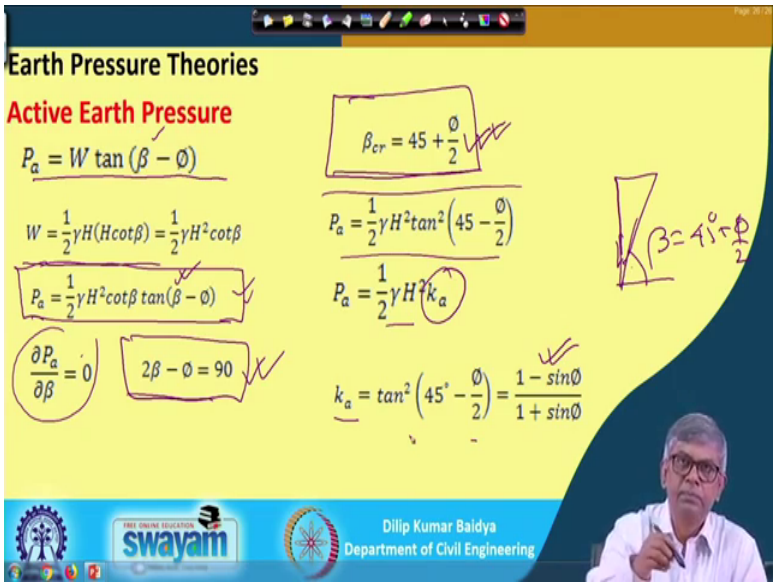

We can see that your from the force polygon I can find out P a equal to W tan beta minus phi, whatever force polygon I have got from there we can get and I can find out W that weight of the that is a wedge actually this is the wedge actually, I can find out the volume and then multiply by gamma then become weight. So, if I substitute this weight to these in these then b P a become in terms of gamma H cot beta phi so P a become this.

So, this P a actually varying the amount of P a, active participation will be varying because of what? Because of the beta only, the beta is the variable now. And as I have mentioned that for different values of beta I will get different values of P a. Now, I have to investigate now, at what value of beta the P a become maximum minimum; maximum or minimum whatever, but we know the active condition it is minimum.

So; that means, the P a is a function of beta, now if I differentiate P a in terms of beta and it set it to 0 that gives you a equation for beta critical and now solving that I will get the value of beta critical and that beta critical if I substitute in this equation I supposed to get the minimum earth pressure. So, that is the concept used here.

So, this is the function, if I differentiate P a with respect to beta and then I will liquid to 0 that is maximum minimum that is the condition we do differentiation we do differentiate the function and set to 0 and that I have do. And then if I simplify then I will see that 2 beta minus phi become 0 become 90 degrees or beta will become 45 degrees plus phi by

2. That means, this angle whatever this beta we have assumed and before qualitatively I have shown that active case it become 45 degrees plus phi by 2.

Whereas, in passive case it is 45 degrees minus phi by 2 that I will show later on, but what the time being from this I can see that that failure plane angle with horizontal become 45 degrees minus phi by 2. So, previously I have only mentioned based on of our knowledge.

Now, you can see mathematically we could find out at what angle these active pressure become minimum or pressure become active that is nothing, but minimum pressure. So, this beta critical now 45 degree plus phi by 2, as I have mentioned that critical angle for which the pressure become minimum that which I can put this beta value to this expression because beta is actually variable here, now I have got the definite value. So, if I put these value now I get the value of total earth pressure on the wall and these earth pressure P a become half gamma h square tan square 45 degrees minus phi by 2 and that can be retained small form P a equal to half gamma H square ka, where k a equal to tan square 45 degrees minus phi by 2 or this is current retained 1 minus sin phi by 1 plus sin phi.

So; that means, this is the active case mathematically I got angle, failure plane angle that is 45 degrees plus phi by 2 and active thrust will be half k a gamma H squares and k equal to tan square 45 degrees minus phi by 2 or 1 minus sin phi by 1 plus sin phi.

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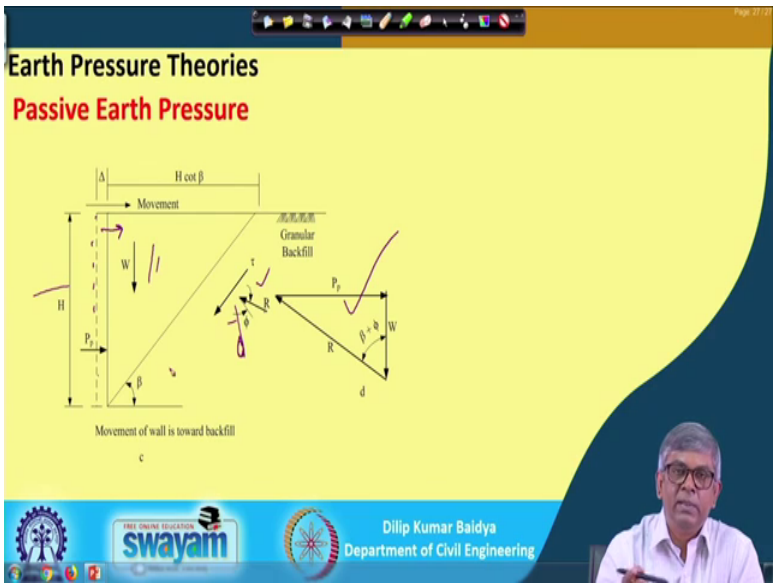

Similarly, if we consider the passive case; that means, the wall was suppose was originally here and disposed towards this. So, this become that delta or movement of the wall and I assume again another wedge here and with the angle beta here and the because of the weight and because then, when wall moves this side Pp will be the passive pressure.

And because of these weight and passive pressure and resistance all because of that this resistance; the resistance earlier was phi with this side phi making phi was this side now phi become other side. So, that is be sensitive the passive case. So, if I draw now the force polygon for this I will get a force polygon like this; that means, P p and W and R and now again similar to previous case; that means, for a particular wedge I have got this polygon.

Now if I change the beta to beta 1 then I will get another wedge and for that I will get another polygon and for that I will get another P a P p; similarly if beta become beta three then I will get another wedge for that I will get another polygon from there I will get another P p. So, like that a number of values of P p I will get for different values of beta and then I have to find out at what values of beta your P p become maximum.

Their actually we have investigated for minimum now it is a maximum. So, that actually that we can do little analysis and we can find out value of beta for which it become maximum and then substitute the beta we can find out the value of maximum passive pressure. So, for these let us do the little analysis.

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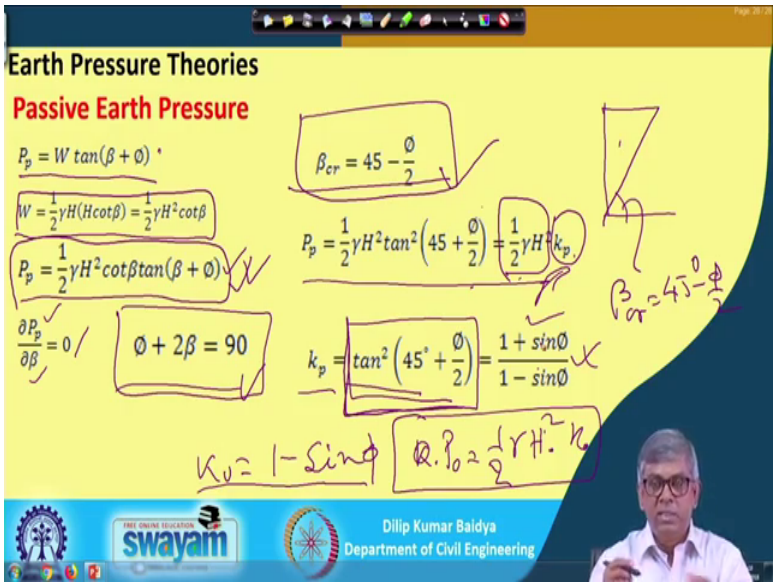

This analysis little complicated you can see may not be; so, the P p can be from the force polygon can be written in this form, P p equal to W tan beta plus phi and you know the W again similarly if this is the wedge then volume of the ways to be obtained. And then multiplied by gamma then that becomes the W this W again substitute to this value then you P p become this.

If you have gamma a square cot beta tan beta plus phi; similar to that when it is the maximum or minimum as I have told you that if the function that is a P p is a function of beta around and this function can be maximized. So, how to maximize? That if I differentiate and set to 0 then I will get the value of function, but particular variable for which it will become maximum. So, if I differentiate P p with respect to beta delta P p by delta beta equal to 0 then and set to 0 and then simplify then we will see that phi plus b 2 beta become 90 degrees or beta critical become 45 degrees minus phi by 2.

Now that means, this angle that angle will be beta critical will be equal to 45 degrees minus phi by 2. So, whatever I have told in the previous module that is based on our knowledge that passive case your failure plane will be making an angle with horizontal 45 degrees minus phi by 2 whereas, in active case it will be 45 degrees plus phi by 2.

Now mathematically we could prove the what is the angle of inclination for the failure at failure plane at by specific case. Now, this one if I substitute into the value of a P p earth pressure at specific condition then here I will get the earth pressure value P p equal to

half gamma H square tan square 45 degrees plus phi by 2 or half gamma h square multiplied by k p where, k p is the coefficient of earth pressure at passive condition which is nothing but tan square plus 45 degrees plus phi by 2; tan square plus 45 degrees plus phi by 2.

So, these one also can be written as 1 plus tan phi by 1 minus sin phi so; that means, we have got k, we have got k p and we have got the total thrust for active case total thrust for passive case. So, all three, all two we have both the things we have got at an at rest condition as I have told you that earth pressure at rest condition coefficient of earth pressure is 1 minus sin phi. That means, sometimes question can be asked that when find out the thrust behind the wall when the wall is prevented from yielding; if this is the question asked then how we find out? That means, it is neither a passive case nor an active case it is prevented from yielding.

That means, that is at rest condition then that case actually your ka k naught become 1 minus sin phi, once you know the one k naught and then you will having k P naught actually at rest condition half gamma H square multiplied by k naught. So, like that unconfined out; otherwise if it is normal condition if it is mentioned that soil phi value is this, a wall height is this then find out the active earth pressure behind the wall. Then simply you find out k a first by using this equation or this equation and then use half gamma a square into k p for passive pressure and half gamma a square k for passive pressure. So, these are the things h awe are getting.

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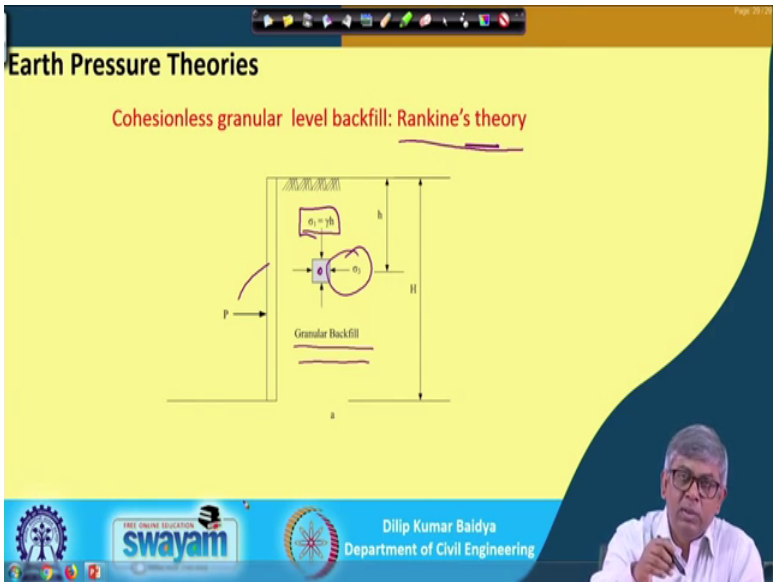

Now, this is actually based on some geometry and failure which, now Rankine's also given a theory for the active earth and passive earth pressure from there also we can see that how it can. So, the Rankine's theory number of assumptions are there in Rankine's theory, actually wall is smooth and vertical backfill is level. So, these are the major assumption in Rankine's theory and based on that he has given earth pressure coefficient earth pressure k a and k p.

And according to that actually this is the wall suppose and at a particular point if the your soil element is here then sigma 1 is actually here and sigma 3 will be here, at active earth pressure at rest sorry active condition. And granular backfill then what we can visualize a for a granular backfill that is a envelope and when fully active condition then I can find out the Mohr circle.

And by seeing the Mohr circle and failure envelope I can relate between sigma 1 and sigma 3 and sigma 3 is nothing, but active earth pressure at sorry active earth pressure. That means, pressure lateral earth pressure in active condition. So, let us look into that.

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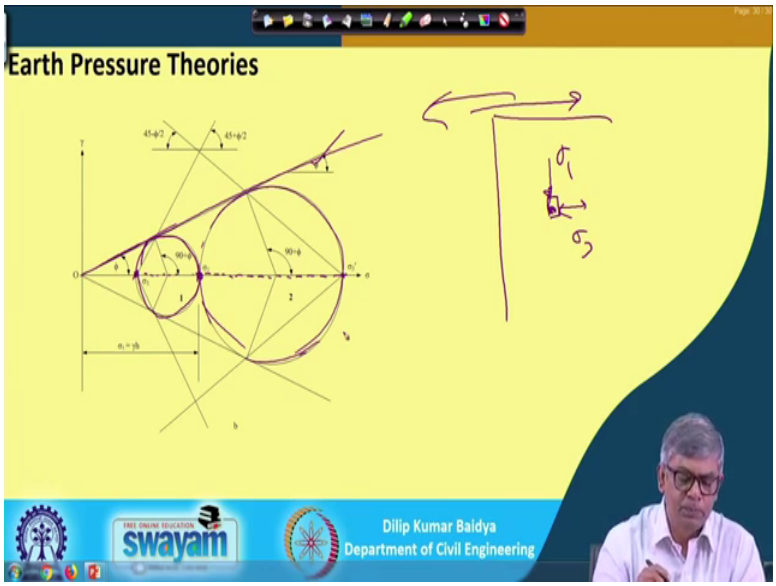

You can see this already we have done this one. So, the whatever shown that point in the previous diagram that at a we have considered; we have considered the same diagram again I draw here, if I draw an element here then this will be sigma 3 and this is sigma 1 ok. So, this one actually so, normally at rest condition this is sigma 1 suppose and when wall moves this way then sigma 1 reducing; reducing; reducing and finally, it become this. Then these become Mohr circle and again this was at rest condition sigma 1 and then the wall move this direction then sigma 1 is increasing; increasing; increasing and becoming this value.

And then so, sigma one and this is sigma 3. So, these become actually Mohr circle and you can see both the cases the Mohr circle will be below, with a tangential to the envelope you can see this is the Mohr envelope. So that means, I can when I will consider active case I will consider this Mohr circle and this envelope, when will consider the passive case I will consider this Mohr circle and this envelope and geometrically I can find out the relationship between sigma 1 and sigma 3.

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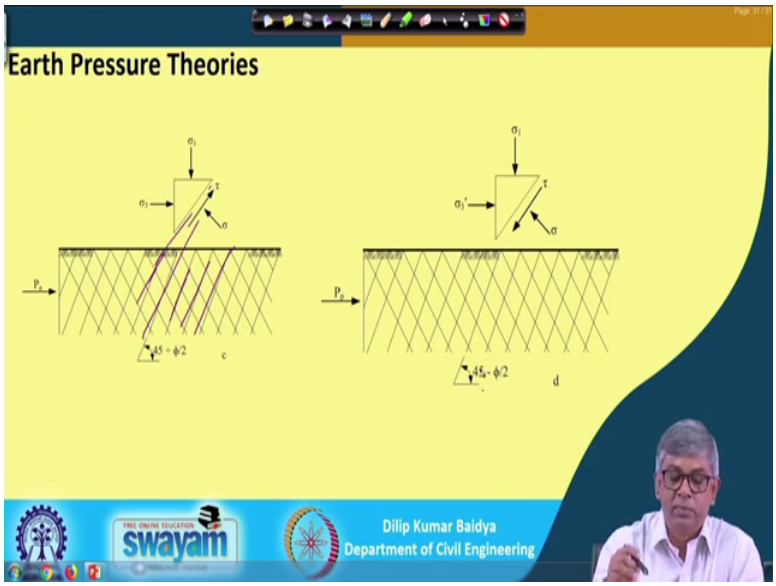

Now, we can see that different cases that is active earth cases the failure in failure plane will be like this you can see and it makes with horizontal 45 degrees plus phi by 2. And so, these are the failure planes this is failure plane actually 45 degree phi by 2 whereas, for passive case your failure plane will be 45 degrees minus phi by 2.

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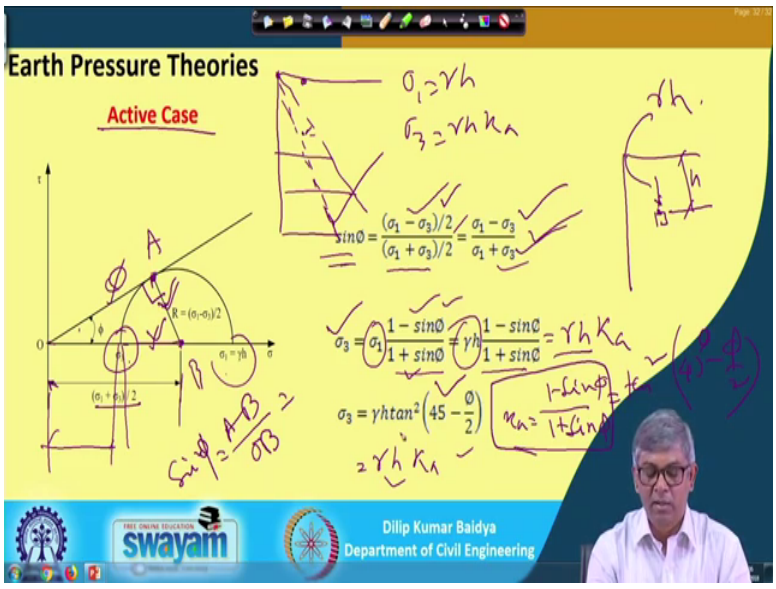

And you can see as I have told that active case, let me take the active case first. If I take active case and this is actually sigma 1 and this is sigma 3; sigma 3 will be the minor sigma 1 we generally call in the major and sigma 3 is the minor. So, sigma 1 here and

sigma 3 will be here and we know that this is small. So, this is the envelope and this is the Mohr circle.

Now if I see that if I consider this is center of the circle and this is tangent so these become right angle. So, now, if I call this is phi angle phi. So, if I consider express sin phi from this triangle; from this triangle if I express sin phi. So, these become actually these by this, sorry these by this ok.

So, this will be that is this is hypotenuse and this is perpendicular. So, sin phi will be so, these this is radius nothing, but the radius is was sigma 1 minus sigma 3 by 2 and this one actually this will be actually you can see sigma 1 plus sigma 3 by 2. So this is actually sigma 1 minus sigma 3 by 2 is this one and this is actually this portion is actually your and from here to here then if I take then it become sigma 1 plus sigma 3 by 2. So, sin phi; that means, these divided by this if I do sigma 1 minus sigma 3 by 2 divided by sigma 1 plus sigma 3 by 2 and if I simplify then it becomes sigma 1 minus sigma 3 by sigma 1 plus sigma 3.

Now, if you further simplify then you can see become sigma 3 will become sigma 1 multiplied by 1 minus sin phi by 1 plus sin phi and sigma 1; that means, when there is wall here sigma 1, if the consider element here sigma 1 is here if depth is h then this sigma one is nothing, but gamma times h. So, it is actually sigma h actually; that means, it will be gamma h k a and where this is also gamma h gamma h k a. So, k a is nothing, but 1 minus sin phi by 1 plus sin phi or tan square 45 degrees minus phi by 2 ok.

So, this is the one (Refer Time: 22:59) minus phi by 2 so; that means, I can find out at any place, suppose this is the wall and if I know the vertical stress here vertical stress at this point is 0, then what is the lateral stress? 0 multiplied by 0; 0 multiplied by 0; see if I considered at depth here the depth h; that means, your sigma 1 is gamma h then sigma 3 will be gamma times h multiplied by k. So; that means, that means sigma v will be some value if I draw the sigma v value like this then your active earth pressure will be if I multiply by k it will be reduced. So, with dept if I find out vertical pressure so, this is the diagram and this is the diagram will be active earth pressure ok. So, this is active condition.

Now, let me consider the so, I hope it is clear. So, this is Mohr circle for active earth condition and this is the failure envelope for the soil. Now considering the circle and if I

draw join center to the tangent point then if I consider these triangles above o A and B. So, your sin phi will become A B by sin phi will be equal to AB by oB and AB is nothing, but radius and AB is nothing, but sigma 1 plus sigma 3 by 2.

So, this you have done. So, sin phi become this and then if I simplify this one then sin phi multiplied by these then you can see ultimately sigma 3 become sigma 1 multiplied by 1 minus sin phi by 1 plus sin phi and sigma 1 you know that gamma h. So, gamma h in to 1 minus sin phi by 1 plus sin phi or gamma h k a where k is nothing, but 1 minus sin phi by 1 plus sin phi and these k a can be also expressed in terms of tan, that is tan square 45 degrees into multiplied by 45 degrees plus minus phi by 2. So, this is actually your active case.

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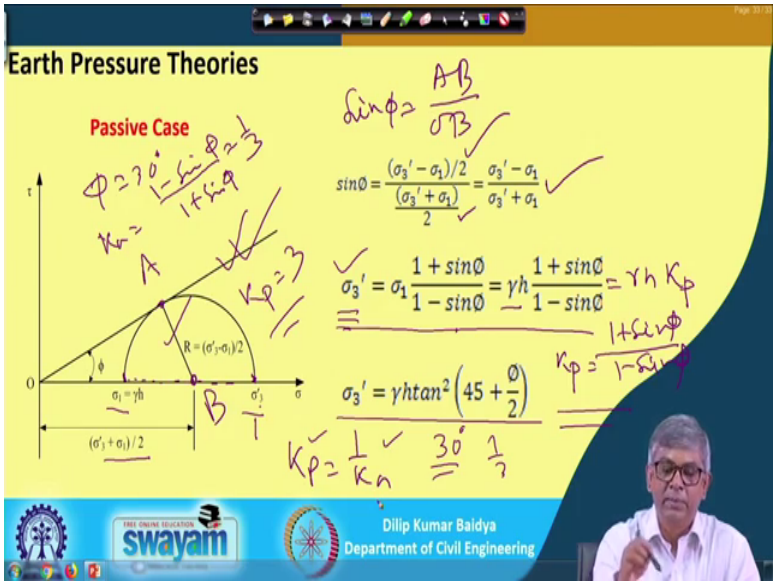

Similarly we can consider now passive case you can see here when it is passive case initially was sigma 1 was here and was here and sigma 3 become here. Now, sigma 3 is the major principles. So, sigma 3 actually lateral earth passive earth pressure sigma 3 is the lateral passive earth pressure and it is B. So, initially sigma 1 here and it was increasing and it become these and when it reaching to maximum that becomes the passive pressure. So, this is sigma 1 and this is sigma 3. In fact, in general we, though we say the larger one as sigma 1 and sigma and smaller 1 sigma 3, here actually little to avoid confusion I will keep this is sigma 1 and sigma 3 irrespective of whatever conventional we follow.

So, this is sigma 1 and this is sigma 3, the initial one was sigma 1 and final one is sigma 3 this is the way if I take and sigma 1 becomes sigma 3 in active case which is small and sigma 1 becomes sigma 3 in passive case which is big, bigger than the sigma 1. So, now, if this is the one failure, this is a Mohr circle and this is a failure envelope and now from the center if I joined to this tangent point. Now again if I take a circle this triangle way oAB and if I again sin phi if I express then sin phi equal to AB over oB and you can see where AB is nothing, but radius and A oB is nothing, but sigma 3 plus sigma 1 by 2.

And if I simplify this become sigma 3 minus sigma 1 by sigma 3 plus sigma 1 and again if I simplify then sigma 3 become sigma 1 1 plus sin phi into 1 minus sin phi you can see this and; that means, sigma 1 is nothing, but gamma h and again sigma and this can be again written gamma h K p where K p equal to 1 plus sin phi by 1 minus. Now, we can see that the K p is nothing, but 1 by K this is also very easy to remember relationship between K p and K a, it is K p is re inverse of k a.

So, and you can see if the K value is, if the if the angle of internal friction is 30 degrees then you can find out 1 minus sin phi by 1 plus sin phi, by that is we 1 by 3 actually. For if the phi equal to 30 degrees then K a equal to 1 minus sin phi by 1 plus sin phi that become 1 by 3. And so K p will become 3; that means, 1 by K 1 by K so it becomes 3. So, you can see know that sigma 3 become larger than sigma 1 here, you can see in the passive case.

So, this is the 1 so, either considering the wedge and mathematically maximize find out the critical angle, then find out the active pressure, passive pressure and then find out the angle of failure plane with horizontal that is the one we can find out. And by Rankine theories actually we know that failure plane that Mohr circles will be tangential to the failure envelope. And from there you considered the that geometrically from here either is active case and passive case we can relate sigma 1 and sigma 3. So, from here actually you can find out that what is the value of K a, what is the K p and from there you can find out what is the relationship between K and K p.

So, with these perhaps I will stop here I will see some more aspect when there will be wall is backfill is inclined and some other complication, then how to handle this will see in the next.

Thank you.