Geotechnical Engineering II / Foundation Engineering
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Lecture – 13
Shallow Foundation (Contd.)

Let me continue with the bearing capacity aspect of Shallow Foundation. And, in the previous lecture, I have just shown that if the same footing is placed on different soil generally even though if we apply through center pressure distribution below the footing will be little different. When it is a clay type of soil then edge pressure is more and center pressure towards the center is less and going to the sand generally edge pressure is less and at the center it is more.

And, some of the it not only depends on the footing soil type it also depends on footing type and some time it is a large footing, flexible footing placed on any soil. Generally, a settlement will be in the middle will be more and at the edge will be less, but surprisingly generally pressure below the footing will be uniform. And, whatever may be these are all some understanding how pressure varies below the footing, but we generally consider when the footing is load is applied through the center of the footing, then we assume generally the footing pressure is average pressure this is equal to load by area.

And finally, load by area and then we by applying bearing capacity theories we find out the ultimate bearing capacity, the net bearing capacity, the net ultimate, net allowable and finally, allowable bearing capacity and then in a particular footing some of the some amount of load and footing size is known then I can find out what could be the average contact pressure. And, in no ways average contact pressure because of the loading should be a should greater than the allowable bearing or other way you can say that that pressure because, of loading should be much smaller than the allowable bearing capacity of the footing.

So, this is the thing we have to do. In fact, by this way where we have find the when you first determine without knowing the footing size we generally in terms of some width and all we find out the ultimate bearing capacity, the net ultimate, the net allowable, then allowable and if the allowable bearing capacity is given for a particular size then what you have to do because of the loading condition and you have to find of the size of the footing. So, that is the keeping the pressure below the footing lower than the allowable bearing pressure. So, that is the thing in the design concept we generally you have to use.

But, as I have mentioned that some time in some condition the footing load may not be through the center of the footing and that may cause actually moment in the footing in addition to the vertical load. And, this moment loading and vertical loading when together acting, then what will happen; below the footing will never be the average pressure. There will be variation in pressure and how, because of the eccentricity of the footing how pressure below the footing varies and based on the eccentricity of the loading that I will show you in the next slide.

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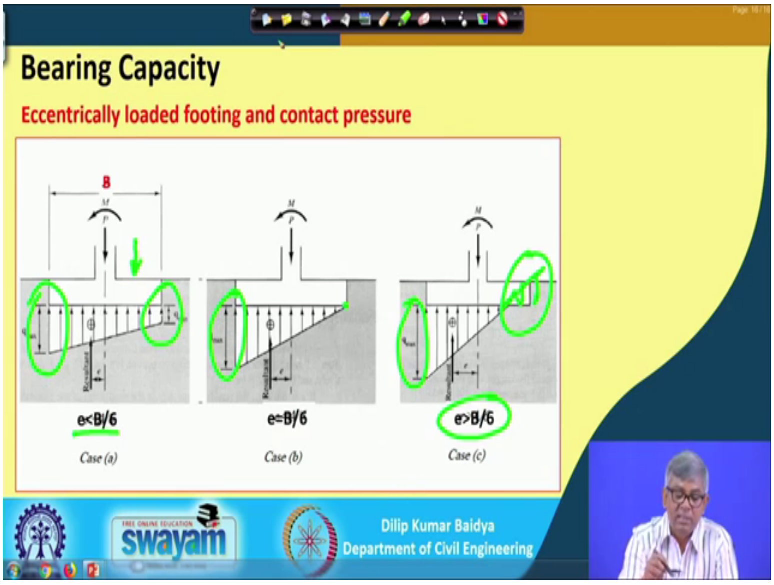

Let see you can see these if this is the width of the footing, if this is the width of the footing this is width of the footing and you can see suppose loading was somewhere here; loading was somewhere here and then I can imagine at the center there is a the same load in addition to this load multiplied by this eccentric distance as a moment. So, I will I can modified this one to in this form. So, initially suppose this thing was not there, only this loading was there and this loading can be visualized the equivalent to a loading at the center and in a mode moment. So, that means, I can assume that eccentrically loaded footing is idealized like this a moment in the moment at the and around axis and a vertical load through the center.

And, depending upon the value of e and e can e can see you can see e less than this I will come later on when e less than B by 6, then pressure distribution below the footing will be like this one side will be pressure will be high one side pressure will be less and in between it will be linear variation and if the e exactly equal to B by 6 then your one side contact pressure become 0 and one side contact pressure is for quite large.

And, if your e become greater than B by 6 then you can see one side pressure become quite large and other side actually it become tension actually it will become it is upward and this side will be downward and downward that; that means, the between the soil and for foundation there is a tension and generally soil cannot take tension that will be separation will take place. This situation should not be arise generally. So, you have to make sure that this type of situation will not occur.

So, that has to that actually in the design process you have to make sure that and if it happens then we ignore generally this portion and we consider only this portion is loaded and any of these and the loading has to be equated in such a way and accordingly you have to find out what should be the maximum value of pressure load can be applied. So, that will come later on.

So; that means, because of the eccentricity of the loading the pressure distribution below the footing will be different, it will not be uniform throughout the length. When it is e eccentricity is less than B by 6 the one-sixth of the width, then one side will be maximum one side will be minimum and they will be linear variation between this and when e exactly equal to B by 6 one-sixth of the width of the footing then one side will be a high value one side will be 0 and in between linear distribution.

And, when e become B by 6 it will become 0 pressure at some location of the footing beyond that this will be tension and tension should be ignored because, we soil cannot take any tension and so that point to be taken as 0 and this point to be taken as maximum value. And, in between is a linear variation since these are the different types of pressure distribution below the footing is below the footing can initialized because of the eccentricity of the loading.

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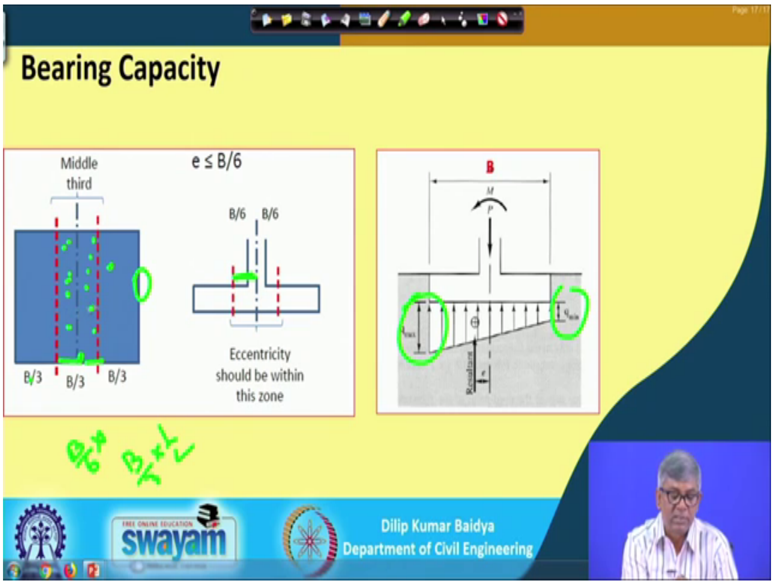

And, you can see now whatever I have shown actually we can see that you can see footing width can be divided by three parts B by 3, B by 3 B by at this is the center and exactly B by 6; B by 6 this is; that means, half of this. So, that is called one third rule, so, that means, the footing eccentric can be maximum can be this much or this much. So, that means, so, from here; that means, eccentricity maximum eccentricity can be B by 6, so, that means, B by 3 multiplied by half.

So, that means, that you have to apply the load if this is the footing if this is the footing either you can apply here, here, here, here, here like that to make pressure 0, because below the footing we generally do not accept the tension. So, if I if you are loading anywhere in this zone your one side will be 0 other side will be maximum when you cross this boundary. When the loading comes here then what will happen then this side there will tension and this side will be too much of compressive load. So, that is the range actually B by 3 B by then if can divided by three parts and so, eccentricity should be within these zone that is what.

So, B by c that means, maximum B by 6. Since one-third is B by 3, so, within these zone to be applied load to be applied and maximum eccentricity will be B by 6 because with respect to centroid. And, if it is if you can keep in general within this zone as they have shown in the previous figure the generalized pressure distribution is q minimum here and q maximum here with any value of e in between this zone.

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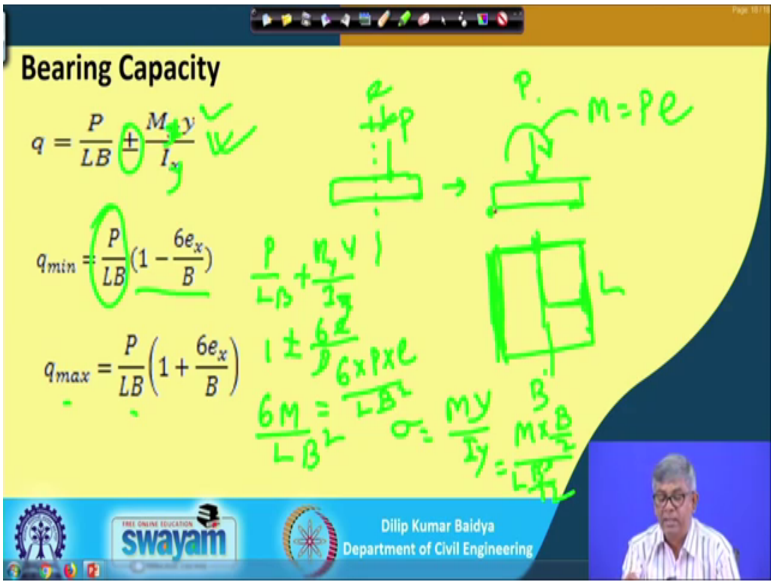

Now, you can see here as I have shown there if this is the footing and this is the centroid load is applied here and this can be considered as a loading and one moment and suppose your this is L and this is suppose B.

So, you can see now because of these loading the two loading one because of this compressive load that suppose P, this is suppose P and this is suppose M is equal to P multiplied by e, if this distance is e and this is P. So, because of this so, q at this point at this point q will be because of this P first. So, p because of the p what we are loading below these at this point? Because, of this loading P when it passing through the center it will be throughout the length it will be P by L by B. And, if I consider at this point since the moment direction is these then this when I consider this side it will be negative pressure and when I will consider at this point it will be positive pressure. So, because of that I can write in general plus minus.

Suppose, I want to find out that this point then it will be minus and or if I want to find out here this will be plus then what would be the. So, the moment is acting with respect to these axis and bending moment theory can be applied here you know that your stress sigma will be equal to M y by I and M is known and y is here how much it will be B by 2. So, it will be ultimately M multiplied by B by 2 divided by I and since the footing is rotating with respect to this axis then you have to find out I y that I y this one. So, I y will become this become then base and this become height. So, this will be L B cube divided by 12.

So, general equation is M y by I by I x and so, if I simplified if I simplify this for this equation if I simplify this equation then I can write once again this is this will become M 6M by LB square. This become 6 M by LB square and this is again can be written as 6 multiplied by P multiplied by e by LB square and. So, P by LB if I take P by LB if I take out so, first component was P by LB plus M y y by I x there should be I y, of course.

So, this is P LB M y by I b and M y M y into Y by I x or I y this is modified to 6 PL by LB square. So, P by LB if I take out from here P by L by if I take out so, it has come out here and then from P by LB if I take this become 1, and this become plus minus and you can see P is taken out LB square. So, it will be 6 times e divided by B will be left over.

So, that is why, so, this equation if I modify accordingly then your equation when you will use minus sign that become minimum q because when because of this direction of moment this has become moment because of moment pressure is negative and because the moment pressure is positive here. So, at this point there will be compressive pressure because of P so it is P by LB minus actually because of that M y M y into Y by I I x I y, this is sorry this is actually M x because moment with respect to this axis you know this is M y this is moment with respect to Y actually. So, this will be M y, this is correct; M y and this distance also Y and when I am considering this moment with respect to this axis this is I y, that is correct.

So, this is the way if I modify so, it become P by LB 1 minus 6 e by B and this side pressure will be maximum P by LB 1 plus 6 e by B. So, this is the maximum and minimum pressure when this loading is applied eccentrically.

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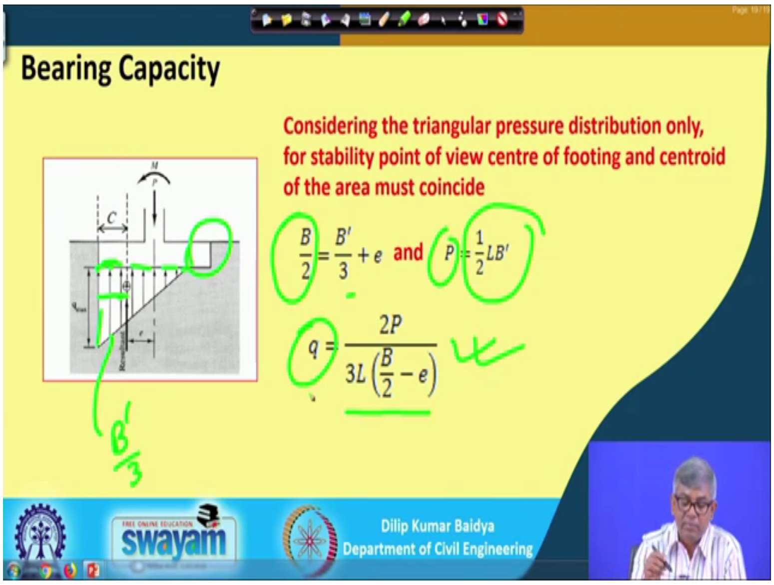

Now, you can see that as I have told you that it can be 0 also, when e equal to B by 6 that can be another condition can be modified. But, when your footing is e is greater than B by 6 so, this side the tension this side pressure is tension so, to be ignored. And, to stable the footing, so, what you have to do? You have to imagine that centroid of this pressure and centroid of this footing should be same. So, to do that you can see here this is the eccentricity and this must be B dash by 3 because the centroid of this triangle the centroid of this triangle will be here. So, this must be B dash by 3, this is B dash by 3. So, B dash by 3 plus e B dash by 3 plus e must be equal to B by 2.

And, and in that case total P whatever applied load must be equal to the area of this triangle that may half L half into L into B dash and so, ultimately in equating this you can find out maximum q will be equal to 2P by 3L into this. So, you have to while in the design stage that q should be allowed this much and this should be compared with the allowable bearing capacity of the footing.

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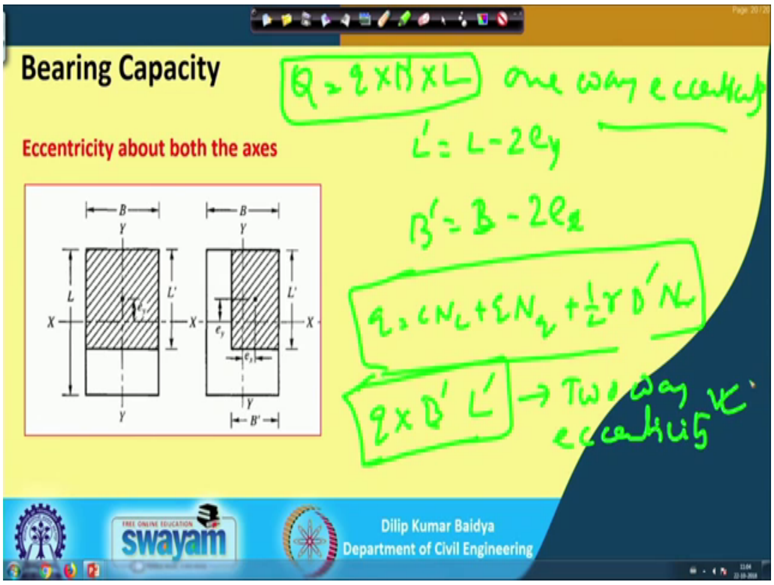

Now, already I have mentioned before that when one way eccentricity and this L become L dashed and B become B dash. And, so, L dash how it will be L dash become L dash equal to L minus 2 e y and B dash will be L minus 2 e sorry B minus 2 e x. So, this is the way we can change and then based on that we can find out bearing capacity is from for a q equal to CN c plus q N q plus half gamma B dash N gamma and based on that you can find out the bearing capacity and finally, to find out the load maximum load in the footing you can do q multiplied by B dash multiplied by L dash.

So, this is the way can be solved this problem; that means, how to find out the carrying load carrying capacity of the footing when either one way when is one way then you have to same way this is actually when two way eccentricity two way eccentricity and when it is one way same bearing capacity equation, but your load capacity q will be equal to q multiplied by B dash multiplied by L. So, this way you can find out this is one way eccentricity.

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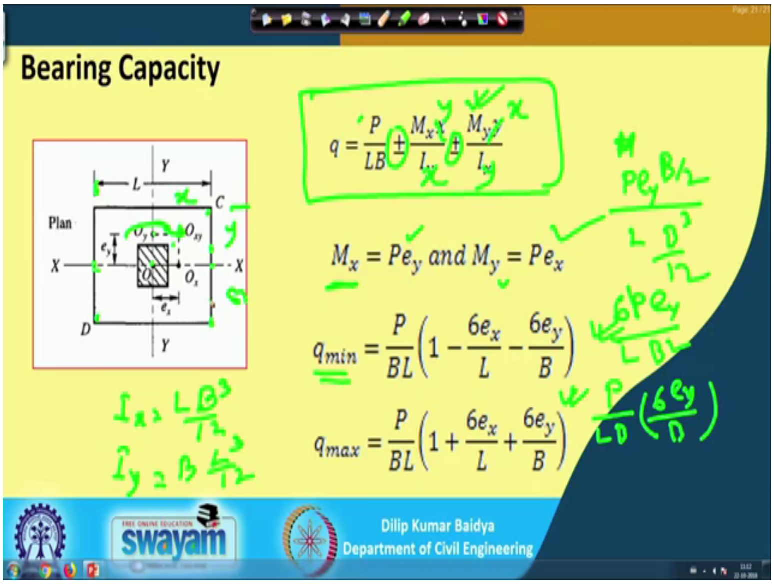

So, this is the way one can find out the loading in the footing, but if it is this is the simple way, but in addition to that similar to the one way eccentricity. For two way eccentricity also we can do this way when instead of load acting here or here if it is here this one way eccentricity if this here one way eccentricity, but the loading acting here that is two way eccentricity. So, two way eccentricity in that case generalized equation for q will be something like this. First of all when the loading is applied here then with around these axis there is a moment like this because of this eccentricity because of these moment because of this moment around this, moment around this and when the loading is acting here like this then what happened your this side will be positive and this side is negative.

So, like that so, depending our load is either here or here I can use a generalized plus minus sign. So, because of because of this so, when the load is here I can consider imagine load is here with a moment here and another moment with respect to x axis that is another moment M x. So, first of all I suppose consider because of these M y are because of M x suppose moment on the x-axis. So, because of the M x M x when I will consider M x multiplied by M x will be M x multiplied by it should be this distance will be Y actually. So, this must be Y and this will be ix M x when this moment with respect to x M x this is this is M x and then this distance is Y and with respect to moment with second moment of with respect to this axis that is I x.

So, M x into Y by I x and similarly when the loading is here then there will be moment with respect to y-axis. So, this is my same and when the my I am considering that pressure at this point and this point it side plus this side minus. So, I can use generalized sign plus minus and when I will take so, when I will take from here. So, this distance become x. So, this is corrected, this should be x actually. So, this will be M x and this will be I y actually with respect to this axis when a moment. So, this will be I y.

So, that means, three components will be there when the moment load is acting both way eccentric then it will you can imagine the load one central load and two couple one couple with respect to x-axis another couple with respect to y-axis. And, so, because of the vertical load this is the pressure and because of the location of the loading it can be plus or minus so, I have used. So, because of the moment we around x-axis, what is the pressure I have added? And, because of the moment around y-axis, what is the pressure I have added? So, these three component will be there.

And, M x will be M x will be equal to P multiplied by e y P multiplied by e y and my will be P multiplied by e x and then finally, similar to the one way eccentricity I can find out by expressing the I x and I y in terms of the L and B and maximum value of x and y by expressing in terms of half the length or half the width and simplify then q minimum reduces two way expression B by P by BL 1 minus 6e x by L minus 6e y by B and so, q maximum when it will be plus.

So, you can see I x, when I will use this one let me show this one how it coming how it is coming. M x actually your this part I will just do M x is actually your P multiplied by e y and this y become your y become y here y is here this is one L. So, this is actually because I have used so far B here, but it is L. So, it is it will be a so; that means, this should be B. So, this should be B by 2 and here actually when I will take moment with respect to this axis; that means, rotating with respect to so, this become so, this become L B cube by 12.

So, this has to be simplified. You can see this become so, this become 6P e y by LB square and you can see that P by LB if I take out P by LB if you take out then you can see 6 e y it become so P and. So, it will become P by LB if I take out then it become 6 time e y and below B will be there. So, you can see second term this term 6 e y by B it is coming for this one for this actually for this it is coming this and for this is coming this one.

So, that means, this I x I y is second moment of area when the moment is taking place with respect to this with respect to this axis then I have to take this I x I x that way when rotation with respect to x-axis it will be LB cube LB cube by 12 and when rotation with respect to y-axis; that means, with respect to this y-axis; that means, I y that will be equal to BL cube by 12. So, accordingly if I substitute in this expression then your q minimum become this and q maximum become this.

So, ultimately you can find out because of the position of this loading point you can find out different corner, what is the maximum pressure, minimum pressure and that has to be equated to the allowable bearing capacity. If it is exceeds the allowable bearing capacity of that then it has to be modified. So, that is one part. Second part is that if I this q minimum and q maximum if I equate to 0, then you can see that I will getting equation of a line of where e x and e y is the variable, ok. If I equate to 0, this if I equate to 0 is equate to 0, then 1 minus 6 e by L minus 6 e y by B become a equation of a line.

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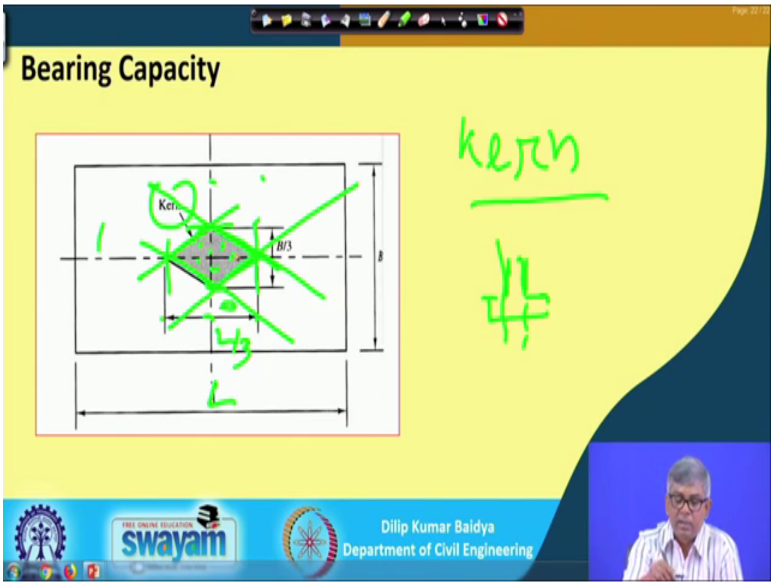

So, I will go to the next slide and you can see. So, this may be the equation of one line, one side, this is one line. This may be another line, this may be another line, this may be another line; so, that means, if your loading varying point of application of loading vary within these zone, it is not going beyond this if it is within this zone then the contact pressure below the footing will be always positive and if it goes out of this then contact pressure may become tension actually.

So, to maintain positive pressure within the footing base what you have to do you have to apply the load within this zone and this zone is called kern that is actually it is called kern; k e r n, kern. So, that way within this area one has to apply the load. So, that what is the distance actually if this is B and this is A? So, this should be so, this B better I will write as L; so, this will be L by 3 and this is B by 3. So, that means, this will be B by 6 maximum, this is B by 6 this side L by 6 L by 6 that is the whatever we have seen before similar thing only there will be a trapezoidal area will get central. Before actually what we have got? So, we have got a zone because footing was now you have assumed in 2-dimension. So, we have got a zone.

Similarly, we have got this is zone when I am considering; similarly, when I consider with respect to this, this is the zone. So, when two zone then I will get a trapezoidal area; this is a rhombus actually, in fact. So, that we L by 6 from here, L by 6 from here, similarly B by 6 from here, B by 6 from here if I join them then we will get a kern that actually; that means, the area within which if we apply the load then pressure with below the footing will be always positive. So, that has to be maintained generally for our design. So, with this I will stop here.

Thank you.