

**Geotechnical Engineering II / Foundation Engineering**  
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**Lecture – 12**  
**Shallow Foundation**

Good morning once again welcome to this lecture series on Foundation Engineering. And we are discussing about Shallow Foundation bearing capacity and I have already discussed various aspects of the bearing capacity and finally, I have discussed about Terzaghi's bearing capacity.

And basically Terzaghi is the first person who has given bearing capacity concept in acceptable form. Before that of course, there were some work and then he has introduced bearing capacity in the three component. Cohesion component, structures component and unit weight component and he has given three factors and those factors of course, later on when compared with experiment and all found little bit of while estimation. And obviously, to improve that and also for the research purpose many people investigated on bearing capacity and there are plenty differences work literature available in the literature.

And perhaps we can discuss all those things in detail in M. Tech level post graduated level and the B. Tech under graduated level I pass bearing capacity it concept is generally is enough. But I want to discuss one more theory where actually we can introduce so many other parameter or factors.

And basically when other theories are developed ; obviously, Terzaghi bearing capacity analysis there are some assumption and it could improved assumptions some of the limitation may be overcome at later stage and may be some other additional assumption was made. So, like that several bearing capacity equations, theories given in the literature and by and large they are similar basically.

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**Bearing Capacity**

Bearing capacity theories given by others: Hansen, Vesic, Meyerhof and many others worked on bearing capacity theories. They are by and large similar concept with minor modification in the assumptions. All gave the bearing capacity equation in the form of the components as it was given by Terzaghi. Only variation is in  $N\gamma$  mostly. Out of the many theories finally Meyerhof's theory is most acceptable. Generalised equation for bearing capacity given by Meyerhof is as given by,

$$q_u = cN_c s_c d_c i_c + qN_q s_q d_q i_q + 0.5B\gamma N_\gamma s_\gamma d_\gamma i_\gamma$$

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Similar in the sense, similar in the sense these bearing capacity equation again given in the, from of three components, one is a surcharge component another is cohesion component another is weight component unit weight of the soil component.

And only difference in and this  $N_c$   $N_q$   $N_\gamma$  from this factors are given and only difference mostly in this  $N_c$   $N_q$   $N_\gamma$  value and when discussing Terzaghi bearing capacity theory that time I have mention that that determination of  $N_\gamma$  is little bit of difficult. And they have done and they have given in the form of chart or graph.

And here what the time people also in this created on  $N_\gamma$  how to simplify it and they might have they have given finally, in the simplified form. And in addition to that there are some additional assumption or multiplication is done and in literature actually there are number of bearing capacity theory is available like Vesic, benzenes then your Meyerhof like that there are many theories are available.

But I will discuss basically on Meyerhof theory and this is the one Meyerhof theory is actually what we has given that I will discuss. And what way it is different from your Terzaghi theory is Terzaghi's bearing capacity concept was something like this, if this is the footing. So, it is like this so bearing capacity concept was something like this and then there are three zone 1, 2 and 3 zone.

So, of course, for Terzaghi bearing capacity, it was the limit and here actually the surcharge and that is all and resistance from this consider. But it Meyerhof equation Meyerhof theory actually this shear plane extended up to also beyond putting base and some additional resistance from this also considered.

So, that is the difference mainly and with this concept and so many other things also observed like though  $N_q$ ,  $N_\gamma$  and  $N_c$  they are function of  $\phi$  it is given, but later on it is seen that with shape of the foundation or with depth of the foundation these also vary little.

So, because of that they have given a correction factor, they are number of correction factors I will come to that and other wise Terzaghi bearing capacity it has given only derived equation for strip footing and he has given  $q_{ultimate}$  equal to  $CN_c$  plus  $qN_q$  plus half  $\gamma b N_\gamma$  this is the form in the given for strip footing. And then he has extended and you suggested for square and circular footing, when it is a square and circular this one multiplied by 1.3, and this one become 0.4 for square and 0.3 for circular footing that was the thing given by Terzaghi whereas the footing can be of rectangular shape of different aspect ratios.

So, because of that because of the shape of the footing and because of the depth of the footing those factors  $N_c$   $N_q$   $N_\gamma$  may vary little. So, because of that to make it very generalized different people actually given the equation of bearing capacity ultimate bearing capacity in this form and you can see in this form when you are doing then this if there are so many earlier it was there only  $cN_c$ .

Now, we can see there is a factor  $i$   $c$  there is a factor  $d$   $c$ , there is a factor  $i$   $c$   $s$   $d$   $q$   $I$   $q$  then  $s$   $\gamma$   $d$   $\gamma$   $N$   $i$   $\gamma$ . So, these are the so many factors are included in this to these are all correction factors and this correction factor for cohesion for actually the safe factor cohesion. Correction in two bearing capacity factor for cohesion for safe.

Similarly, correction factor for depth of embedment, correction factor for inclination of the load so; that means safe factor is what? That means, footing can be of circle, footing can be of square, footing can be of rectangle, footing can be of long street. So, like that these are the because of the different shape, we can have this  $s$   $c$  in the cohesion per similarly for different shape we can get a correction here that is  $s$   $q$ .

Similarly, for different shape we can get a correction here that is actually  $s_\gamma$ . So, like that like that a shape similar to that there can be depth so, footing can be of footing can be placed like this and this depth this is the depth of the foundation  $D_f$  this can be 1 meter, 2 meter, 3 meter, 4 meter like that. So, the depend also so when with the variation of the depth of foundation this  $N_c$   $N_q$   $N_\gamma$  also vary so that actually given in  $d_c$   $d_q$  and  $d_\gamma$ .

And another thing is the load. So, far in this loading is considered as vertical; that means, if the footing is here and the load is considered to be if the vertical direction and. So, this is the vertical loading and then, but this loading can be because when there is a movement or something then resultant force can be inclined.

So, because of that inclination so there is a factor, correction factor for inclination of the load so  $i_c$   $i_q$  and  $i_\gamma$ . So, that like that in a very generalized form this bearing capacity equation is expressed by Meyerhof and others and so finally, so he has suggested since additional consideration is considered in the in the shear itself above the base of the footing. Say  $N_c$   $N_q$   $N_\gamma$  also little varied and in addition to that the interaction of  $a_c$   $d_c$   $i_c$   $s_c$   $q$   $d_q$   $i_q$  and  $s_\gamma$   $d_\gamma$   $i_\gamma$  are given.

So, these are the so, he has given all those in a in the particular form. So, that one can use easily. So, I will go one by one to those thing. So, this is the generalized bearing capacity equation.

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**Bearing Capacity**

$N_c, N_q, \text{ and } N_\gamma$  Bearing capacity factors for cohesion, surcharge and weight of the soil, respectively

$s_c, s_q \text{ and } s_\gamma$  Shape factors for cohesion, surcharge and weight of the soil, respectively

$d_c, d_q \text{ and } d_\gamma$  Depth factors for cohesion, surcharge and weight of the soil, respectively

$i_c, i_q \text{ and } i_\gamma$  Inclination factors for cohesion, surcharge and weight of the soil, respectively

C is the cohesion of soil,  $\gamma$  is the unit weight of soil, B is the width of the footing and q is the surcharge pressure at the level of footing base

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Next is you can see now we are giving you the different  $N_c$ ,  $N_q$ ,  $N_\gamma$  they are actually bearing capacity factor for cohesion surcharge and weight of the soil unit weight of the soil. This is actually same as before and ah; that means, but the value will be little different because the expression everything I will come to that next slide.

And  $s_c$ ,  $s_q$ ,  $s_\gamma$  that is shape factors for cohesion surcharge and weight of the soil; that means, shape factor this will be includes  $N_3$  components. Because of the shape that in the surcharge component cohesion component and weight of the soil component there will be little change. That changes how to incorporate? By it can be incorporated by introducing this shape factor so  $s_c$ ,  $s_q$ ,  $s_\gamma$ . So, because of the shape the factors will be introduce in all three parameter components and when it is introduced in cohesion component that is shape factor for cohesion where it is introduced for surcharge that is called shape factor for surcharge. When introduced in weight for weight of the soil part then that is a factor for unit weight is  $\gamma$ .

Similarly, three more factors like  $d_c$ ,  $d_q$ ,  $d_\gamma$  also introduced and they are called depth factors and as I have shown in the previous slide that when the with the increase of the depth of the foundation your this  $N_c$ ,  $N_q$ ,  $N_\gamma$  or that total resistance because of cohesion surcharge and unit weight of the soil it will it will differ.

And so that differ how much it will be it will be differ it will differ and that actually incorporated by a factor and that factor actually when we are introduce in cohesion part that is called depth factor for cohesion. When introduced in surcharge part that is called depth factor in surcharge, when introduce in weight part then that is called depth factor for unit weight of the soil.

So, this three will be introduced in three different part similarly  $i_c$ ,  $i_q$ ,  $i_\gamma$ ; that means, as I have told you that when the loading is something like this your footing is something like this and loading is something like this ok. So, in that case there is a some inclination and of the applied load because of this inclination the bearing capacity will not be same so, that has to be modified. So, that all three components will be getting modified.

So, how to modify? You can introduce some factor that is  $i_c$ ,  $i_q$ ,  $i_\gamma$ . Then  $i_c$  means, when inclination factor when given to cohesion  $i_q$  means inclination factor to the surcharge and  $i_\gamma$  is inclination factor in the unit weight part.

So, all three so; that means, total 9 here 3 already there so 12 parameters and. So, c is the cohesion of the soil gamma is the unit weight in the previous bearing capacity whatever was there. So, c will be cohesion and gamma unit weight b is the width of the footing and q the searchers pressure at the level of the footing; that means, if you think of the footing like this. So, this much depth of the soil will give you some surcharge here if this is D f. So, they are actually surcharge will be gamma times D f.

So, this that has to be considered. So, that is the way the introduced. Now let me see the next slide.

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**Bearing Capacity**

As suggested by Meyerhof

$$N_q = e^{\pi \tan \phi} \tan^2 \left( 45^\circ + \frac{\phi}{2} \right)$$

$$N_c = \cot \phi (N_q - 1)$$

$$N_\gamma = (N_q - 1) \tan(1.4\phi)$$

Relation between Bearing Capacity Factors and  $\phi$

$\phi^\circ$	$N_q$	$N_c$	$N_\gamma$
0	5.14	1.00	0.00
2	5.63	1.20	0.01
4	6.19	1.43	0.04
6	6.81	1.72	0.11
8	7.53	2.06	0.21
10	8.34	2.47	0.37
12	9.28	2.97	0.60
14	10.37	3.59	0.92
16	11.63	4.34	1.37
18	13.10	5.26	2.00
20	14.83	6.40	2.87
22	16.88	7.82	4.07
24	19.32	9.60	5.72
26	22.25	11.85	8.00
28	25.80	14.72	11.19
30	30.14	18.40	15.67
32	35.49	23.18	22.02
34	42.16	29.44	31.15
36	50.59	37.75	44.43
38	61.35	48.93	64.08
40	75.32	64.20	93.69
42	93.71	85.38	139.32
44	118.37	115.31	201.41
46	152.30	158.51	289.74
48	199.27	222.31	526.47
50	266.89	319.07	873.89

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And at the beginning actually I have shown that bearing capacity equation and there all these are almost similar. And this one I have told that itself actually since N gamma is difficult to estimate where the a lot of mathematical complexities are there. So, Meyerhof also did that and for different values of phi he could calculate N gamma and finally, he has treated that N gamma in the form of this equation.

And finally, this is the form of equation is given for it calculation of N c. N q, N gamma also he has given the table form; that means, with the variation of phi how N c N q N gamma varied that also is given. So, you can see for 0 when phi equal to 0; that means, q cohesive soil the value of N c is 5.14 and N q is 1 and N gamma is 0.

Now, one thing to be noted here that Meyerhof  $N_c$  value for cohesive soil when  $\phi = 0$  condition  $\phi$  equal to 0 condition  $N_c$  become 5.14. Whereas, in Terzaghi bearing capacity equation this when  $\phi$  equal to 0 the  $N_c$  was 5.71 this is one difference has to be noted. That means, sometimes if the question is asked that apply bearing Meyerhof equation and find out bearing capacity on the clayey soil. Clayey soil means that was  $\phi$  may be 0 in that case your bearing capacity will equation will be equal to  $c N_c$  and sometime you may divide that what is the value of  $N_c$ , but since 0 for  $\phi$  equal to 0 called the  $N_c$  value has to be remembered.

If it is Terzaghi's theory then it is 5.771 and if it is a Meyerhof theory this is 5.41. So; that means, directly I can write  $5.14 c$  as per your Meyerhof equation author theory and  $q_{ultimate}$  I can write  $q_{ultimate}$  equal to  $5.71 c$  as per Terzaghi's bearing capacity theory.

So, these are the one difference and similarly if you compare with Terzaghi's table with the Meyerhof table we can we can compare at any angle of  $\phi$  and you can compare these three values and you will see there are some changes, somewhat if by different they are different actually. So, particularly a when it is a 30 degrees is this value of was 37 this value was sometime like 20 something this was 18 something like that there are little little different. But ultimately the Meyerhof bearing capacity was developed later on, but more logical and because of that is more acceptable to people actually in the industry may use either Meyerhof or basic, basic we are not discussing they are almost similar.

So, Meyerhof theory only I am discussing. So, if it is not mentioned you have every right to use either Terzaghi or Meyerhof, but if it is mentioned that is the use Meyerhof equation then you have to use this table. And if it is mentioned Terzaghi used Terzaghi's theory then you have use Terzaghi's table and sometime it may be value of  $N_c$ ,  $N_q$ ,  $N_\gamma$  for  $\phi$  value may be given in you have to directly to be used whether it is Terzaghi or Meyerhof we do not have to worry about it.

So, this is in the tabular form and this is in the equation form  $N_c$ ,  $N_q$ ,  $N_\gamma$  one can obtain. Next part actually, there are additional 9 components.

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**Bearing Capacity**

$s_c, s_q$  and  $s_\gamma = 1$  for strip footing

$$s_c = 1 + 0.2 \frac{B}{L} \tan^2 \left( 45^\circ + \frac{\phi}{2} \right)$$

$$s_q, s_\gamma = 1 + 0.1 \frac{B}{L} \tan^2 \left( 45^\circ + \frac{\phi}{2} \right) \text{ for } \phi > 10^\circ$$

$$s_q, s_\gamma = 1 \text{ for } \phi = 0$$

For Rectangular footing

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The next slide are given you can see here that  $s_c, s_q$  and  $s_\gamma$  when it is strip footing it is 1, when the strip footing it is 1; that means, you can see; that means, it is reduce it to Terzaghi's bearing capacity. If the it becomes  $cN_c$  plus  $qN_q$  plus; that means, there is no safe factor and for rectangular footing you can see, for rectangular footing he has given  $s_c$  equal to 1 plus 0.2  $b$  by  $l$  tan square 45 degrees plus  $\phi$  by 2; that means, if there is a footing something there rectangular footing. So, this is actually your  $B$  and this is your  $L$ . So, you have to put  $B$  and  $L$  and tan square 45 degree plus  $\phi$  of the  $\phi$  of the soils suppose 30 degrees then you have to use 30 by 2 15, 45 plus 15 60. So, tan square 60 multiplied by  $B$  by  $l$  multiplied by 0.2 last one.

So, like that if it is the. So, if it is suppose  $B$  over  $L$  if it is  $B$  over  $L$  is suppose a it is 1 and it is 2 then it will be suppose for  $L$  by  $B$  equal to 2 then in that case the value will become  $\phi$  equal to 30 degrees it will become 1 plus 0.2 multiplied by 1 by 2 multiplied by tan 60 degrees a tan 0.3 square. So, a root 3 square. So, it become 3. So, you can see 1 plus 0.6 by 2 so; that means 1.3. So; that means, when there is a rectangular footing of  $L$  by  $B$  equal to 2 then your shape factor  $s_c$  become 1.3.

So,. So, that is the thing is as far Meyerhof equation and  $s_q, s_\gamma$  he has given two sets of equation  $s_q, s_\gamma$  he has given two sets of equation one for when  $\phi$  equal to greater than 10 degrees these are all actually empirical equation. So, it is fitted equation. So, because of that this, the fitting may not be applicable for what the full range of  $\phi$ .



So, because of that he has divided in two parts up to phi greater than 10 degrees this is the equation. That means, if B by L 1 by 2 then this will be your 0.1 1 by 2 times square for 60 degrees again that can be done. So, it will be 1.1, in that case it will become 1.15.

So, s q and s gamma become 1.15 and when phi equal to greater than thirty degree that were 30 degrees I have assumed initially. So, it become 1.1. So, how it will become? 1 plus 0.1 multiplied by 1 by 2 multiplied by 3. So, it will 0.3 by 2 there were 0.15. So, it become 1 plus 0.15 equal 1.15.

So, the s q s gamma for a rectangular footing ratio L by ratio 2 and 530 degrees it become s q s gamma become 1.15 and suppose if the phi is equal to 0 a phi equal to 0 then sq s gamma equal to 1 for phi equal to is. That means, of course, it is not given between 0 and 10 degrees what will be the value one can use their judgment and little change can be, little less than this and little greater than 1 can be used. So, this is the two ranges given. So, in between one can interpolate (Refer Time: 23:05).

So, this is the way so; that means, that how many we have given sq s gamma for strip footing is one and s c value there is a universal value with this for to phi is given and sq gamma for phi greater than 10 degrees this given value and if it is 0 this given another value. So; that means, s c s q s gamma this 3 quantities already explained.

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**Bearing Capacity**

$$d_c = 1 + 0.2 \frac{D}{B} \tan \left( 45^\circ + \frac{\phi}{2} \right)$$

$$d_q = d_y = 1 + 0.1 \frac{D}{B} \tan \left( 45^\circ + \frac{\phi}{2} \right) \text{ for } \phi > 10^\circ$$

$$d_q = d_y = 1 \text{ for } \phi = 0^\circ$$

$$i_c = i_q = \left( 1 - \frac{\alpha}{90} \right)^2 \text{ } \alpha \text{ in degrees}$$

$$i_y = \left( 1 - \frac{\alpha}{90} \right)^2 \quad \alpha = \tan^{-1} (Q_h / Q_v)$$

The slide also features a diagram of a footing on soil with parameters  $D$ ,  $B$ ,  $L$ ,  $Q_h$ , and  $Q_v$ . At the bottom, there are logos for 'swayam' and 'Dilip Kumar Baidya, Department of Civil Engineering' along with a small video inset of the presenter.

Now, next three components you can see  $d_c$ ,  $d_q$  and  $d_\gamma$  here also you can see  $d_c$  again expressed  $1 + 0.2 d$  by  $b \tan 45^\circ$  is not square here,  $45^\circ$  plus  $\phi$  by 2 and  $d_q$  equal to  $d_\gamma$  equal to this expression when  $\phi$  equal to greater than  $10^\circ$  degrees and  $d_q$  (Refer Time: 24:00) 1 for  $\phi$  equal to. So; that means, again  $d_c$  that is universal value there is no limit for  $\phi$  value, but  $d_q$   $d_\gamma$  if it is  $\phi$  greater than  $10^\circ$  degree this is a value and if the  $\phi$  equal to  $10^\circ$  degrees this is the value.

Now, in between 0 to  $10^\circ$  degree values are not given, one can use that judgment to use the value in between values if it is  $\phi$  equal to  $8^\circ$  or  $9^\circ$  or  $6^\circ$ . So, in between values can be use so; that means, by this expression the  $d_c$   $d_q$  and  $d_\gamma$  are given. So,  $d_c$  is the unique equation at  $d_\gamma$   $d_q$   $d_\gamma$  for depend on  $\phi$  greater than  $10^\circ$  degree this is the equation and if 0 if this is the equation.

Similarly,  $i_c$   $i_q$  given in this by this equation and this is given in the form of equation  $1 - \alpha$  by  $90^\circ$  and  $\alpha$  in degrees and  $\alpha$  equal to  $10 \tan^{-1} Q_h$  by  $Q_v$ . This is actually  $Q_v$  and this is  $Q_h$  and  $Q_h$  by  $Q_v$ ; that means, vertical component and horizontal component ratio  $\tan^{-1}$  if you do that you get the  $\alpha$  value and that  $\alpha$  should be expressed in degrees. For example, if it is a this  $\tan^{-1} Q_h$  by  $Q_v$  become  $40^\circ$  or  $50^\circ$ .

Then I have to re use  $1 - \sin$  suppose  $30^\circ$  then  $30$  by  $90$ . So, it will become  $1$  by  $3$ . So,  $1 - 0.33$ ; that means  $0.67$ . So; that means,  $i_c$   $i_q$  will be reduced to  $0.67$ ; that means, if I keep the vertical load vertical whatever bearing capacity if the load is inclined then your bearing capacity has to be reduced, that is what factor is becoming smaller than 1.

Similarly,  $i_c$   $i_q$  given by this equation and  $I_\gamma$  given by this equation that is  $1 - \alpha$  by  $\phi$ ,  $\alpha$  by  $\phi$  here  $\alpha$  as it is and both are in the same unit to be expressed in degrees. So,  $\alpha$   $30^\circ$   $\phi$   $30^\circ$  then you have 0 actually otherwise depends on the value  $\alpha$  equals  $10^\circ$  and  $30^\circ$ . So, it become  $1$  by  $3$   $1$  was like that whatever value it comes based on this  $i_\gamma$  to be calculated. And then this can be introduced in the generalized equation to get the bearing capacity.

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**Bearing Capacity**

$$q_u = cN_c s_c d_c i_c + qN_q s_q d_q i_q + 0.5B' \gamma N_\gamma s_\gamma d_\gamma i_\gamma$$

**Effect of eccentric load**

For two ways eccentricity

$$B' = B - 2e_x$$
$$L' = L - 2e_y$$

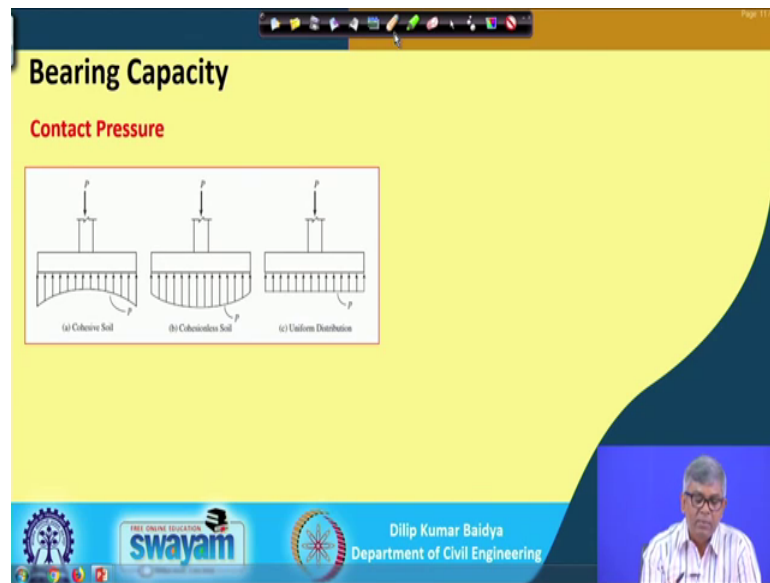
Eff width,  $B' = B - 2e_x$

Now sometime your assent load is ascetic like if I, if this is the area and this is the centroid this is the centroid, but your instead of load applied through this point it is applied eccentrically somewhere here and. So, two ways it can be done, it can be reduced the area become. So, what have to do the, if the load is applied eccentrically then that will be movement.

So, to overcome that difficulty what you can be done the footing can be imagined in such a way that point of application and center of the area will be same. So, if I want to do that then what I have to do I have to reduce this area. So, how to reduce this? The if the eccentricity is e and width of the footing is b then that effective width become effective width b it will be b minus twice e and so, that is the only way the b you can; that means, if the load is eccentric; that means, there also bearing capacity has to be reduced. How to that same equation can be taken, only here instead of B B dash can be that effective unique effective weight will be used.

And if the if the eccentric is in both the direction then; that means, if the footing is somewhere here like this and load is applied here. So, there will be eccentric this direction in this direction. So, effective weight can be a calculated by the B minus 2 e x, effective length will be can be L minus 2 L e y and then we can best first we can calculate the bearing capacity based on width. And then capacity of the footing can be used by q ultimate multiplied by B dash multiplied by L dash. So, like this.

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Now, this contact pressure distribution here actually below the footing when load is applied, depending upon type of soil and type of footing different we have distribution will be same. When the cohesive soil generally a is pleasure will be more and center pressure will be less and if there is purely cohesive soil generally theoretically this is actually very high value and it middle actually is quite less value.

When it cohesion less soil is generally pressure is less and the middle pressure is maximum and if it is a flexible footing then what happens generally the putting settlement become uniform non uniform, but pressures become uniform. And most of the time we will try to use the uniform pressure distribution whatever may be the condition where in the in the safer side we use uniform pressure distribution.

And when you do this uniform pressure distribution and bearing capacity and based on the loading on the footing whatever pressure contact pressure is coming that contact pressure suppose  $q_c$  must be less than the  $q$  all over. That means, bearing capacity to the soil should be greater than the pressure because of this loading. So, because of that actually we have to see now the contact pressure how for different loading condition, how the contact pressure changes.

First of all if it is a concentric loading because of the soil type some variation will be there, but most of the time we take this, but if the loading is not concentric or to the center then the variation of the pressure over the footing width will be different. And for

that how to solve this problem I will have to see the as a next lecture. With this I will close here.

Thank you.