Matrix Method of Structural Analysis
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Lecture - 08
Review of Structural Analysis - I (Contd.)

Hello everyone, welcome to lecture 8 of second week.

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We have been discussing or reviewing the concept of force and displacement method in the last the first two classes of this week we reviewed the force method; method of consistent deformation also discussed the method of consistent deformation retaining terms of moment; which is 3 moment equation. Now today we will first review the concept of displacement method.

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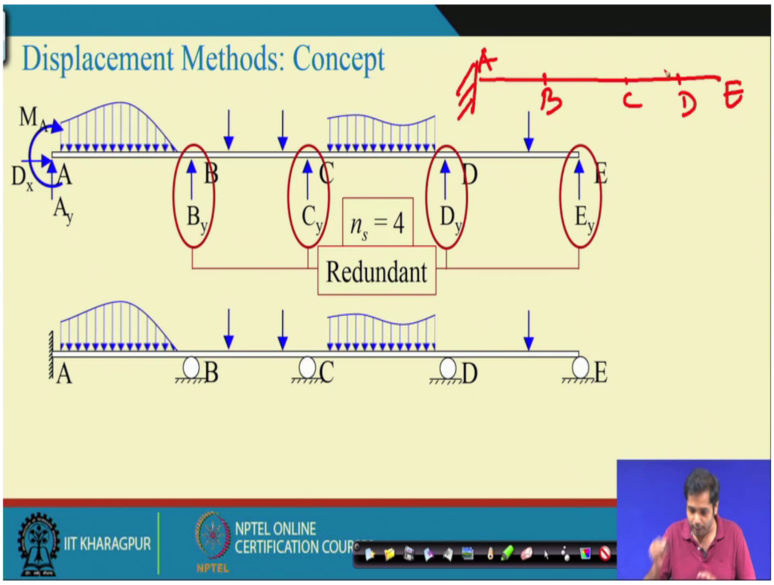

Now, you see if you recall the force method is a as name suggest force are the primary are known; though we have to once we have compute the static indeterminacy then identify the redundant which is the forces and a which is the number equal to the number to the static indeterminacy and once then the redundant is a redundant forces are identified then release those redundant forces, remove those redundant re release those corresponding carminative variables, corresponding displacement associated with the with the with the forces, redundant forces.

And then you get the primary structure, the determinant structure then you solve the determinant structure for both sub the external and also for the redundant forces and then apply the compatibility equation to get the additional unknown.

Now here a here the in terms of a we will be doing the displacement method is also based on the similar philosophy, but the, but instead of forces it is the displacement as the name suggest; the displacements which are taken as primary are known ok. So,, but rest of the thing is the your the all these steps that you have force in the force method we have the similar steps in displacement methods as well, but in the force method the forces the all the methods are methods are a all the steps are due to the due to the fact that due to the forces are taken as redundant and here the similar method similar steps we have, but the steps is different because the displacements are taken as primary unknown.

Now let us try to understand the this to the same examples ok, now this is the indeterminate structure now by just looking at this structure we can say that this static indeterminacy of the structure is 4 and again we also know how to determine kinematic determinacy and again if you look at these structure we can see the kinematic in determinacy of the structure is also 4 all right.

Now, then what we have is actually when we see the kinematic indeterminacy is 4 we neglect the axial deformation in the in this structure if the axial deformation is neglected if we if we only if we and only allow 2 kind of deformation one is a transfers deformation and the rotation at any joint then only we can see the kinematic indeterminacy of this structure is the structure is 4 otherwise the axial deformation also we need to we need to take into account ok; which is we are not taking into account right now.

Now so if we in the force method if we substitute if we if we if we write the entire all this all the reaction all the supports if we replace them by their by their equivalent forces these are the equivalent forces now so static indeterminate structure is 4.

Then suppose these are redundant forces. So, if we take these as redundant forces then your structure becomes your basic primary structure becomes a cantilever beams right cantilever beams which is A and then B C D and E this is the primary structure. Now we need to solve these structure 4 times, 5 times one is under the external load and then 4 times for different redundant forces 4 different redundant forces and then apply the com compatibility at 4 different joints to get the 4 additional equations.

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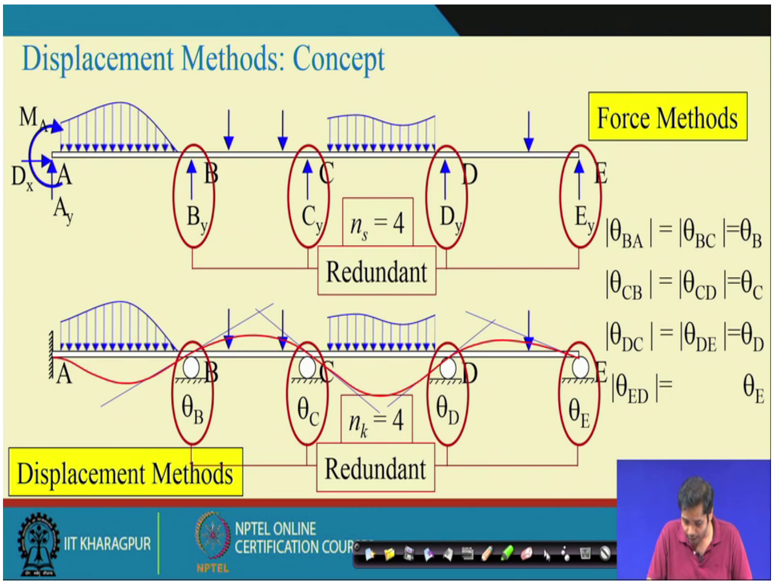

Now so this is the primary structure for force method, now displacement method as displacement plays the major roll so let us take let us assume the red one is the de deformed shape of the of the structure it could be a it could be a it could be the deformation could be different, but it could be just for the demonstrations suppose this is the deform shape of the structure of this continuous beam.

Now, let us take slope at various joints B C and D are and E are this; suppose theta BA theta BA theta BA is the slope in slope at B in segment BA and theta BC is the slope, slope at B in segment BC; means theta BA is this and theta BC is this. Similarly theta CB theta CB is this and this is this is theta CD as for the compatibility conditions as com compatibility that every joints because of the continuity we can say that these 2 has to be have to be same. Suppose we can write that theta B, theta C, theta D, theta E are the slope at various joints there is no theta A at theta there is no rotation at a because if joints a is fixed so therefore, rotation is 0.

Now, then so theta B, theta C, theta D are the corresponding rotation and here the kinematic indeterminacy is 4 there is kinematic there is no indeterminacy here it is 0 there is no displacement there is no rotation here so only this so only 4 displacements which can which can which can which a which can represent the configuration of this structure is rotation at 4 joints therefore, kinematic indeterminacy is 4 ok. Now these are the redundant so in the force method we take B y, C y, D y, E y at the redundant the forces at the redundant.

Now, in this case theta B, theta C, theta D, theta E the displacements at the joints or rotations at the joints or the redundant. So this is force method and this is displacement method.

Now, this is the difference between force and displacement method rest of the things it is the main differences between the force and force method and displacement method and the rest of the things are the just consequence of this difference ok.

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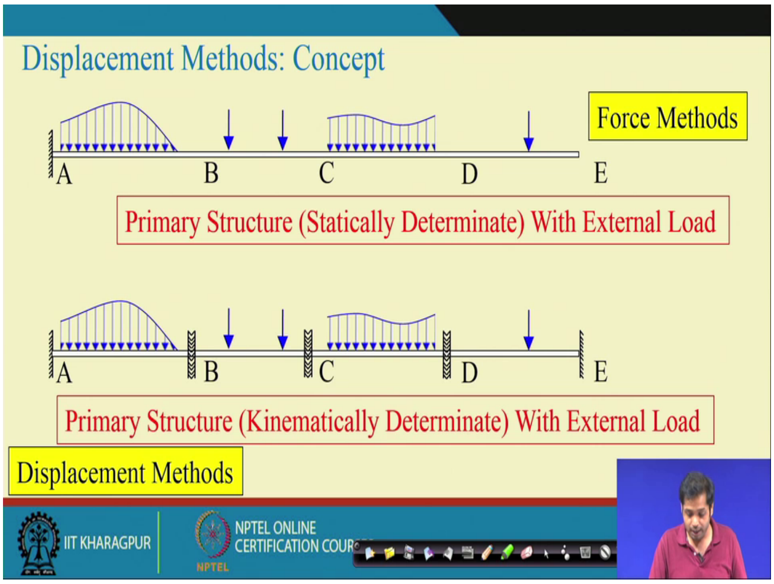

Now so let us see what is so in the force method what do we do this becomes are primary, this becomes are primary variable and then this is the this is the primary structure with the external load and then how do we get the primary structure or the basic indeterminate structure basic determinate structure in the case of force method.

If we have a once we identify a redundant force we have to remove the redundant force right removing the redundant force means we need to allow the associated displacement allow the displacement to take place in this structure associated displacement to take place in this structure.

In the case of displacement method once you have identified the redundant displacement; what we need to do is we need to constraint those displacement. So, that displacement needs to be removed the in the force method the corresponding forces need to be removed in the displacement method the corresponding displacement need to be removed may for instance here the dis correct displacement where theta B, theta C, theta D and theta E, but then if you remove them remove them means if you do not allow these structure to have rotation at B C D E it means that rotation at B C D E are 0.

We are making the rotation at B C D E at 0. How can you make that by assuming there is a fixed support there is a fixed end at joint B C D E which is presenting rotation at B C D E to occur ok.

Now, now next is once this is the primary structure kinematically determinate, this is primary structure statically determinate for the force method in the case of displacement method the structure is kinematically determinate. There is no deformation at any end so it is kinematic; the kinematic indeterminacy of the primary structure the basic structure is 0 with external load ok.

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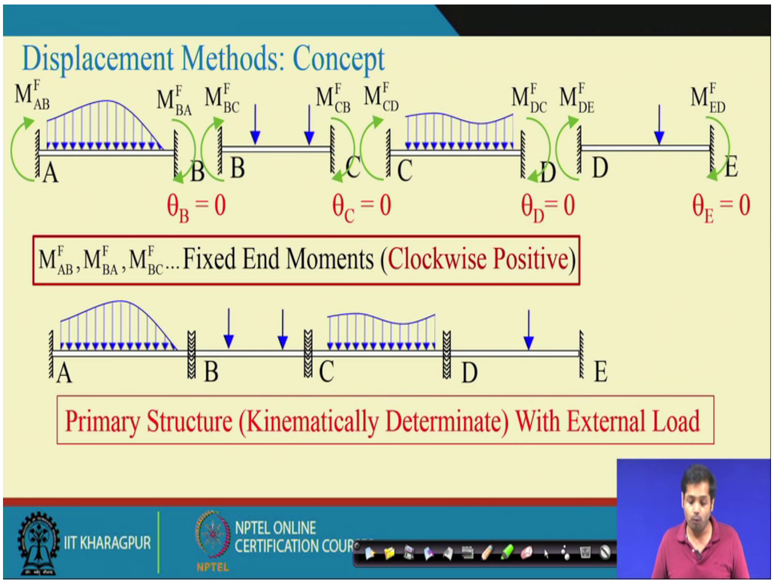

Now, let us once we have that then what we do in the force method we need to we need to solve that problem we need to solve the basic determinacy structure for the applied force and for the redundant forces as well we have to do the similar thing here. Now we make this entire we divide this entire structure to different parts we can make it different parts because this ends are any way fixed.

Now, once we do that once we do that then suppose so theta B, theta C, theta D, theta E are 0 everywhere, because these are we are we are making it to B 0 and then suppose M AB is the moment at a and this is M BA is moment at B in segment AB. F stands for fixed end moment the if it is they are called fixed end moments because this ms are assumed to be fixed similarly we have fixed m moments for BC CD and DE.

Now, this is the primary structure subjected to the external load, now we have to solve the primary structure subjected to the redundant forces or displacement as well and then apply the corresponding equation.

Now this is the M these are the fixed end moments the sign convention taken as clock wise positive it is very important the sign convention that you might have understood in your structure analysis one that what sign convention you take you can take any sign convention sign convention has really it is very subjective, you take your sign convention something is positive for you may not be positive for me may be negative for me, but what is negative for me I should be consistent with that definition.

Similarly, whatever sign convention you take is that your choice, but whatever you choose you have to be consistent with that sign convention great.

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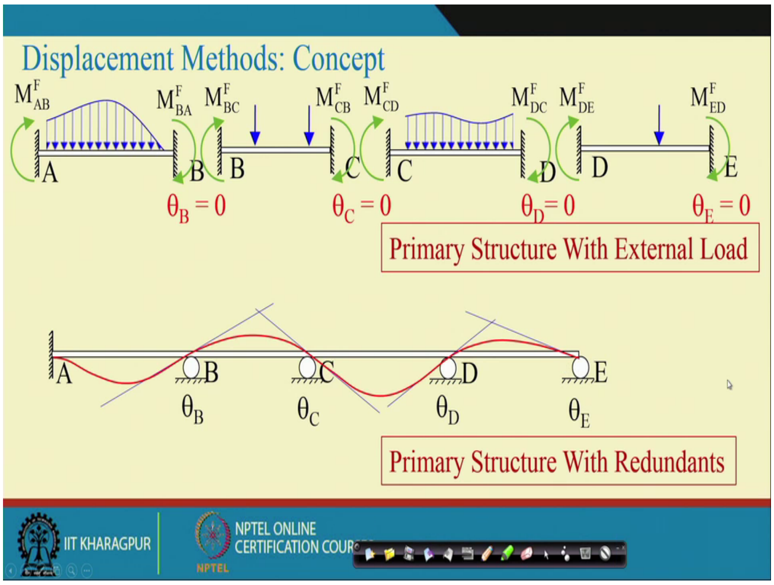

Now once we have that this is the primary structure with external load now next is the primary structure subjected to the redundant forces in the case of force method. Now in this case displacement method the primary structures are subjected to redundant displacement; now redundant displacement are this.

Now, these are the with redundant displacements. Now if we now if we if we now these displacements is these displacement occur only when if we apply some amount of forces in this structure.

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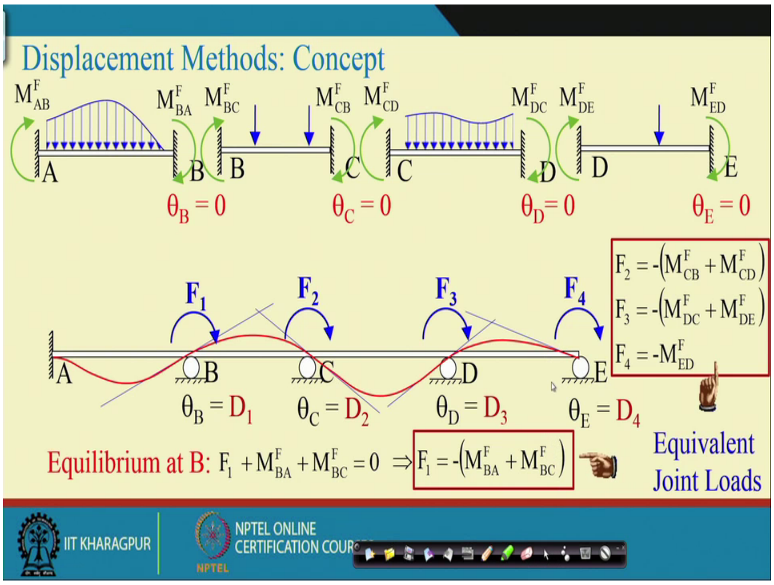

Suppose those forces are these displacements are retained in terms of D 1, D 2, D 3, D 4 suppose and suppose the corresponding moments at B C D E are F 1, F 2, F 3, F 4 by application of that moments we can have a deform shape like this, we can have a deform shape, we can have a deform shape like, this we can have a deform shape like this correct.

Now, these step is very important because it is not only for this week rest of the rest of the rest of the entire course we will be using this concept again and again. Now suppose now next is we need to next is this is if this is the primary structure subjected to external load this is the primary structure subjected to redundant displacement right.

In the force method once we have the primary structure once we have solved the primary structure subjected to external load and the primary structures subjected to redundant forces; next step was apply the compatibility conditions at various joints, but here once we solve the struct primary structure subjected to external load, primary structure subjected to redundant displacement or rotation next is to use equilibrium equation to get additional unknown. These are two these are the major difference we use compatibility equation to get the additional equation, here it is the equilibrium which gives us additional equation; equilibrium at various joints ok.

Now what are the equilibrium at various joints? Here equilibrium at B is you see total moment total moment at B is one total moment as B is this plus this plus this F 1 plus this plus this is not it; and in actual structure there was no external moment therefore, these plus these plus these should be equal to 0. That is the equilibrium condition so F 1 plus fixed moment BA, fixed moment BC should be equal to 0. Similarly we can apply equilibrium equations at all other joints so these gives us F 1 is equal to this.

Similarly, we can have equilibrium equation at a all other joints and those equilibrium equations are this is F 2, F 3 and F 4 and these are called equivalent joint loads you might have you can recall these are called equivalent joint loads ok.

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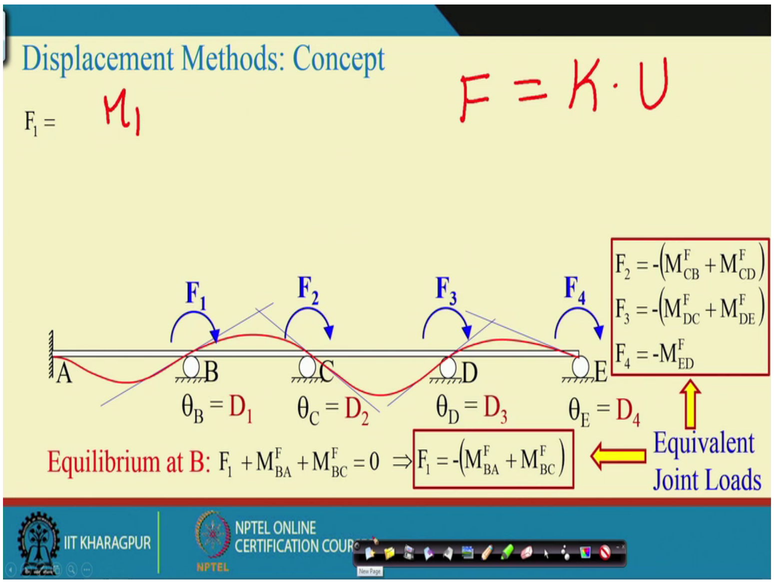

Now once we have equivalent joint loads the next is we can write F 1 this is the important step if we recall we discussed flexibility we discussed stiffness and we discussed that their primary role of flexibility of stiffness of they essentially tell you the similar story, but probably the narratives are slightly different. Your whether it is flexibility or stiffness they relate force and the associated displacement right.

Now, but then whether you compute force with a known displacement or you compute the displacements with the known force depending on that you have the you have the stiffness or flexibility. If the force method we used flexibility matrices, flexibility definition because the forces where unknown and the, we need to compute the forces based on the known kinematic condition.

So, use flexibility definition forces is equal to flexibility into displacement, but in the displacement method the displacements are displacements are unknown; we need to compute the displacements with the known kinematic known static condition and therefore, you use the stiffness definition. Now what is this stiffness definition? Stiffness definition is if you recall if you have a force if you have a force that force is equal to stiffness into displacement ok.

Now, you see F 1 is equal to from this definition here if you write F 1 is equal to we have say we have M let us write it first and then we discuss.

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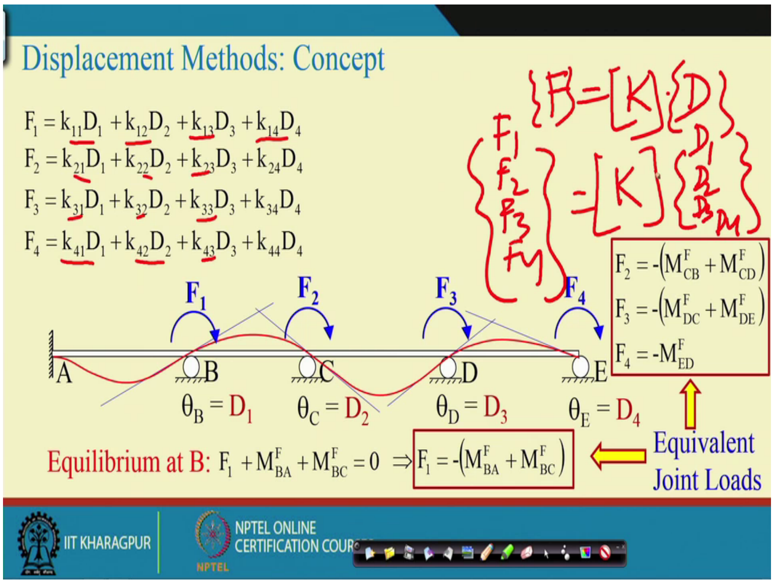

Let us write F 1 as k 1 1 into D 1 plus k 1 2 into D 2 plus k 1 3 into D 3 plus k 1 4 into D 4. Now again let me write these stiffness definition was force is equal to force is equal to k into displacement instead of U let us write it is D, because that is the symbol we are using now. F is equal to k into D now if it is if we recall this spin problem, where you have just one force and one associated displacement it is a scalar equation.

But if any (Refer Time: 17:05) recall the other problem where you had several forces and several associated displacement then these equation is not a scalar equation this equation is essentially a matrix equation right. The force is a vector then the displacement is a vector and the stiffness is a matrix; which goes stiffness matrix ok. Now in this case there are 4 components of forces say F 1, F 2, F 3, F 4; so F here is F 1, F 2, F 3 and F 4 and this is D here are the 3 components 4 components one is theta B, theta C, theta D, theta E or D 1, D 2, D 3, dt D 4 so D 1, D 2, D 3 and D 4.

So, naturally if you have to relate F 1, F 2, F 3, F 4 and to D 1, D 2, D 3, D 4 we need a matrix k which is the 4 by 4 matrix and suppose the elements of that 4 by 4 matrices matrix is k 1 1, k 1 2, k 1 3 and k 1 4 ok. So, the similarly we can write F 3, F 4, F 2 in the same way so D 1, D 2, D 3, D 1, D 2, D 3, D 4 are the displacements or rotation and k 1 k 1 2 and all these are the elements all are the elements of the this stiffness this stiffness matrix k ok.

Now but once see what is very important that you will un that will that I will we will keep on telling you throughout the entire course you see learning a method is important absolutely important, but until unless we understand what that method physically tells you what is the physical interpretation of that method any step we do here there is a physical interpretation until and unless we understand that there is no point in remembering this steps blindly when we say that k 1, k 2, k 3, k 4 are the element of that stiffness matrices what those elements essentially mean.

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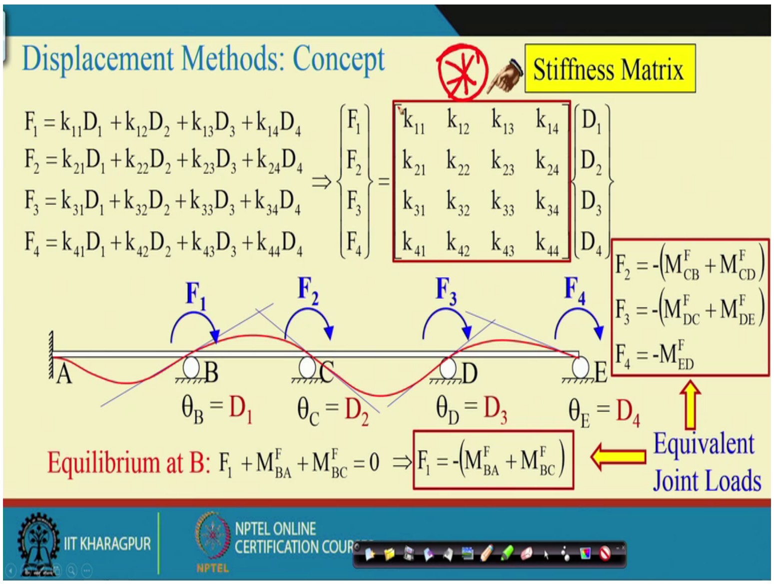

Now, they mean so we can write that in a matrix form like this just now we discussed; now let us now this is stiffness matrix this are very important this is very important for this course and these is the this is the building block of the entire course the matrix method of structure analysis. In this course will we will be will be forming this stiffness matrix again and again for different kinds of structure ok.

Now, going back to the going back to my previous point the physical interpretation of this these stiffness coefficients ok.

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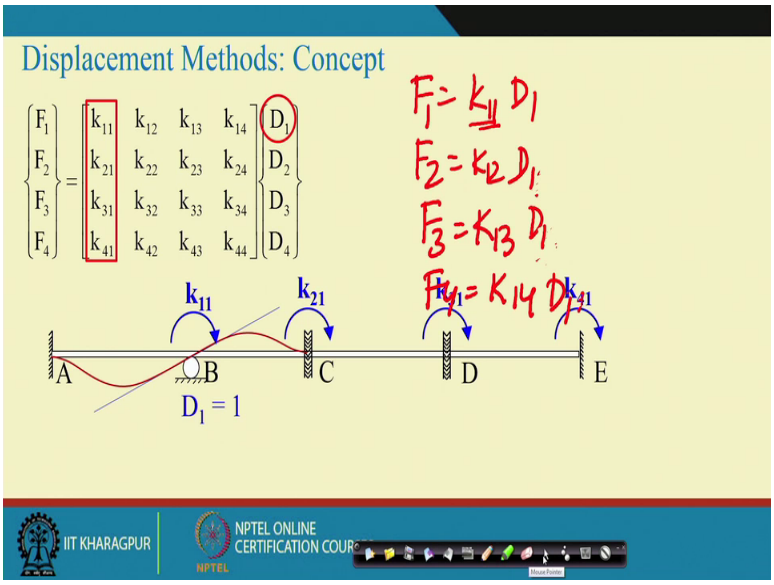

Let us see what is the physical stiff interpretation of these this way this column; now this column essentially gives you see when k 1 1 is multiplied with D 1 it gives you F 1 so essentially it gives you say F 1 if you just consider this F 1 is equal to k 1 1 into D 1 F 2 is equal to k 1 2 into D 2 F 3 is equal to k 1 3 D 3 and F 4 is equal to k 1 4 D 4 right; what does it mean?

Now, this is the stiffness definition these stiffness definition tells you that if F 1 if we apply D 1 D 1 displacement or rotation I mean every whenever I say displacement in this context it is rotation only. When D 1 is the my displacement or deformation then and then an F 1 is the force these D 1 and F 1 is related to k 1 1 and k 1 is the associated displacement.

If you recall when I say if you recall in the last class I said in the last week I said whenever we say stiffness or flexibility they do not carry any meaning a alone, the always there have to be a force and the displacement associated displacement along with the definition of stiffness ok. So, when we say stiffness then these stiffness relate to which displacement to which force that is very important.

So, k 1 1 relates F 1 and D 1 2, k 1 relates F 2 and D 2 or other and k 1 3 relates F 3 and D 3 and k 1 4 to F 4 and D 4. So, means if we apply an unit D 1 is as a D 1 is the rotation at B; so if we apply unit rotation at B unit sorry this is there is a mistake these this should this should all be 1, this all should be 1, this is these should be 1 great this all should be 1 right yes.

Now, if we apply unit displacement at unit displacement at B means D 1 1 is equal to 1 so F 1 1 becomes k 1 1 k 1 2 k 3 and F 2 becomes k 1 2 means F 1 F 2 F 3 becomes their corresponding stiffness ok.

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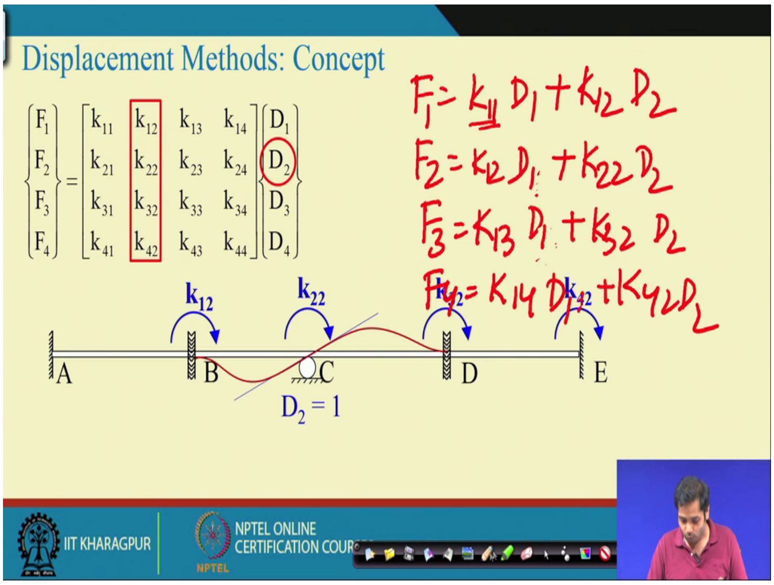

Now similarly if we take if we take D 2; D 2 means if we apply unit rotation at C then F 1 1 becomes means contribution at F 1 1 then F these becomes then contribution at F 1 1 becomes k 1 2 into D 2. So, total k 1 becomes k 1 2 into D 2 similarly these becomes k 1 k 2 2 into D 2 plus k 2 3 2 D 2 plus k 4 2 into D 2. So, this is the contribution for D 1 this is the contribution for D 2 and similarly we have if we make D 3 these will be the unit rotation if we apply this is the contribution for D 3 and similarly if we apply unit rotation at E then this will be contribution for the D 4.

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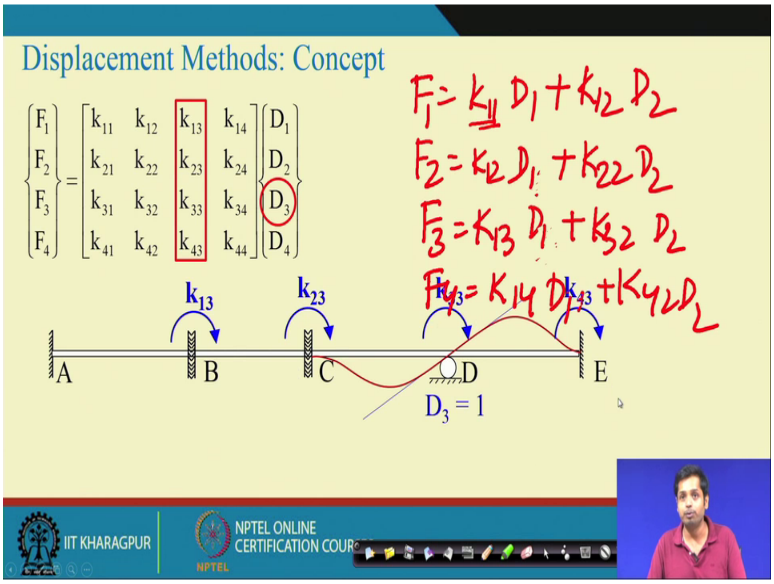

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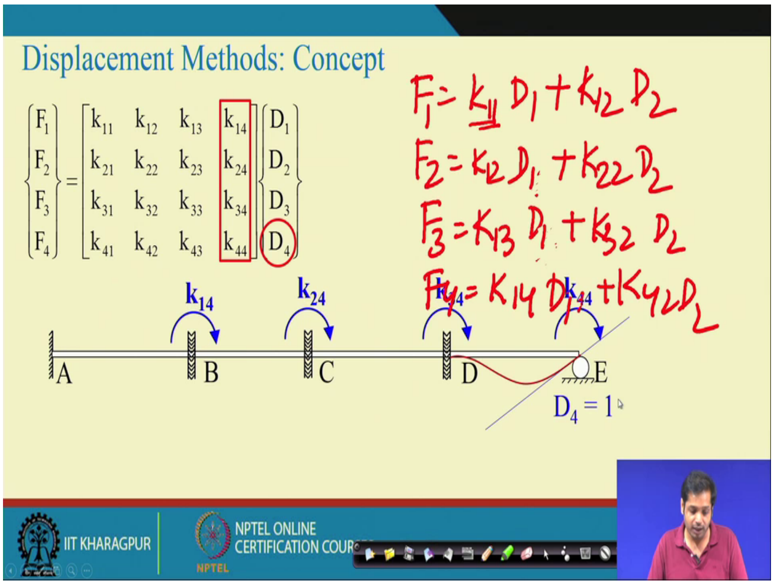

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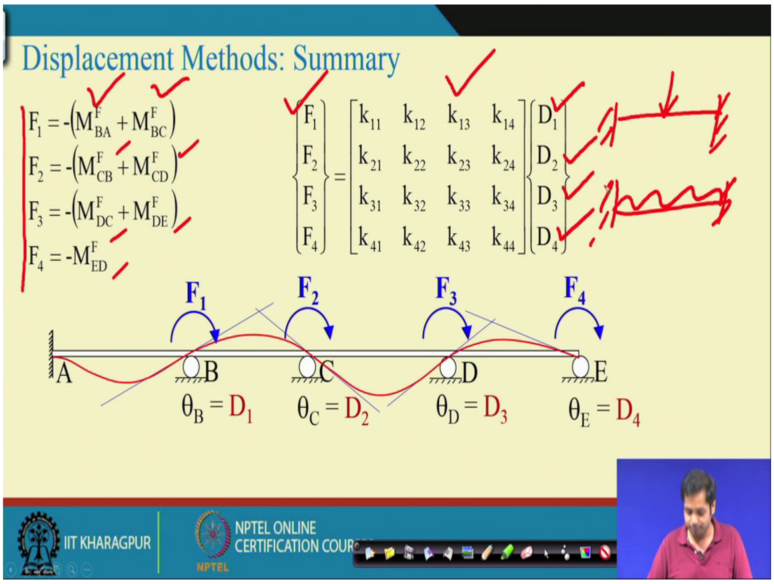

And then finally, we have an equation like this. So, this is this now in this expression what are known; in this expression we calculate the fixed end moments for a given for instance for a given structure given if we have a structure like this and subjected to any arbitrary load we can calculate fixed end moment. For a given structure if is subjected to uniformly subjected load any other load we can calculate fixed end moments. So, all this fixed end moments are known.

Now, once this fixed moments are known then equivalent joint loads are also known means this vectors these vectors F 1, F 2, F 3 are known; this stiffness k 1, k 2 this stiffness all this elements stiffness matrix they are also known because they depends on the material property and the geometry of the structure. So, they are also known only unknown here is the displacement.

So, displacement can be computed so we have a set of linear equations and if you solve this equations we can compute we can solve for D 1, D 2, D 3, D 4 once you have D 1, D 2, D 3, D 4 then like in the force method once we had forces the redundant forces once we have solved for the redundant forces then we use that redundant force the information of the redundant force to solve the rest of the part of the structure.

Here similar thing we can do once we have the deformation then rest of the things we can easily compute this is displacement method and this is very important because this is the concept that we will be taking that that will be taken further to or that would be to form a very general method which is the subject of this of this of this course ok.

So, in the if in last in the last 2 lecture of this week we will once again we will discuss as a various aspects of displacements methods, how to compute this stiffness of different kinds of structure, what are the and also see what are the stiffness matrix and then finally, form the bases the of the our platform will be ready to move further ok, then I will stop here today see you in the next class.

Thank you.