Matrix Method of Structural Analysis
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Lecture - 07
Review of Structural Analysis - I (Contd.)

Hello everyone, welcome to lecture 7 of week 2. We have been reviewing the various methods for solving a indeterminate structures, force method and displacement methods. In the last class we discussed the basic concept given force methods, where the force are essentially the primary unknown. And then when we say indeterminate structure means the equations of equilibrium are not enough to solve this structure completely. So, we need additional equations and in the force method additional equations are formed based on the compatibility condition.

And what is the general procedure that we discussed in the last class. The method which is a va the method which is come under the category of force, method is the method of consistence deformation and the different steps, we discussed in the last class. Now, what we do today is based on the similar concept, there is an important equation called 3 moment equation, which is very useful for solving continuous beam, a we will just review the concept of 3 moment equation ok.

You see a one important thing again I want to make here and, probably I will be a repeating these thing in next few lecture. The methods that we have learned in structural analysis one and what we have being reviewing in the first two week. All the methods a we will really do not use them as it is today, for analyzing real life structures, but and that is not the purpose of these two weeks also. We are not going to demonstrate those methods to through an example, but the concept that constitute the basis of this method, those concepts are important because the similar concepts. The same concept will be used for formulating, the more advance structure analysis structure analysis method matrix method of structure analysis.

So, here we will concentrate only on the concepts behind different methods like, force method we discussed and the three movement equation, we will discuss today and, then displacement method and various aspects of displacement method that will be discussed in the subsequent weeks. So, there will be no demonstration through any example, it is the basic concepts which form the method ok.

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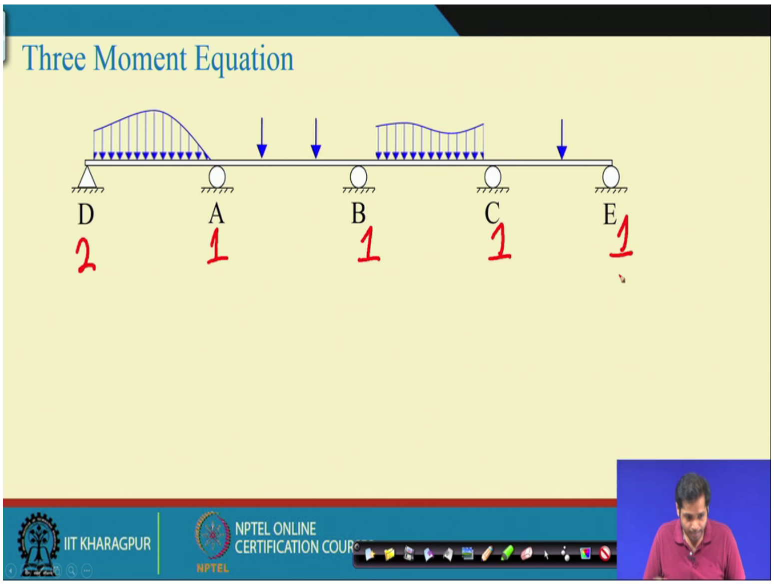

So, 3 moment equation let us understand or review rather, the three moment equation through an example, this is an example this continuous beam, which is statically indeterminate structure. We can easily compute the static indeterminacy of the structure here, it is 3 react 2 reactions here, it is 2 2 reactions 2 2 2 horizon 1 horizontal reactions, 1 vertical reactions here it is 1, then 1, then another vertical reaction another vertical reaction.

So, total number of unknown the reactions are a 6 and, the number of equations total are unknown 6 and the number of equations available is the number of equations available is 3. So, it is statically indeterminate structure ok. Now, you see; what is three moment equation as name suggest, these equation has moment, the equations are written in terms of movements and every equations there are 3 moments constitute an equation.

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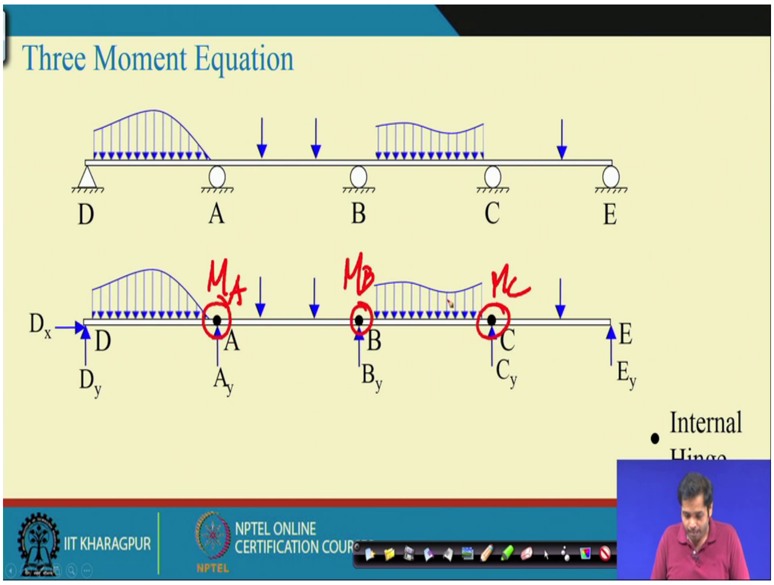

Now, you see so, let us write this is the entire structure all the forces are expressed, all the reactions are expressed in terms of their equivalent reactions. Now, you see if you remember the in the force method, the first is a first step once we understand once we calculate the number of static indeterminacy, then the next step which is very important step is to identify the redundant forces.

Now, redundant forces it could be anything, but the at end of the day, what is important is once you choose the redundant forces and create the basic, or primary structure the basic determinate structure, your structure should be stable, determinate and stable. We demonstrate it rather the E x the examples that we not the example, the concept that we use concept was demonstrated by taking reaction, as the redundant force, but not necessarily that we always have to take the reactions as the, or vertical reaction as the a redundant force for instance in 3 moment equation, instead of the force the reactions by C y E y it is the moment at various joints as taken as redundant.

Now, once you take the so, we have we have in this structure the static indeterminacy is 3. So, redundant forces has to be a 3. So, let us take the moment M A M moment at A moment at B and moment at C instead of taking A y B y and C y are the redundant forces, let us take moment M A moment at B and moment at C are the redundant are the redundant ok. Now, we can take moment M A, M B, M C are the redundant because, M A, M B and M C are continues at the beam is continuous at A B and C therefore, there are there is moment at A B and C. Now, the next step is once you choose the redundant force, next step is we have to we have to release that corresponding constraint.

Now, had it been had it been A y if we choose A y as the redundant force, then we have to release these constraint. So, there this support is to be released. So, therefore, the deformation in that particular direction is allowed. So, if you if you take by as the redundant. So, this support is to be released. So, therefore, the vertical deformation the transfer displacement in direction B will be allowed, but now since the moments as taken as redundant. So, the associated displacement associated kinematic variables for the moment is the rotation ok.

So, since moment at A moment at B and moment at C are the redundant therefore, corresponding kinematic variables is the rotation which needs to be released. And therefore, rotation at A, rotation at B and rotation at C be will be now allowed that is no constraints, against rotation at B rotation at C and rotation at A. It means that we provide that now the instead of the continuous beam, now we provide some internal hinge at point A B and C right. If you recall that was that was the first step we do, in while deriving the equation for three moments.

So, once we insert the internal hinge at A B and C, then automatically there is no constraint against rotation at point A B and C therefore, and which is same as releasing the moments B M A M B and M C ok. Now therefore, these are the internal hinges, these are all internal hinges this is internal hinge, internal hinge, internal hinge ok.

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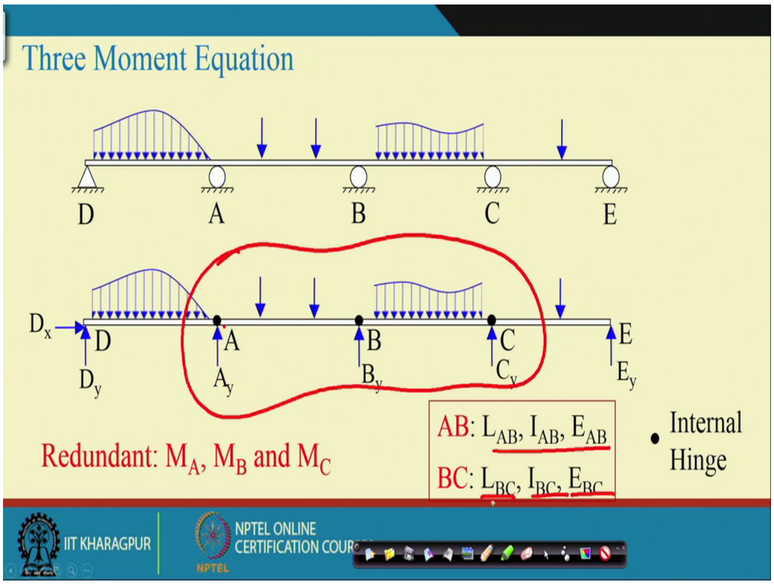

So, the redundant are now the moment M A, M B and M C ok. Now, let us see once we have taken moment M A, M B, M C and let us take as let us consider only this part. Let us consider only this part and so, because these part cons these parts includes the moment at A moment at B and moment at C and 3 moment equation is essentially and the equation, which relates, which tells us how the moment at A, moment at B and moment at C are related to each other ok. Let us take L AB, L I AB, E AB are the length of A B second moment of inertia, second moment of area of cross section at A B and the Young's modulus of A B.

Similarly, L BC, L I BC and ABC are the length second moment of area and Young's modulus of BC ok. Now, once we have identified the redundant forces and, release the corresponding force a by allowing the corresponding associated displacement and, the next step is to make the structure A, or create the basic determinate structure ok. The basic determinate structure now here is if you consider this part there are this is the basic determinate structure right; now, the basic determinate structure which is subjected to the external load.

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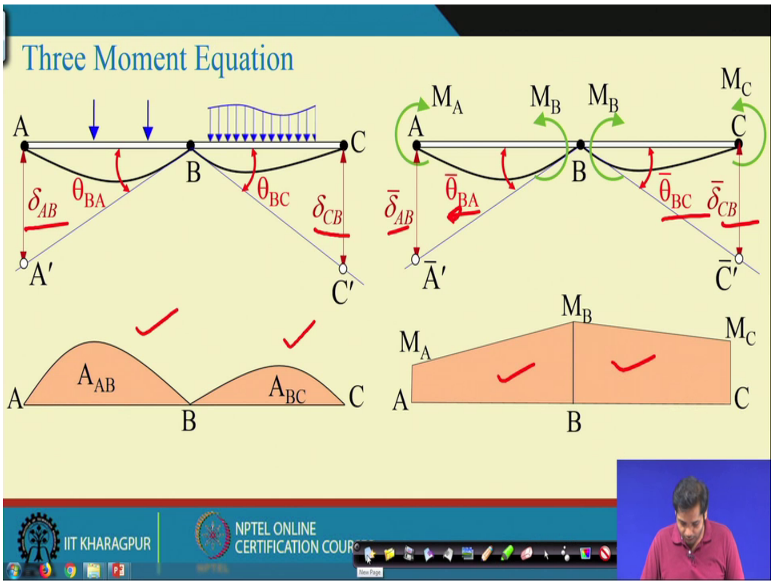

Now, suppose under the action of these external load, this is the deflection. And now by looking at the deflection pattern, you can easily make that the though there is a that that these a at the deform shape is continuous at B, but it is not differentiable at point B, the slope is not continuous at point B ok. Because the rotation at this point rotation of segment A B and rotation of segment BC, they are they could be different ok.

Now, suppose theta BA is the rotation at the rotation at B in segment B BA and the segment BC is the rotation at B, at segment BC. And then this is this is the basic determinate structure subjected to the subjected to subjected to external load. And then we also need the basic determinate structure subjected to the redundant forces, redundant forces are the moments here. So, this is the basic determinate structure subjected to the redundant force. And then the linear super position theory is says our actual structure, the solution of the actual structure should be equal to the solution of this two; so this plus this that is what we did in method of consistency deformation ok.

So, compatibility between these deformation and the between these deformation needs to be maintained and, that compatibility gives us the additional equation. Now so, deform shape of these is suppose these and theta bar represent the rotation at B for the redundant due to the redundant force ok.

Now, so we can solve this using any method, let us do it using the moment area method, moment area method first you have to ca calculate the bending moment diagram.

Suppose this is the bending moment diagram for A B and this is the bending moment diagram for BC again. Look at the bending moment it should be consistent with the problem. The moment at B is 0 in this case because rotation is not constrained here rotation at B is allowed. So, moment will be 0 here.

Now, similarly the bending moment for this is this where M A, M B, M C are the moments at B moments at A B and C ok.

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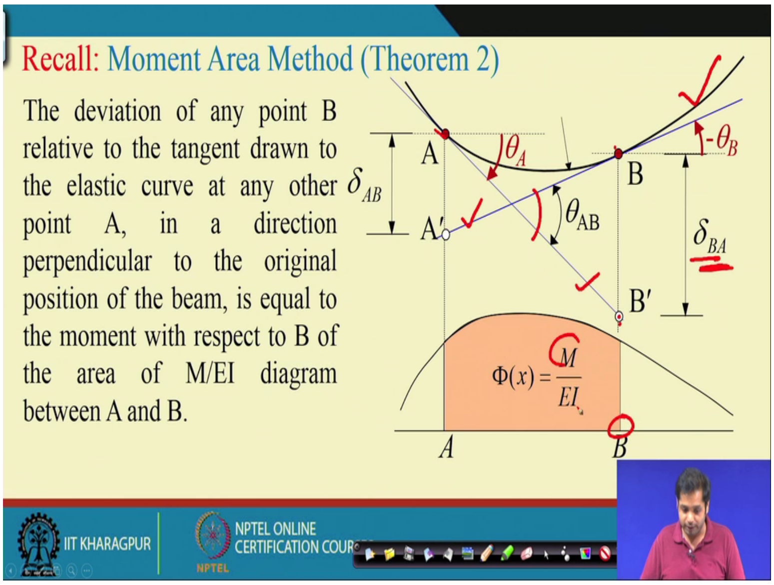

Now, we need to recall one important theory the moment area method, the moment area method there are two theorem of moment area method, theorem one says that how to compute the slope at a given point and then theorem two tells you how to calculate the deflection at a given point.

The theorem two is which is which is important in this case. It sees that if you have a if you have a de deform shape, it is an elastic carve of it is an elastic card and, if you take two points in a elastic card say point A and B. And the bending moment between M is the bending moment and E I is the flexural rigidity and the M I, E I diagram between two point is this, then the and if you have a slope at point A is this and slope at point B is this and the angle between this two slope is theta A B.

Then the theorem one says that theta A B will be the area of this bending moment M I M by E I diagram and theory 2 says, theorem 2 says that the deviations of point B this is point B and, it is deviation is this is the deviation of point B, with respect to the slope at point A. So, this is the slope at point A and the deviation of point B is a prediction of point B on to slope at point A see this deviation is given by the moment of this M I M by E I diagram about point A, that is the moment area theorem A.

If you can recall so, same theorem you can be used here. So, if you use suppose a A A B is the moment a the area of this diagram and A B C is the area of this diagram. So, pa then what we can take is so, this is the deflection at a diff this is the deviation of point A, with respect to slope at B similarly deviation of point C, with respect to slope at B. And similarly we can have deviation of point A and B with respect to slope at B in this case. So, these values these we can obtain these we can obtain from the M I this diagram and these and these can be obtained these, sorry these and these can be obtained from these two diagram right.

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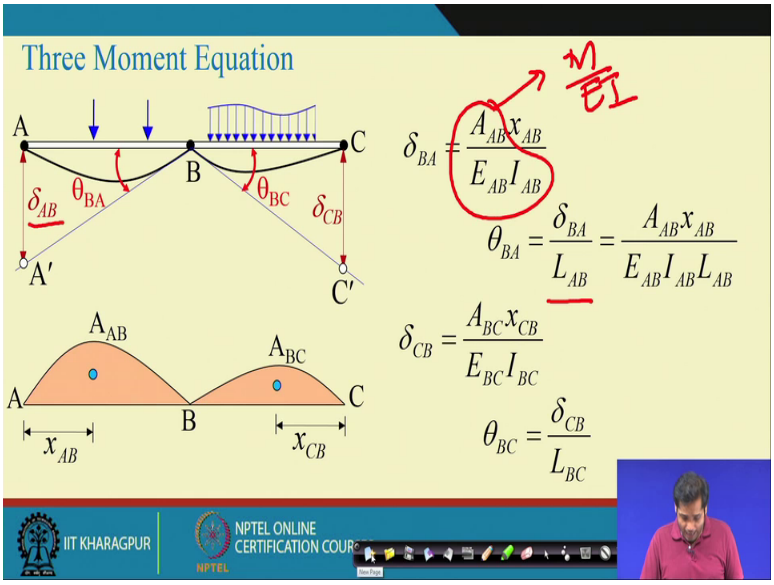

Now, then if you do that then, this is the suppose x AB, x CB are the centroid distance of centroid from A and C and, then delta B A is equal to am this is the movement M by E I diagram, this is the area of the moment and this is M I E I diagram M by M by E I diagram.

And this is the centroid of that diagram. So, similarly once we have delta B A. Now, once we know this value, this if this is known if this is known this angle can be obtained as delta B A by this length which is L AB and so, theta B A is equal to this. Similarly we can have delta C B and theta B C is equal to this.

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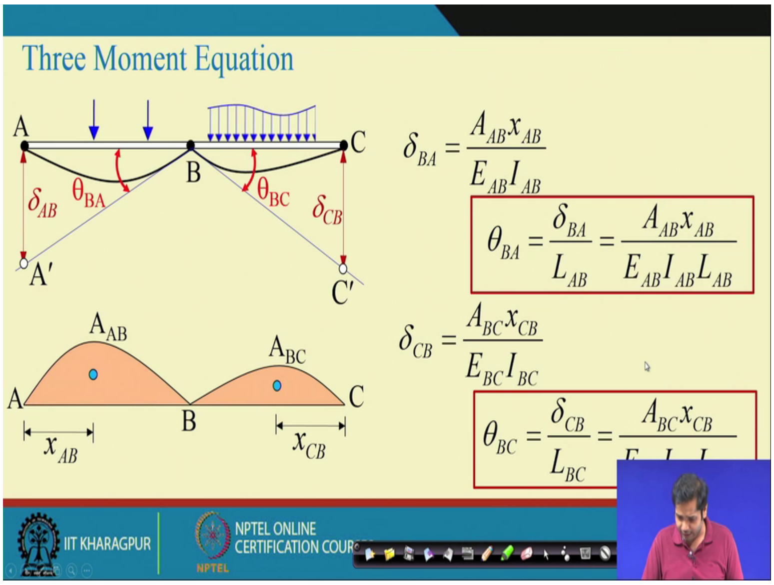

For the second case second ba a diagram second basic indeterminate structure subjected to the redundant force, we can do the same exercise and if you do that.

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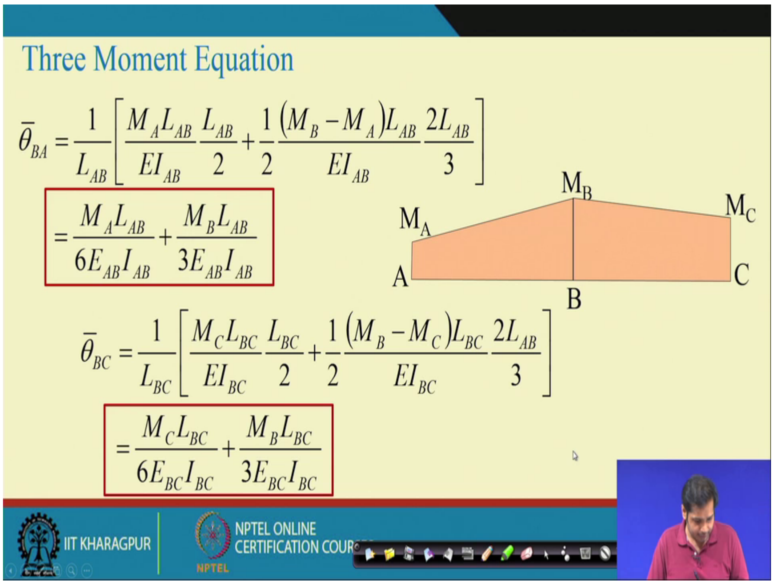

Then the theta B A bar means the angle at B A rotation at B A in segment rotation at B in segment B A can be obtained like this. And similarly rotation at B, in segment B C can also be obtained like this.

So, once we have all the rotations the determinate structure, the primary structure the basic determinate structure subjected to external load and, the basic determinant structure subjected to a redundant force. Once we have the deformation in both the cases, then the next step is to calculate apply the how the deformation should be related to each other. So, that the compatibility in the actual structure is satisfied right.

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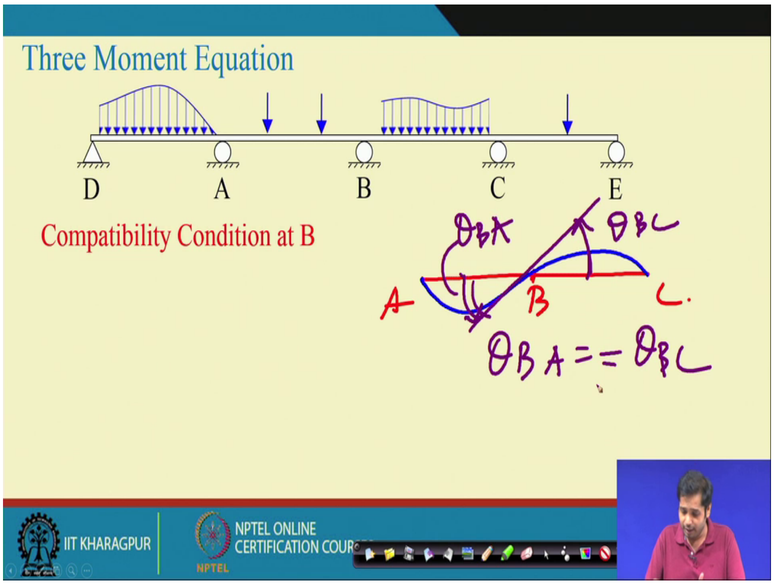

Now, this are the theta B and theta B C, now let us see what is the compatibility in this case you see. The compatibility conditions will be suppose if we a if we take any continuous beam, suppose it is a this is A B and this is C. And suppose deform shape in these structure deform shape is suppose something like this something like this ok.

Now, what is important here is a and the slope is like this, slope is like this, what is the compatibility at B is if this is theta 1, this is theta B A and, this is this is theta B C, the compatibility conditions says that theta B A should be equal to minus theta B C ok. So, this angle should be equal to this angle the minus because, the way you measure this angle clock wise anti clock wise.

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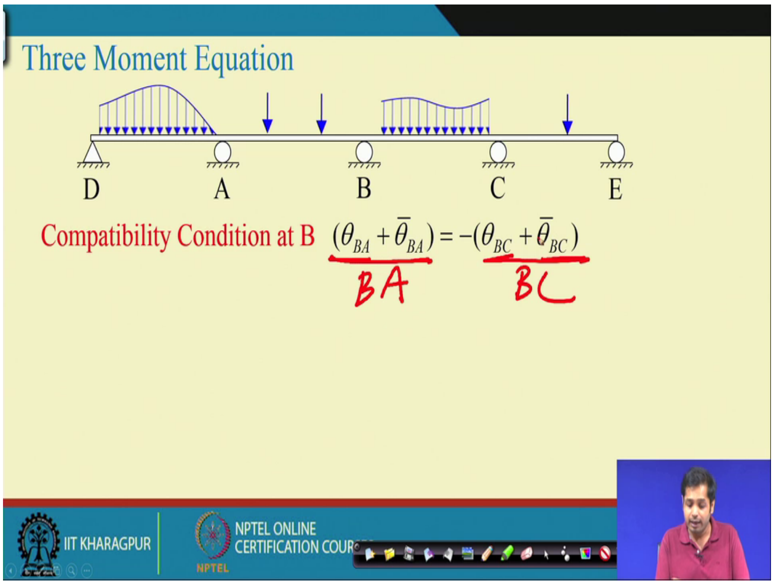

Now, if you have this the compatibility condition will be at point B will be so, total angle total rotation at point B will be this is the rotation due to the a basic rotation, in the basic determinant structure, due to external load basic determinant structure, due to redundant force basic determinant structure external load basic determinant structure ex a redundant force. So, this is the total rotation in B A at B in segment B A and this is the total rotation in B C. So, this should be this should be equal to minus of that this, that is the compatibility condition and if we then.

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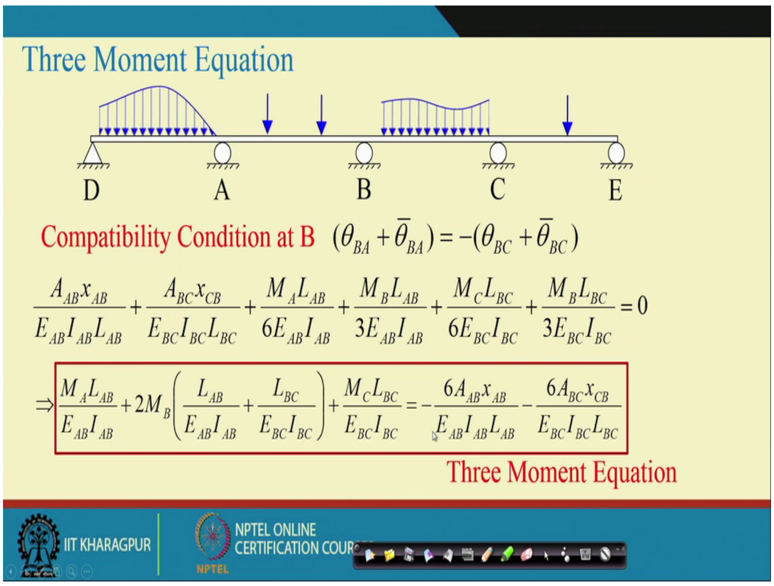

If we substitute the correspondent values then, we have these equation So, these is called three moment equation the reason is three this is called three moment equation, you see these equation actually relates moment M A and, then moment M C and, then moment and then moment M B. And other then this three moments M A, M B and M C all other parameters you look at this equation, all other parameters are known. So, only none known unknown in this equation M A, M B and MC.

Now, now so this is essentially what this is essentially the compatibility equation only, this is in consistency deformation the basic concepts of force method is same thing here, but the equation is written in terms of moment ok. Now, similar equations you can have now, we can have we can apply this three moment equation for point a point B point C every time we have a set of equations and, then solve all this equations to get the unknown ok. So, this is the concept of the three moment equation. If you remember now this is three moment equation.

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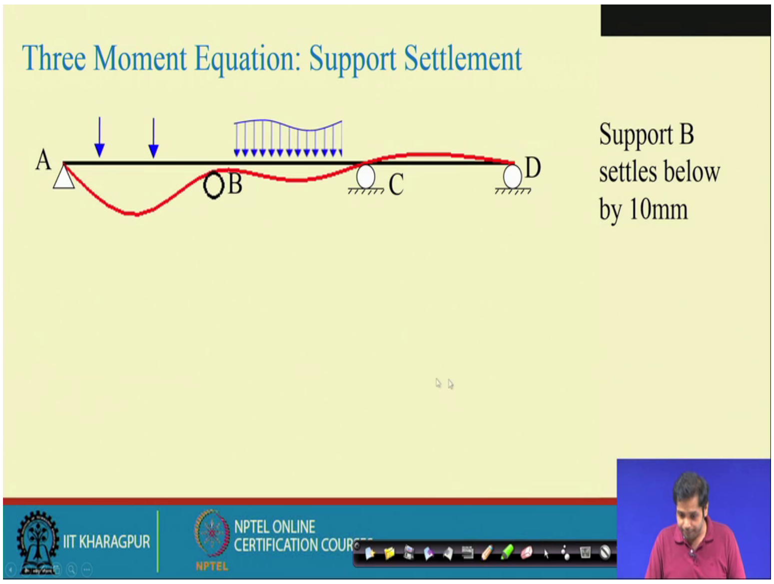

Now, another important addition in or in important part in three moment equation is the it supports settlements, you see suppose in a continuous beam, if any support settle any support settles means, if there is a displacement in support that could be because of anything, because of the construction problem, because of the of the sinking of the structure, because if the settlement of the soil, it could be anything. But it any case if there is a settlement of a support, then that settlement will cause moment in this structure.

So, three moment structure allows you to consider settlement of support structure settlement of the support. So, the same equation that we derived here, we can extent it far further we can modify, it in order to account for that support settlement. Suppose considering this case the support B, which was originally at this point. Now, a it settles like this and a we now position is B dash and, then suppose delta B is the settlement of this settlement of support B.

Now, if the settlement of support B, I mean just to look at this could be the deform shape the red one is the deform shape, you can see that B in the deform shape the support B is also a is coming down it is. So, because if these settlement of this support, the moment that is caused that also need to be accounted for ok.

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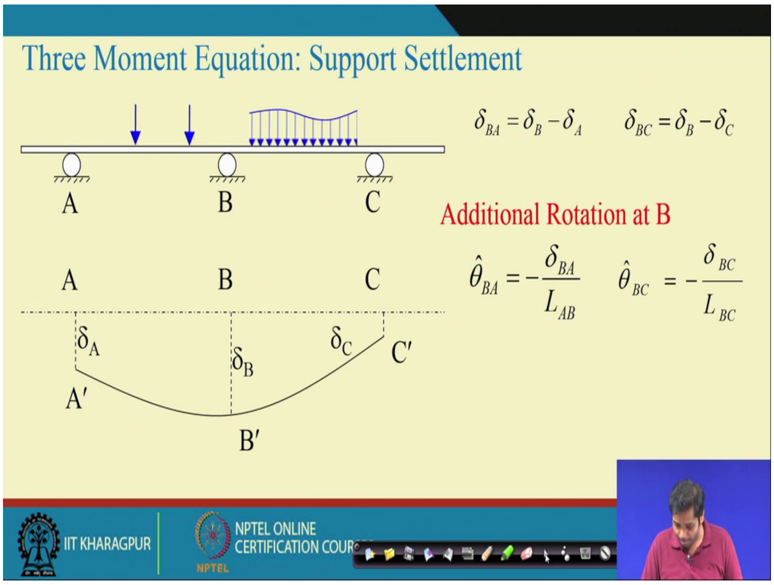

Now, next is take an take a very general expression, again we consider the same part C same segment A B and B C, support delta B is the settlement at the at the support B delta and delta C at the settlement of support A and C respectively. These are general equation, if we see that there is settlement only in support B there is no settlement, in support A and C. So, we can we can we can make delta A and delta C, 0 in the expression ok, but the expression that we derived is for general.

Now, once we a now so, additional rotation because of this settlement of support the additional rotation takes place, at point B and that additional rotation suppose delta B a at delta B is the relative displacement, in segment B A and delta B C will be the relative segment, in relative displacement, in segment B C. Once we have the relative displacement and the lengths we know, then we can calculate the relative the additional rotation at point B, because of these because of the relative displacement in A B is equal to this.

And similarly, the additional rotation at B due to the relative displacement in segment B C is this. So, this and this, then the total displacement will be the total rotation will be the rotation that we already calculated two rotations, basically determinant structure under the external load, basic determinant structure under the redundant force. And, then the third component in this case will be the rotation due to the support settlement ok.

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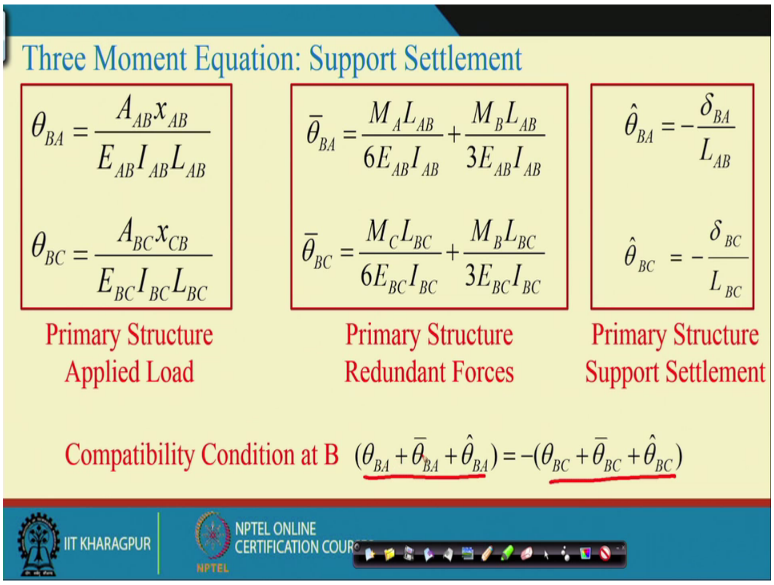

So, if we have this, then there are three components this is the component primary structure, or basic determinant structure, subjected to external load basic determinant structure is subjected to subjected to the redundant force. And then the basic determinant structure primary structure, that is the subject subjected to the support settlement.

Now, once we have that once we have all the components then we have to apply the compatibility conditions ok. Now, compatibility condition, but please note one point here, all these are theta B A theta C that we obtained, that we obtained based on the based on the based on the based on the based on the methods that we studied E for determinant structure right.

Now, then compatibility equation con condition at B will be what, compatibility condition will be at B will be this is again this total rotation, at B in segment B A and, then total rotation at B in segment B C and, if we add them now. Next is we have to substitute these all these variables from these from these expression and if you substitute that.

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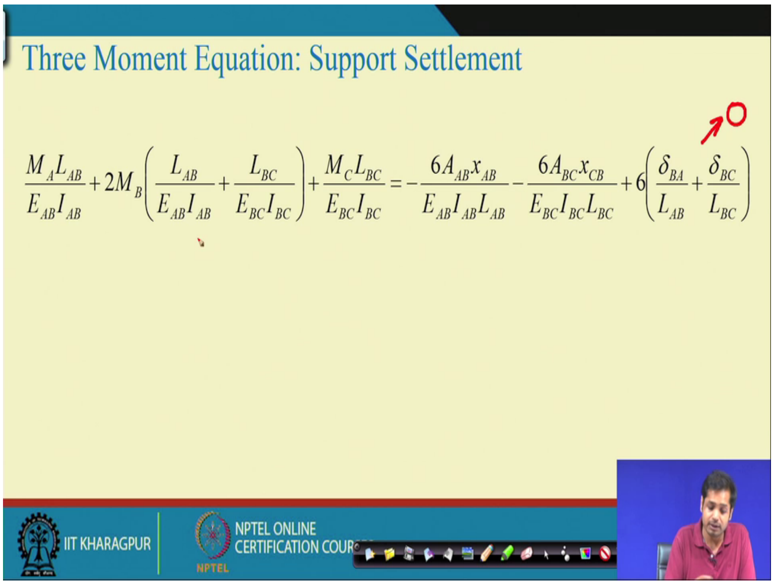

Then what we have this is the expression for the general expression for general expression for three moment equation, when there is a support settlement, or if now there is no support settlement see these parts can be these parts can be 0. So, the expression becomes this same expression that we just now derived.

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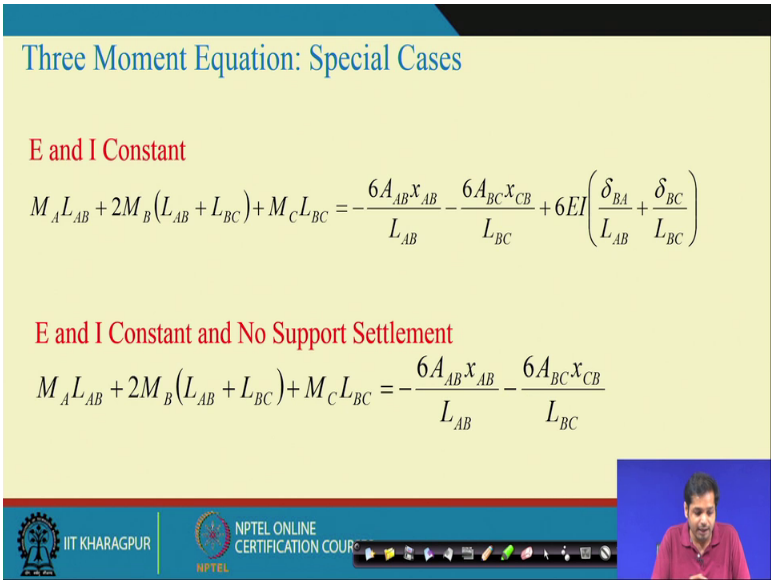

Now, slightly simplify this equation simplifying this equation is you see, if Young's modulus E I is constant means reflection rigidity of the entire beam is constant, then in from the previous expression this E I may get cancelled, this all E I if they are same, then this may get cancelled. And if we do that then what we have is this is the expression, this is the expression what we have. Now, in addition to that if E I are constant, but there is there is no support settlement, then these is the this makes this can be made 0, this part can be made 0, this part can be make 0 and the equation becomes this.

So, in these expression you see only thing is M A, M B and M C are the unknown. So, these equation gives you how these three moments are related to each other, moments at three consecutive supports are related to each other. And then we write this equation at every point and get the system of equation and in that syst and then solve it, just to quickly demons quickly demonstrate that. We can tell you with this same example, if you take this same example, if you take the same example here you see so, ok.

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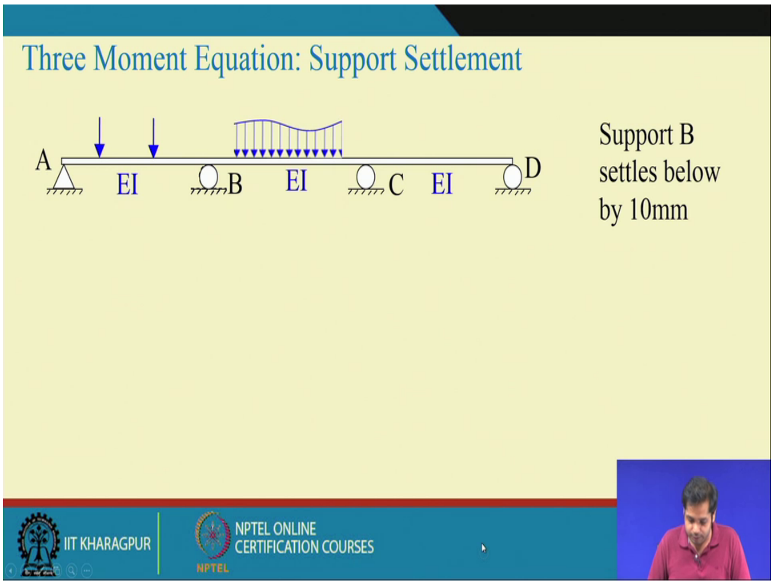

We can take the previous one we can go back, yes.

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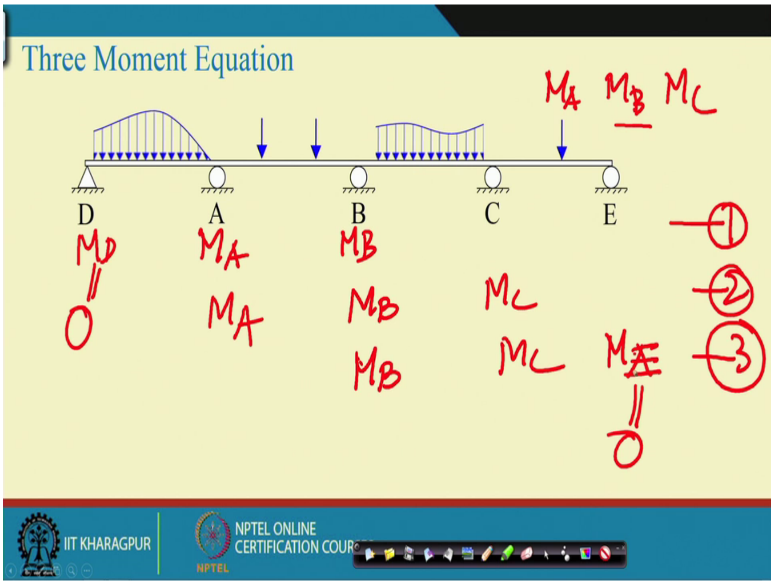

Now, we have the relation between M A, M B and M C. Now, we have relation between M A, M B, M A, M B and M C right. Now, M A, M B, M C are means we have a relation between three cons moments in three consecutive, three consecutive supports. If we are considering the compatibility conditions at B, then we have relation between M A, M B and M C, but if we compute the compatibility equation at A, then we have a relation between M D, M A, M A and M B that is for compatibility equation at A.

If we consider compatibility equation at B, then we have a relation between M A, then M B M B and M C. Similarly if we consider the compatibility equation at C, then we have a relation between M B, M C and M A so, me. So, essentially we have three equations the equation 1, equation 2 and equation 3. And three equations we obtain by satisfying compatibility equation at A B and C right, but these compatibility equations are written in terms of moments ok.

Now, you see since this support is the hinge support. So, automatically M D become 0 and, this is a roller support. So, automatically me becomes 0. So, in these three equations only unknown we have M A, M B and M C three equations three unknown we can solve for M A, M B and M C. Once you solve for M A and M B once M A, M B, M C are known, then the rest of the things can be obtained can be obtained easily ok.

So what we do now is we stop here today. So, this is again we discuss the concept of force method today and, it is based on the concept force method, or method of consistency deformation the three moment equation. And next class we will review the concept of displacement based method. And, then see how the displacement method can be taken further to formulate, again the same thing I am repeating, how the displacement method can be taken further with the knowledge that we learned in structure analysis one, how the displacement method can be taken further to form a very general analysis procedure.

Thank you. See you in the next class.