Matrix Method of Structural Analysis
Prof. Amit Shaw
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture - 06
Review of Structural Analysis - I (Contd.)

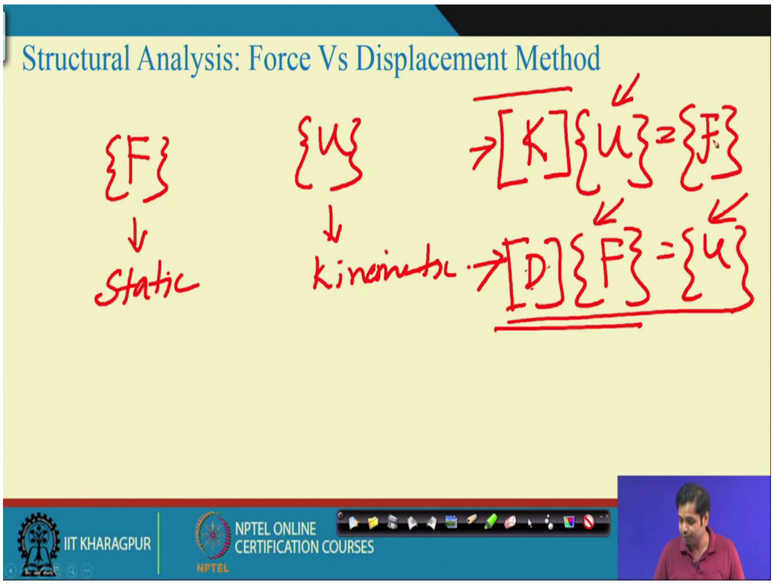
Hello everyone; welcome to the second week of the online course on Matrix Method of Structural Analysis. What we have done in the last week is we briefly reviewed some of the concepts of equilibrium equation, then the compatibility conditions, we talked about what is stiffness, what is flexibility, we also talked about static determinate structure and statically indeterminate structure, kinematic indeterminacy and then what we do today is a this week will review the some of the methods for solving indeterminate structures

There are two class of methods one is force method and one is displacement method, will see what are the, we will review rather. What are the concepts of force method and displacement method what are the different kinds of force method and displacement methods and briefly review them ok.

(Refer Slide Time: 01:13)

So, today it is the 6th lecture and it is the statically indeterminate structure, Review of Statically Indeterminate Structure Force and Displacement Methods ok.

(Refer Slide Time: 01:22)



If you recall what we have done so far you see suppose a we have a structure, suppose we will discuss that suppose this force when we say that force it accounts for everything force moments everything is a vector. Suppose it is a force and this is displacement this is say displacement and again when we say displacement it accounts for displacement rotation everything.

And then this forces this is this are static variable this is static variable and this is kinematic variable right kinematic variable kinematic variable ok. Now what; for given structure we know that force and displacement they are related through stiffness and flexibility matrixes. For instance if we if we can write say K, which is a stiffness matrix and then displacement this is force we discussed that in the last class, similar relation we can have that say another matrix say F we have used F so say it is let us take its let us take is say instead of F you take say it is D D and then F is equal to u; we have seen that where k is the stiffness matrix and D is the flexibility matrix ok.

Now, when we the main purpose of structural analysis is to find out either forces for a given displacement or a either forces for a given displacement or displacement for a given force. Now stiffness and stiffness and flexibility they are the properties of the structure for a given structure it depends on the material of the structure it depends on the geometry of the structure so for a given structure stiffness and disp or flexibility which was inverse of stiffness they are known.

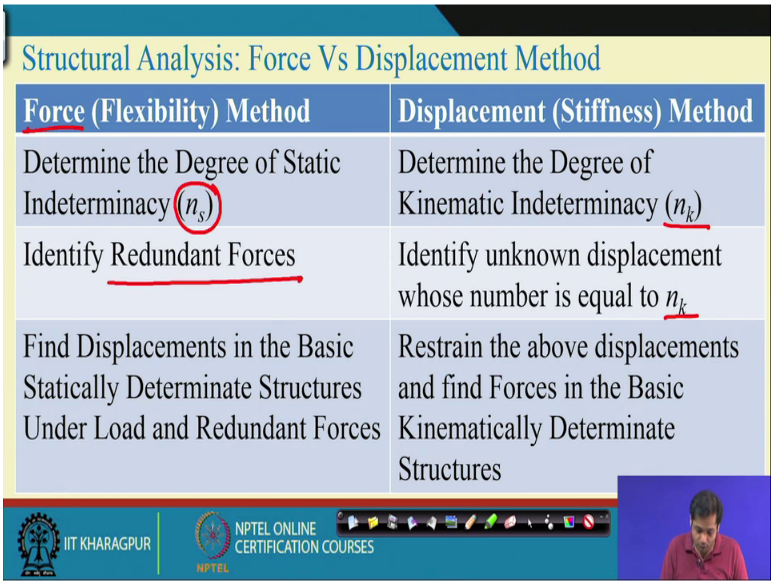
Now, when the when the, that structure is subjected to certain amount of load, then we want to find out the displacement of the; if the displacement are known or the kinematic variables are known we need to find out what are the static variables. Now depending on that; so when we so either you can be so we the main purpose is to find out u the kinematic variables for a given, with a knowledge of static variables or find out static variable F with the knowledge of kinematic variable u.

So, how we frame the problem, whether we use this definition or this definition both definition tells you the similar thing, but the, these definitions are the different in the sense that, you are unknown are different in this case the first definition is unknown is u, kinematic variable and in second definition unknowns are unknown are force. So which we take as our a primary unknown; depending on that we can frame the problem in a two different way, one is the force based method and one is displacement base displacement based method.

As name suggest force based method if the forces are the primary unknown where we use these definition where you use the flexibility matrix and for a given for a given kinematic condition with a knowledge of kinematic condition we determine the unknown forces. And for displacement based method where the static conditions are known and we determine the unknown kinematic condition kinematic variables using the a stiffness relation.

So, that is the main that is that is way that is how this force and displacement methods are different, but then since their definitions are different we are using the we are using a different a different representation of the equation, naturally the steps will be different there will be diff there will be different in the steps ok. So let us try to understand or rather try to review what are the different steps that we follow in force and displacement method ok.

(Refer Slide Time: 05:43)



Force method as I say; since it is the flexibility definition it is sometimes called flexibility method and also displacement method which is which uses a stiffness definition which is also known as stiffness method ok.

So, since in the force method; in the force method since your force is the primary unknown so the number of unknown depends on the number of additional forces or additional constraint additional reactions or additional internal forces that we need to find out. So therefore, the number of ad unknowns depends on the, what is the static indeterminacy of the structure n s.

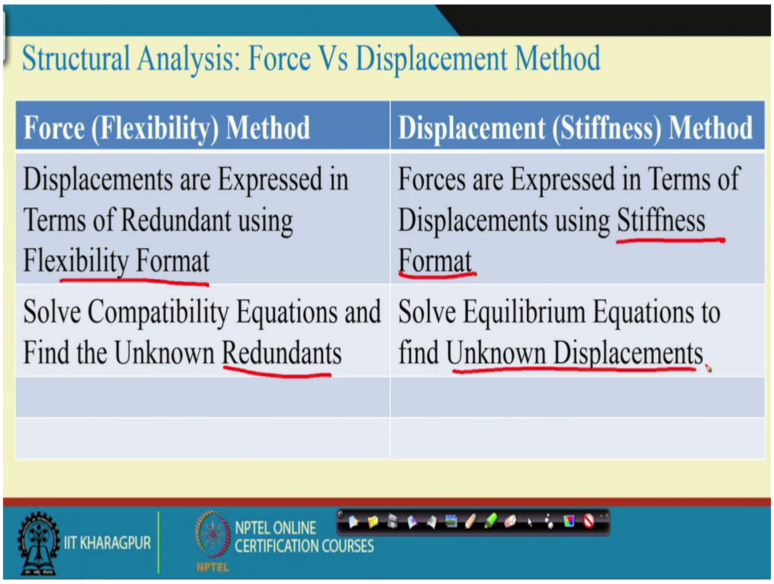
Now, the other hand if the displacement based method then we have need to find out displacement then the number of displacement need to be found out in order to in order to a in order to define the configuration of a structure or the configuration of the deformation is the kinematic indeterminacy of this; that is called kinematic indeterminacy.

The number of unknown in this case is kinematic indeterminacy, the force based method we need to find first step would be to find out a static indeterminacy and the displacement method find out the kinematic indeterminacy. Then identify the redundant forces, redundant forces means the forces that need to be computed that need to be determined which are which are to be considered as unknown.

Now, in this case the again we have to need to identify the unknown displacement, the number of the unknown displacement are equal to the number of static kinematic indeterminacy. Now then in the force method the third step is find displacement in basic statically determine structure we will just demonstrate this both the methods through one example we need to once we have the redundant identified the redundant forces then we need to split the entire in determine structure into several statically determined structure and then solve them.

And in these case the displacement method what we have to do is; we have to restrain the above displacement the displacement which are unknown and find the forces in the basic kinematically determinate structure. So in this case we need to we need to divide the problem into several sub problem that these sub problems becomes statically determinate and the displacement method we need to make the problems static kinematically determinate ok.

(Refer Slide Time: 08:20)



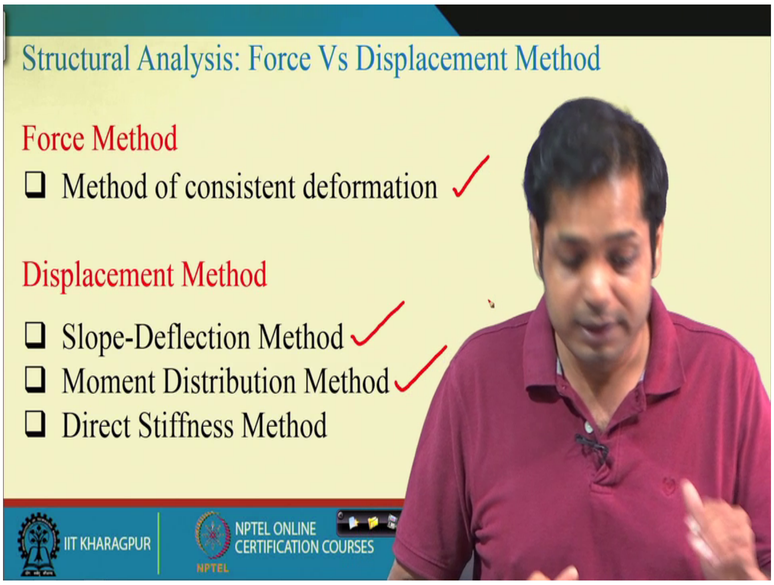
Force (Flexibility) Method	Displacement (Stiffness) Method
Displacements are Expressed in Terms of Redundant using Flexibility Format	Forces are Expressed in Terms of Displacements using Stiffness Format
Solve Compatibility Equations and Find the Unknown Redundants	Solve Equilibrium Equations to find Unknown Displacements

Now, in the next is next is once we have once we have decompose the divided the entire problem in basic determinate structure and calculated the internal forces and the displacement, then the force based method displacements are expressed in terms of redundant using flexibility format that we just now discussed and in these case it is the forces are represented or related through displacement using stiffness format.

Now, once we have a representation the next step is a solving system of a linear equation in the force based method then we solve for the redundant forces and in the displacement based method we solve for the redundant unknown displacement.

Once we have solved for the unknown forces and the unknown displacement rest of the things becomes then straight forward, rest of the thing is substituting those unknown forces the additional forces which are unknown into the static equilibrium equation then solve them all. So you can you can you can you can recall the steps that you followed in force and displacement method and tried to find out the relation between the dist the differences that that just now we discussed.

(Refer Slide Time: 09:38)

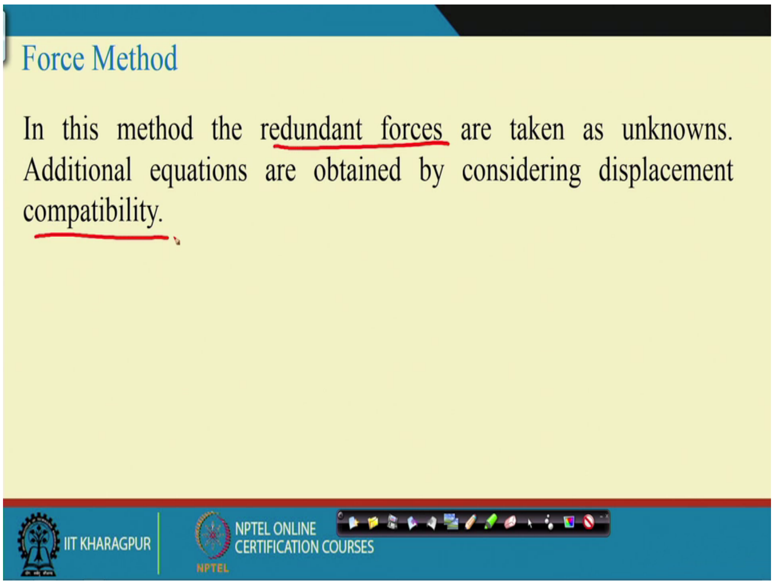


What will do to now is we will just before that so the force based method you might have studied in structure analysis one that method of consistency deformation is a force based method, then you started a displacement in structural analysis one slope deflection method, moment distribution method they are all displacement based method. (Refer Time: 09:54) stiffness based meth method if you are not done in structure analysis one direct stiffness method then it is also fine because we are going to discuss the direct stiffness method in detail; which is essentially the basic concept of matrix method of structural analysis ok.

So, what we do in this week a we just briefly review all this method the method of consistent deformation and slope deflection, moment distribution and then set the platform; so that we can go further and formulate the basic purpose the objective of this course matrix method of structural analysis ok.

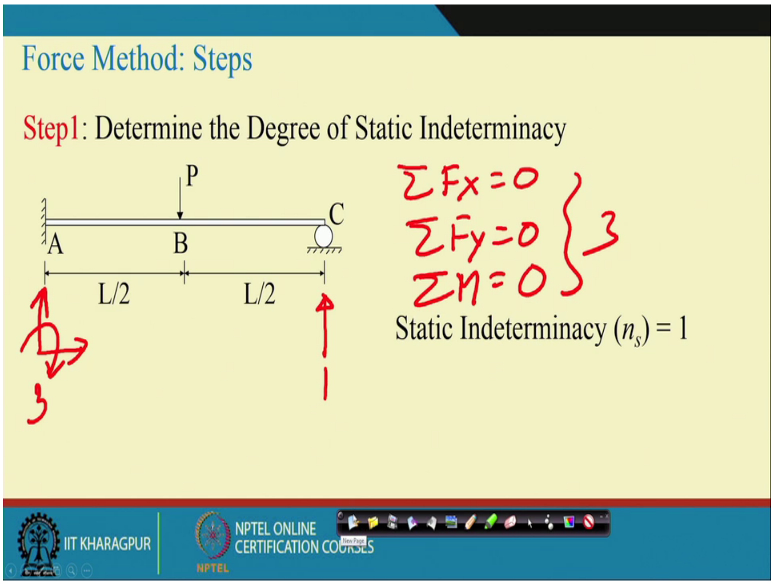
We will start with today we will start with force method of consistent deformation; so we are going to we will be discussing the steps followed in force base meth method through one a through one examples ok.

(Refer Slide Time: 10:51)



Now let us take one example so force based method let us first define force based method the force based method is essentially we take the redundant forces; the forces are unknown and the additional equation are obtained when we say the statically indeterminate structure or any indeterminate structure it means that the there are some unknown which for which you do not have equations and we have to formulate those equations and how those equations are formed based on that we have different methods. Now in this case the forces are unknown and the additional equations are formed based on the compatibility and then we solve them ok.

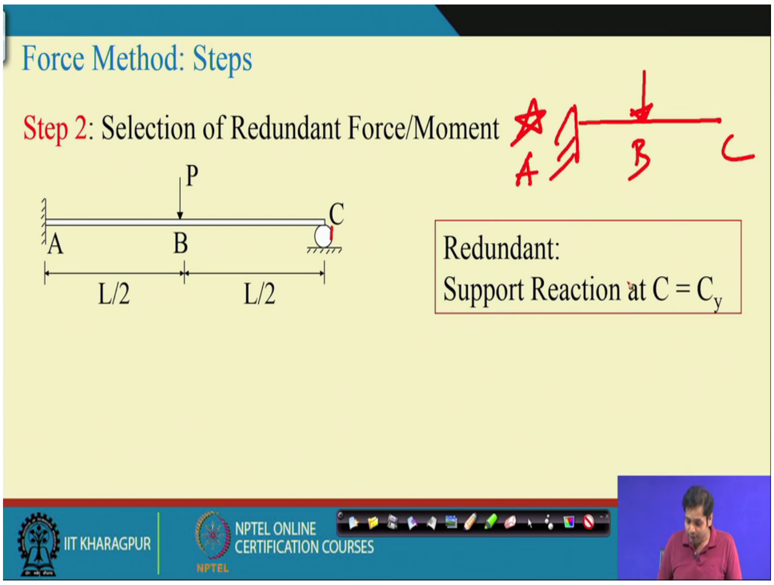
(Refer Slide Time: 11:30)



So, let us take one example this is a statically indeterminate structure we need to find out what is the we need to analyze the structure when means we need to find out the internal forces and displacements ok. So as we know since it is a statically indeterminate structure just equilibrium equation not enough to a completely analyze the structure to come to find out all the unknowns, we can probably partially solve partial some of the unknowns can be determined through equilibrium equation, but not all for indeterminate structure.

The first step is to find out the static indeterminacy, so in these case static indeterminacy is one eh this is a fix support we have three reactions here, if you recall the characteristic of suppose and then we have roller support we have 1 so here it is 3 at it is 1; so total number of unknown is 4 and then number of equations equilibrium equation available is a 3 summation of force is equal to 0, summation of F x is equal to 0, summation of a force in y direction is equal to 0 and summation of moment at any point is equal to 0; so total three equations (Refer Time: 12:41) in this case static indeterminacy is 1 ok.

(Refer Slide Time: 12:52)



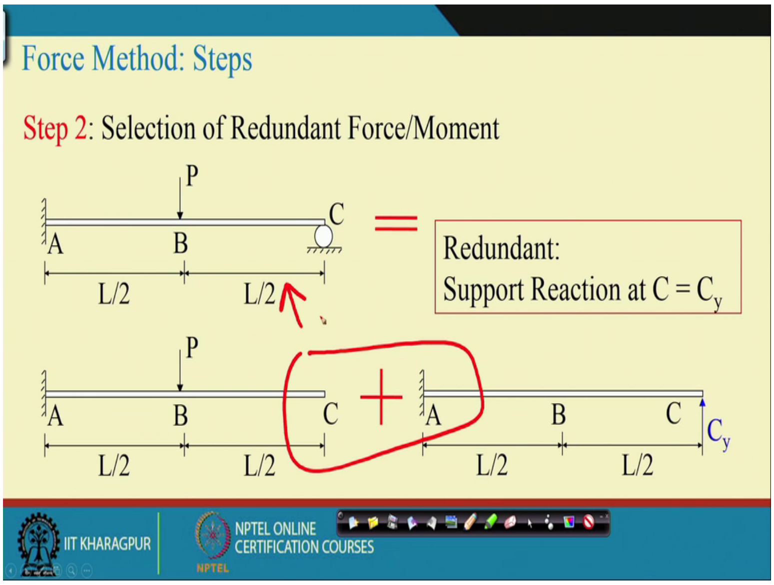
So, once we determine the static indeterminacy next step is to next step is to selection of redundant forces; this is a very important step this is the very important step you know if you again recall this we need to make this structure what is the purpose of redundant selection of redundant force because that is that the force you are going to be is going to the unknown.

Now and then based on that force we are going we have to decompose the entire structure into a basic sums into basic determinant structure ok. Now redundant force should not be such that when we decompose the structure into basic deter basic determinate structure your basic determinate structure become unstable; and based on that I will give you what does it mean when I say that when I say when I make this statement through one example ok.

So, in these case you see, in this case there are there are two possibilities we can choose redundant force one is take the support reactions at support reaction here as redundant force; means you see the eh it is a prompt cantilever beam had it been like this, had it been like this and then we have a load like this is A and this is C and this is B, then the structure is determined it is a cantilever beam ok.

So these additional these propt a at point C is extra so the reactions at point C is additional reaction. So therefore, we can assume we can take a reaction at C which is C y as the redundant force, so this is the one case ok.

(Refer Slide Time: 14:48)

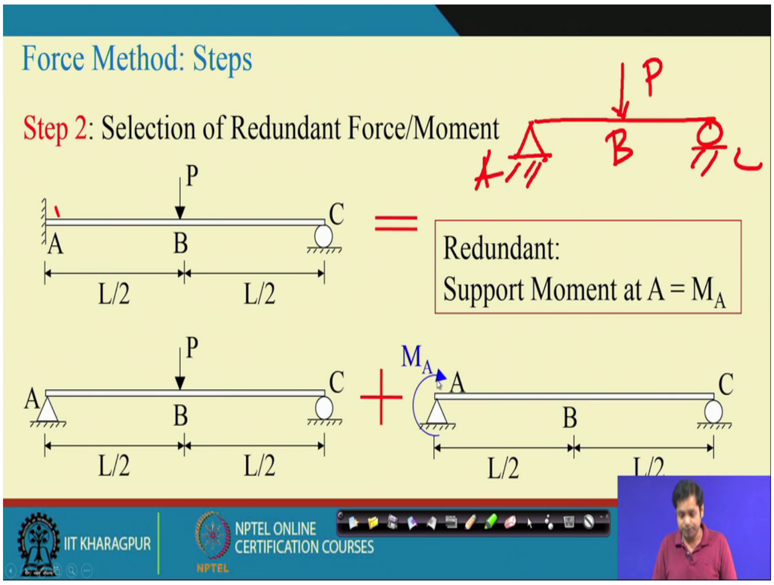


Now if you do that if you take this as a redundant force so F you do that this as a redundant force next step is we have to use it based on this redundant force we need to decompose the entire structure into basic indeterminate structure.

Now so these structure can be now decompose into two part using linear super position one is the cantilever beam; where there is no support at all and then this another cantilever beam where we have just the support reactions. So together this two gives me a so these two these two gives me this; that is the super position right linear super position, as long as your force displacement relations are linear we can have the linearity in this system this super position is a applicable ok.

So, this is case one a case one this is the one way we can choose we can choose the cy as redundant forces. Let us see if there any other possibilities another possibility would be a say again ins these structure if we remove the re remove the instead if we remove these fix support here and make it hinge the sup these becomes the it is hinge support and this is roller support, these structure is still this is P, this is A, B and then C.

(Refer Slide Time: 16:03)

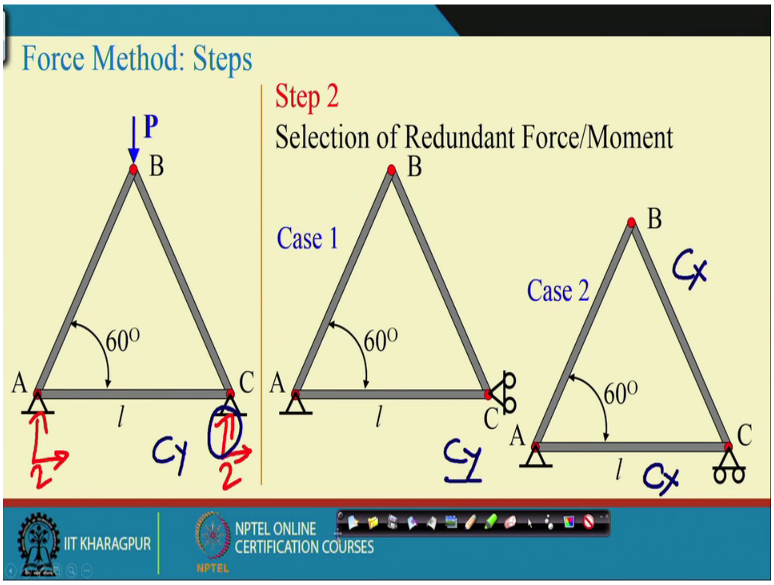


This structure is still determinate structure and not only that it is stable structure so we have removed the moment at the moment constraints at A so in this case the moment at A in A is taken as redundant forces.

So, either we can take support reaction at C as redundant forces or moment at A as redundant force that is you have to decide ok. So once we take this as a redundant force then basic is basic determinant structure becomes this; where there is no reactions, no basic determinant structure becomes your cantilever simply supported beam in these case, but in this case there is no rested, no moment and in this case it is only subjected to external load and then it is only subjected to the redundant force.

Now, this is how we can take there are many possibilities you can check whether other reactions can be taken as redundant force without compromising with this stability of the structure. Now just give you an example consider a truss I will go back to the this problem once again just see this step this is very important let us spend 1 or 2 more slides on spec 2 on step 2.

(Refer Slide Time: 17:20)



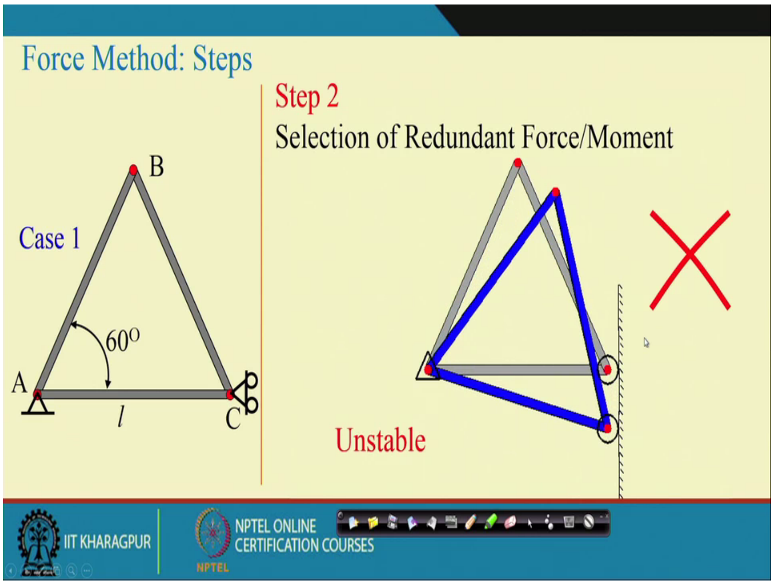
Take another structure another truss; which is again determine indeterminate structure if you can see here the number of reactions are 2, here also number of reactions are 2, so here it is 2, here it is 2; so total number of reactions are 4, but as a per as the number of reactions are concerned we can have 3 equations so it is redundant forces 1 static indeterminacy is 1 ok.

So now, one case is case 1 is we can take this as redundant force we can take say these as redundant force um, we can take these as redundant force means this was that redundant force the vertical reaction means C y, C y as redundant force if you take C y as redundant force basic in determine indeterminate structure is 1; so in this case C y as is redundant force there is no no restraint in at C in y direction.

Now another possibility is we can take C x as redundant force C; C x as redundant force so there is no restraint there is no restraint this is C x as redundant force so there is no restraint in x direction ok.

Now, let us see whether both the cases are valid or not, we can take we can continue the analysis by taking any of the cases or not are applicable or not. Now let us see what happens in the case of in this case means case 1 ok.

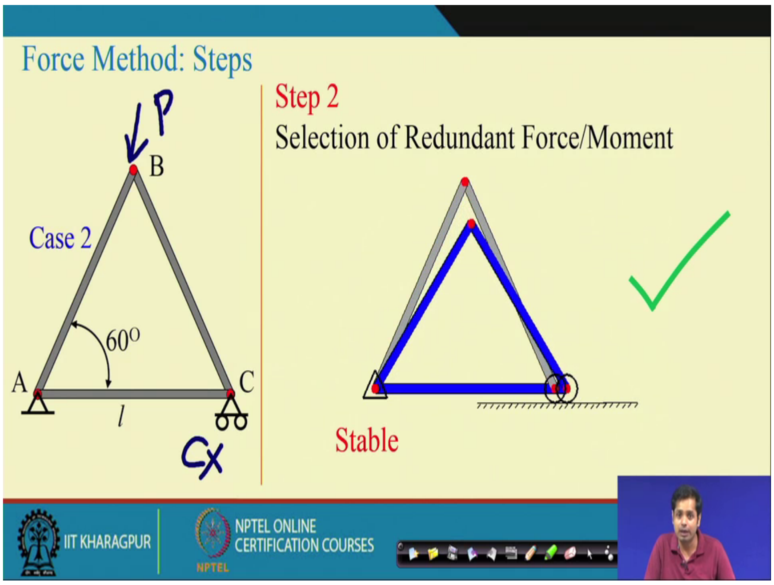
(Refer Slide Time: 19:13)



Case 1 is this one is the case 1 where C x C y is the redundant forces; now if it is subjected to a C x is a redundant forces; now you see if it is subjected to if it is subjected to some load some load P then this may deform like this you are this may this may collapse entire structure may collapse like this.

So what happens so this is not a stable structure so case one we cannot continue our simu our analysis with case 1, but see what happens in the case of case 2, case 2 this is unstable structure so this is not this is not this cannot be taken.

(Refer Slide Time: 20:01)

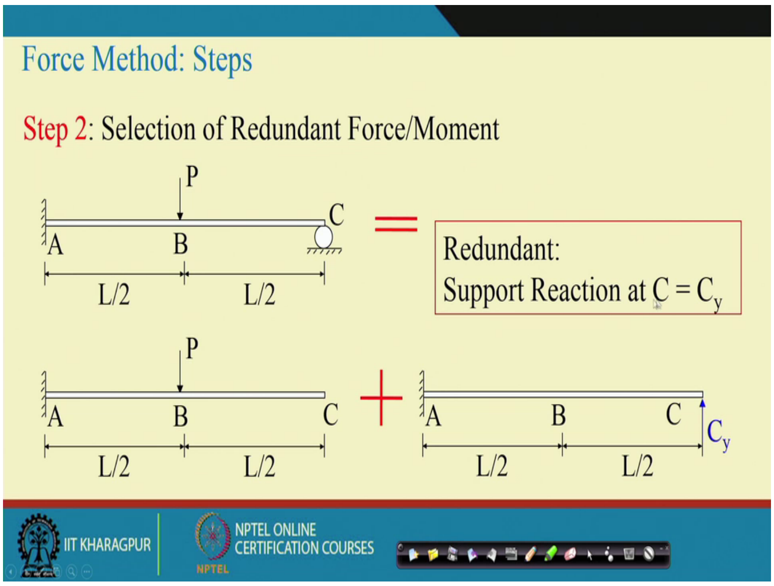


Take the example of case 2; in these case your redundant is C x is the redundant right. Now if you take C x is the redundant and then apply if we subjected to some load some load P here then it may be deform like this, so in this case it is stable and therefore we can take C x as redundant force.

So, when you choose a redundant force it is very important to have some understanding about the stability of the structure and the stability of the structure if you recall in your structure analysis 1 stability of the structure something that we have to we have to we have to judge based on based on looking at the configuration of the structure is there are equations, there are we can we can see what is the indeterminacy of the structure; whether positive or negative based.

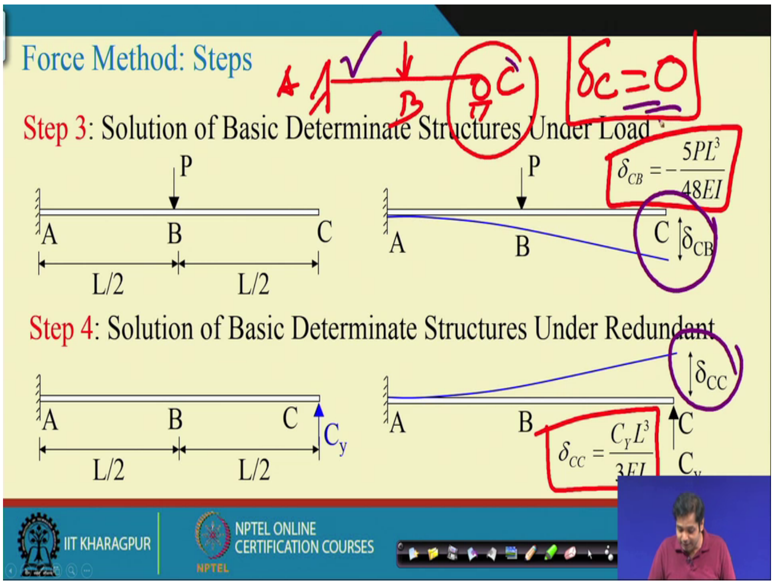
On that we can comment on the stability of the structure, but there are some example where we can show that just your indeterminacy is eh static indeterminacy is negative, but the static indeterminacy is positive the structure is indeterminate, but still this structure is not stable. The stability of the structure is something in that we have to we have to we have to judge based on our engineering sense ok; that is a very good thing that we should we all must practice ok.

(Refer Slide Time: 21:33)



So, once it is done again go back to our original problem the this problem; so let us continue the analysis by taking C x as the redundant C y as the redundant. Now if I take C y as the redundant this are the basic indeterminate structure; now the next step in these case is to solve this basic indeterminate structure ok. Now when you solve this means you have to find out displacement moments and everything.

(Refer Slide Time: 21:55)



And then first is this the basic indeterminate structure 1; where it is cantilever beam subjected to the external load and if we if we solve it then your displacement at the this is the displace this is the deflection profile and this is the deflection at point C, point C and these deflection will be a something this. So that we know to how to determine we know how to determine we have learned it structure analysis 1 right.

Now, the another basic indeterminate structure is the second one, which is cantilever beam subjected to the redundant force and if we your deflected shape will be like this and the deflection at point C will be this; deflection at point C will be this ok. Now once we have determine the deflection for the all the basic indeterminate structure then the next step is to is to use the compatibility condition to have an additional equation.

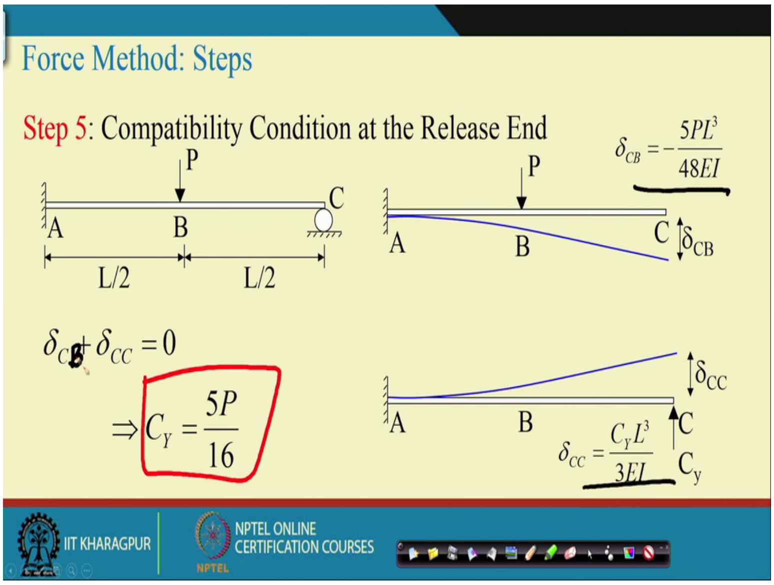
Now, let see what us the compatibility condition? Compatibility condition if you recall that it says that it says that it the deflection profile should be such that it should be it should satisfy all the continuous continuity requirement differentiability requirement and also the boundary conditions, all the constraints then only we can say the deflection is continuous or deflection is compatible.

Now, you see in these case 1 condition we know for these structure is actually the original structure was like this original structure was a cantilever beam a appropriate cantilever beam right a appropriate cantilever beam this point is C and this point is A and then you have a load and B point B.

Now, one condition is for sure irrespective what may be the deflection of the beam what may be the forces moments, but one condition with absolutely certainty we can say that; the deflection at the vertical deflection at point C has to be 0 because that is the support condition ok. So we have given a constraint in vertical direction so C delta C has to be 0 ok.

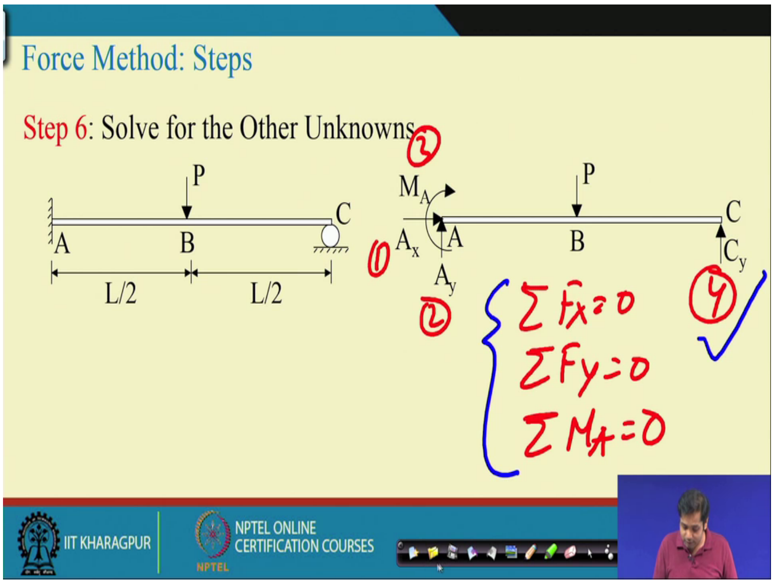
Now, a the super position say the linear super position theory says theorem says that; these the response of the entire structure is now can be obtained by response of their response of the individual sub problem. So if we add them the we get the response of the eh response of the actual structure; so the response of this structure response of response of response of this structure or the deflection or at point C should be equal to deflection obtained from this basic determinant structure plus the deflection obtained from this basic determinant structure and that total deflection has to be 0; so that is the additional equation that we have ok.

(Refer Slide Time: 25:05)



Let us see then what is the compatibility condition? Compatibility condition says that delta CC delta CC this should be a this should be C here this should be CB here this should be CB. So this plus this has to be equal to 0 right. Now if we make it 0 then what it becomes your becomes C y is equal to this, C y is equal to this so your reaction at C is equal to this.

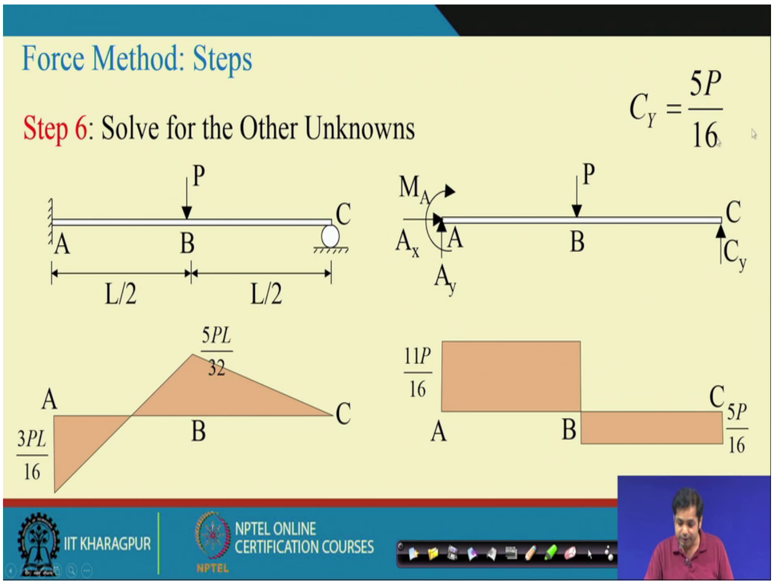
(Refer Slide Time: 25:47)



So, now once we know the reaction at point C the next step is to next step is this is now known; if you apply the free body diagram this is the free body diagram in this free body diagram initially before doing all this exercise initially we had four unknowns A x A y 1 2 3 and then 4, we had 4 unknown, 3 equations summation of F x summation of F y summation of F x is equal to 0.

Summation of F y is equal to 0 and let us say summation of moment at a is equal to 0; 3 unknown 3 equations 4 unknown cannot be obtained, but now after doing all these exercise this is C y is no longer unknown. So C y is known so C y is now known so essentially we are left with 3 unknown A x A y and M a and applying this 3 equations we can obtain this 3 unknown ok.

(Refer Slide Time: 26:45)



Now, once we obtain this then rest we can up we can we can we can draw the bending moment and shear force diagram, so C y is equal to this which is known and with this known C y we can obtain A x A and A y can draw the bending moment and shear force diagram this is the bending moment and shear force diagram. Once you have the bending moment and the shear force diagram then you can determine the deflection, slopes at any point; if any method you studied in studied for statically determinate structure ok.

So, this was has been a very small demonstration or other review of force method or method consistent deformation. Next class what we do is we will have a brief introduction to displacement method and review some of the displacement methods that the list that we give and then we then we see how this displacement method can be taken forward to form a very general method which is the topic of this course matrix method of structural analysis then see you in next class.

Thank you.