Matrix Method of Structural Analysis
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Lecture - 05
Review of Structural Analysis - I (Contd.)

Hello, welcome to the fifth lecture of Matrix Method of Structural Analysis. What we are going to do today is; we will review some of the concept of stiffness and flexibility. So, this is lecture 5.

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Before jumping on to the definition of stiffness and flexibility and what does it mean in the context of structural analysis, let us try to understand through some demonstration.

You see take one strip on a channel, if I apply some load for instance if I apply some bending, then it may bend like this that is the maximum bending that I can give.

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Now take the similar thing a piece of a paper and apply the similar bending and you can see that this is the kind of deformation it experiences. Then we say that this is more flexible than this. So, it is more stiffer which is stiffer than this. The stiffness and flexibility these two words we have used in different context, but and the meaning of stiffness and flexibility that we use in different context, here also the meaning remain almost similar.

So, it stiffness means. So, essentially stiffness and flexibility of anybody any object is essentially is a measure of the measure of its ability to deform when it is subjected to certain amount of load. In this case it stiffness is less its flexibility is more, but in this case compared to the previous one it is stiffer, but its flexibility is less ok.

Now, let us start today's discussion with the last slide of the previous class.

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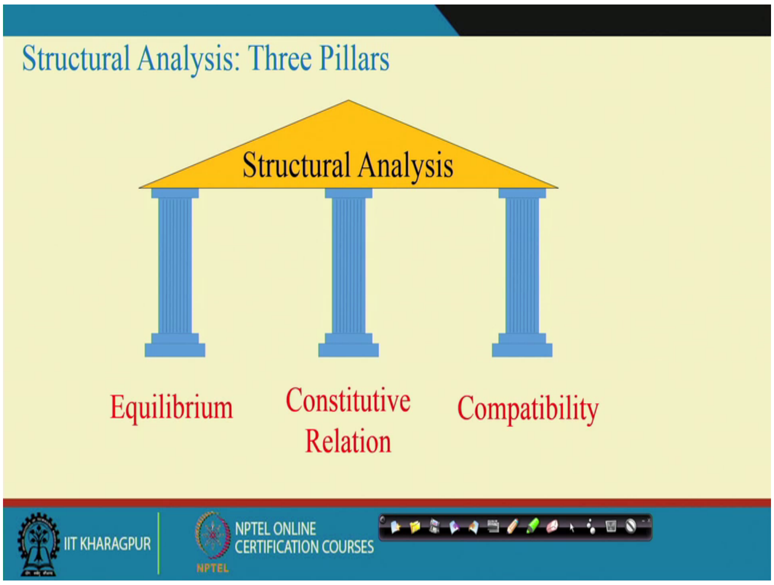

You remember in the previous class we discussed there is a there are three pillars of structural analysis, one is equilibrium which tells you how the different forces are related to each other internal and external forces are related to each other, then we have compatibility on the other end. Compatibility is now when a structure undergoes deformation, then how the what is the relation between the deformation and various and various point of the structures, how this deformation is related deformation at a particular point is related to the deformation is the other point and whether the boundary conditions are the constant that is given to the structure they have been satisfied or not. So, that is something we called compatibility.

Now, the third pillar which essentially find is a relation between equilibrium and compatibility is the constitutive relation. So, now the con when we talk about constitutive relation, there are two important ingredients of the constitutive relation one is stiffness and another one is flexibility and that is what we are going to discuss during today ok.

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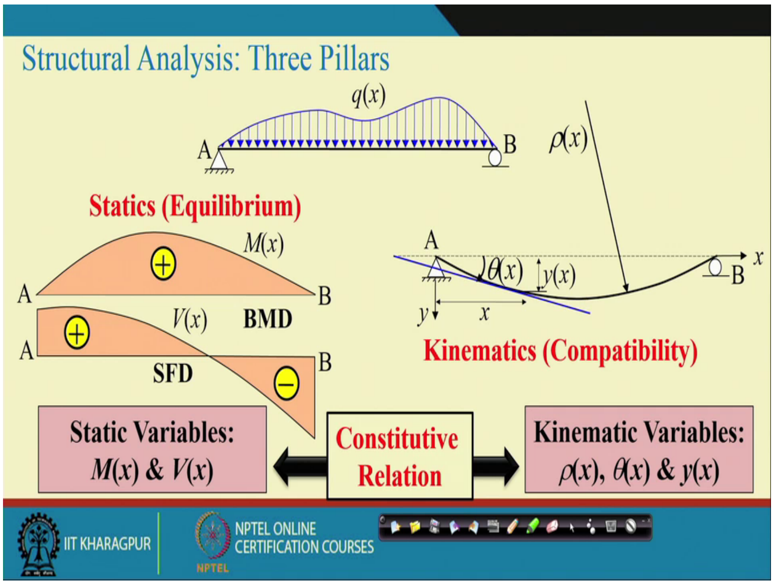

Last, let us consider one example, let us one is a simply supported beam subjected to any distributed load, then we know then this is the bending moment and this is Shear force of the of the of the beam. Now this steps finding this step this is statically indeterminate structure. So, we can just apply the equilibrium conditions to find out bending moment and shear force. So, this is the problem of statics ok. So, the problem of statics the equation that we used is equilibrium equations and what are the variables static variables? Static variables are here in this case bending moment and shear forces ok.

Now, all the force variables; now then suppose under this load the beam deforms like this, now the deformation at say for given at point A rotation is theta, at point B rotation there will be another rotation theta b then at any arbitrary distance from a deflection is y. So, now, theta xy and theta B, they should be such that the conditions and continuity that the beam must have these values of theta x y x and all these deformation variable they should satisfy them, and this is called compatibility and in this the process of finding this deformation is called the kinematics. And the kinematic variables are in this case you see it is what is the radius of curvature then what is the deflection at any point, what is the what is the what is the rotation at any particular point.

Now what? So, this is one pillar this is another pillar and then what constitutive relation does is the constitutive relation gives a relation gives how this two fa two different variables mean the static variables and the kinematic variables are related to each other, and that is the main purpose of constitutive relation ok. So, constitutive relation is essentially a relation between your static variables in this case moment and shear force and the kinematic variables.

Now, at this point one point I should make the constitutive relation or constitutive modeling is a separate branch of mechanics or I would say the very large branch in mechanics. So, we would not go into detail of those constitutive relation, here whenever we talk about whenever we whenever we say constitutive relation it implicitly means the relation between force and displacement. When we say force it accounts for everything force and moments and when you say displacement it accounts to everything displacement rotation everything. So, force is a static variable and the displacement the kinetic variable and constitutive relation at least for the present context of this course is the relation between force and the displacement ok.

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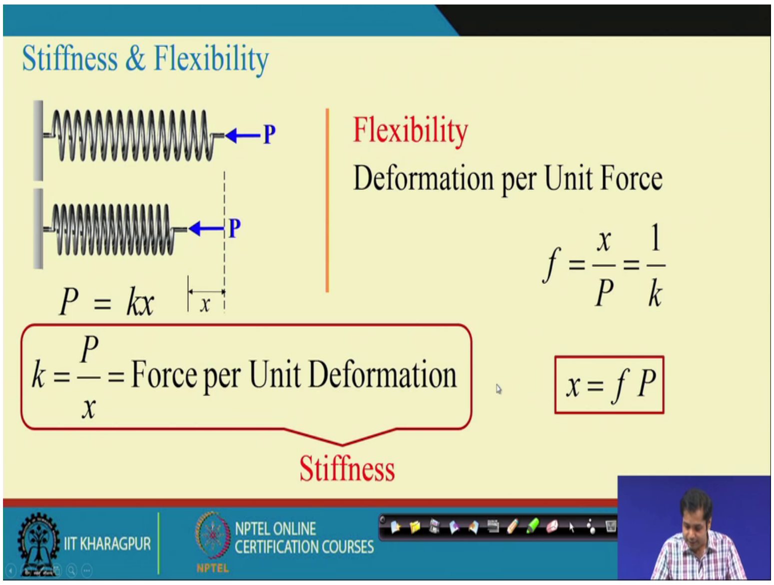

Now, take some example take a spring linear spring and which is applied to a force P and then under the action of this force P suppose x is the displacement and then we know the relation between force P and the displacement, and the relation is P is equal to kx. Where k is essentially P by x which is force per unit deformation and then k is called stiffness of the spring.

Now, if we plot the load displacement curve this load displacement curve, the load displacement as a before that what is flexibility? Flexibility is the just opposite of stiffness. So, in this case flexibility will be deformation per unit force and for if in this P the flexibility f is equal to 1 by k. So, this is stiffness and this is flexibility.

Now, this is very straight forward for a now then one thing is very important here, whenever we talk about stiffness and flexibility, they are not they do not carry any meaning until and unless we say until unless we mention that this stiffness or this flexibility they relate which force to which displacement. For instance in this case the force is this and if in this case say force is force is this and displacement is force is this and the displacement is this. So, when we say that this is force into displacement force into displ when we say the k is the stiffness and flexibility f is the flexibility. So, it means that they relate f when k relate P and x.

So, every time when we talk about flexibility and stiffness as I say they are not just absolute what they do not carry any meaning by itself, always we have to mention along with them that these stiffness and flexibility relate which force to which displacement ok.

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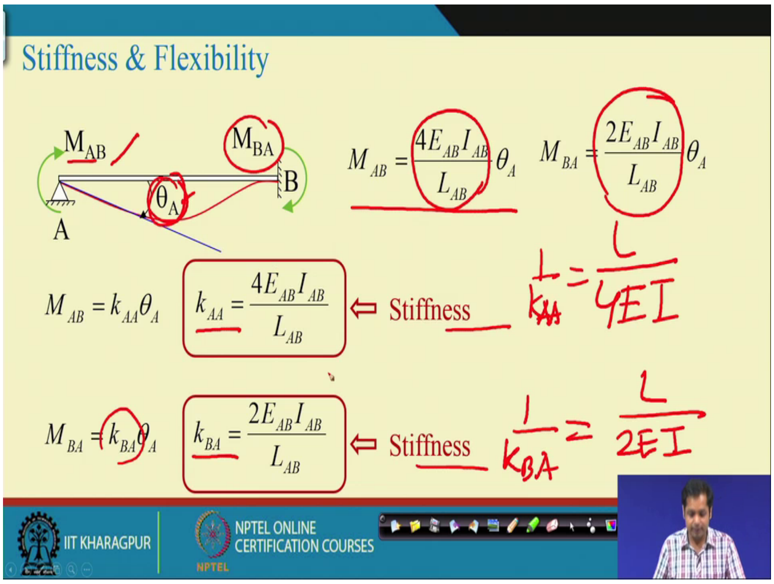

Now let us see some example take propped cantilever beam and suppose we apply a moment M A B moment M AB because moment is applied at A in a segment AB. Now due to the action of this moment this beam may undergo deformation like this. So, we have theta A is the rotation at end A B being the fixed end there is no displacement no rotation at B. So, theta A is the displacement, but also we know that when we apply a moment a M AB there will be an re resisting moment at B suppose that is M AB ok.

Now, we already know how to how to solve this structure, we have learnt that in structural analysis 1. Suppose and we the solution is something like this, M AB the relation between this force M AB and relation between this a this and this theta is this. Where E is the Young's modulus of AB, I is the moment of second moment of area of the cross section of the beam AB and L AB is the length of the beam AB.

And the similarly we have we have moment BA is equal to this that is how M BA is related to theta A. Now then what we say is then M AB can be written as this and where k is equal to the entire thing, k is equal to the entire thing similarly M BA can be written as k B into theta A, where k BA is equal to the entire this entire thing this entire thing and then we say that k AA and k BA they are stiffness.

Now, now when we as I just now I say it just stiffness is a term do not carry any meaning stiffness term did not carry any meaning. For instance in this case the stiffness k there it relates force M AB and theta A, but in this case k BA this relates theta A and M BA. So, that is why it is written as k AA means it relates displacement at A and the force at a it relates displacement at a due to the force at force at B ok.

Now, similarly then what would be what would be the flexibility? Flexibility would be just in this case the flexibility will be 1 by K A, 1 by K A which will be l by 4 E I and similarly in this case it will be 1 by k BA this will be L by 2 E I. Now this is stiffness and this is flexibility ok

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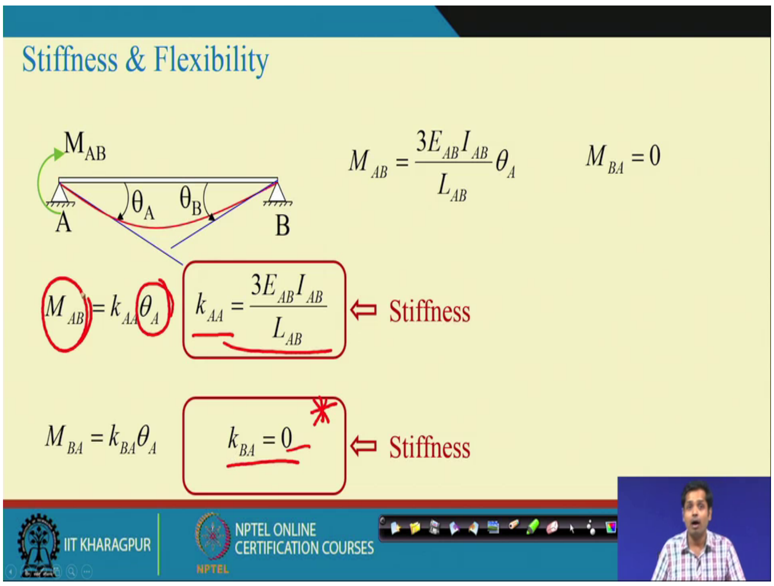

Now, now let us go let us see some more example. This is another example suppose we apply a load M A B for moment M A B here, then it undergoes deformation like this theta A theta B the deformation rotations at A and B respectively and then again from structural analysis zone, these are the solutions ok. There will be no there will be no bending moment at A generated because this is hinged support. And M A B and this theta and this theta are related to related like this.

Now, we write M A B is equal to k A into theta A, where k A is equal to this and then similarly M A B M B A is equal k BA into theta A, where k B is equal to 0. So, these are stiffnesses this is stiffnesses this is stiffnesses this is stiffnesses.

Now, you see at the there is a reason why in we know that M B A is equal to 0, but in spite of that there is a reason why a root k BA is equal to 0 explicitly put a star marks here, will come what happens when stiffness become 0 what is the physical significance of that ok. So, similarly by 1 by stiffness you can get the flexibility, but do not you may ask this question that 1 by k this give flexibilities in finite as I say, there is a purpose why intentionally wrote k BA is equal to 0. We will come to this point again in this course and we will see; what is the physical significance what is the interpretation of stiffness being 0 ok.

So, now again in this case the when we when we say that it is k AA, k AA always means that it relates force theta the displacement theta A and moment theta BA theta AB.

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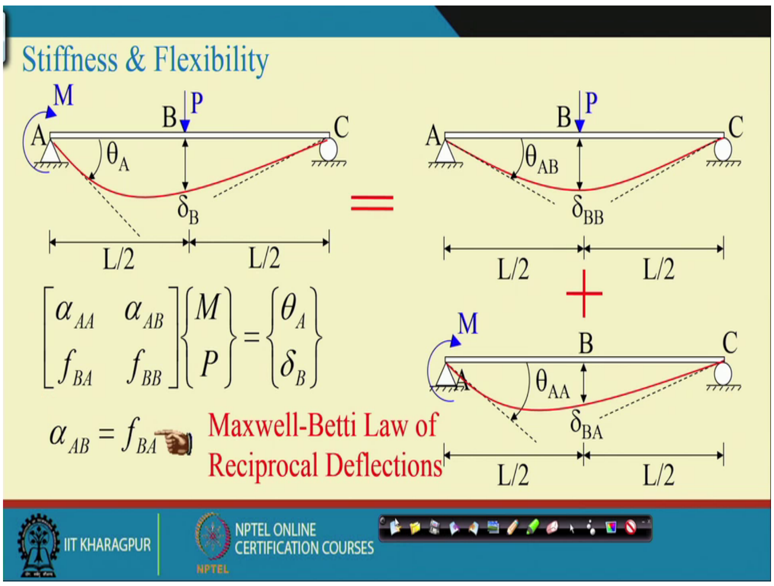

let us see one example where this example where in the previous example we had just one force, now here suppose we have two forces. In the same beam we have moment at a and the and force P at the at the mid span ok

Now, if we know just before writing the flexibility and stiffness coefficients for this for this problem.

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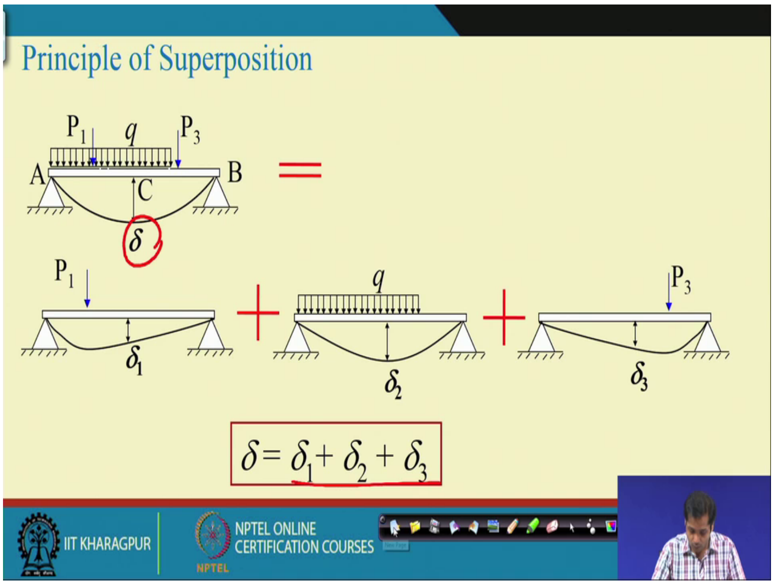

Let us review the principle as super position, if we know the principle as super position is as the name suggest that, if we have a problem like this and again it is only applicable for a linear system. As long as your force displacement relation is linear you can apply this and then these entire problem can be divided into three parts and then. So, you can take the effect of individual load separately and then combine them to get the total effect of the entire loading system that is what the principle of super position is say.

So, we just superimpose the effect of different components to get the component of the exact actual loading system. Now we do the same thing here, and suppose in this case it is a delta 1 delta 2 delta 3 at the three a displacement for three separately computed, then the total displacement at total displacement at c due to the effect of inter load will be d 1 plus d 2 plus d t and it is applicable as long as your displacements as small load deflection in relation is linear ok.

Now, again go back to the go back to the previous problem similarly we can apply the principle of super position in this case and divide the entire problem into two part.

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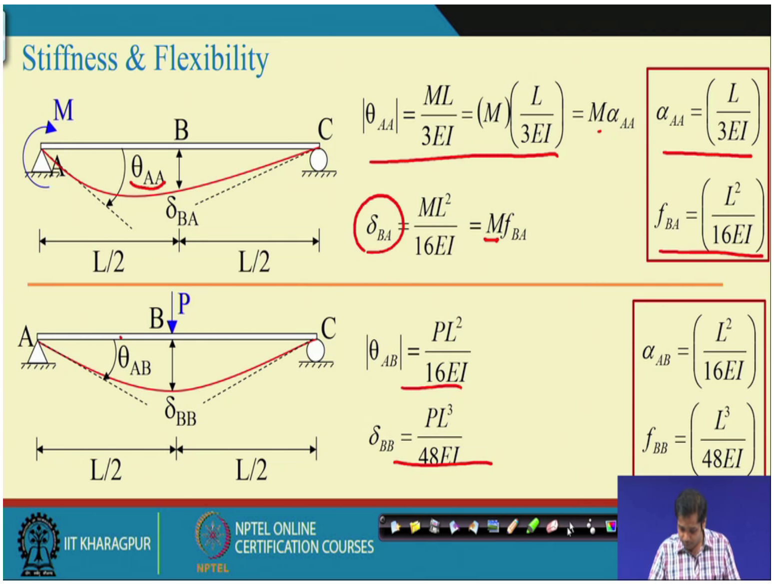

One is just the moment applied to this and this is the theta AA and M is related theta MM, theta A and M is related with this. And then we can have and suppose that that is written as m into alpha a where alpha a is equal to L by 3 I alpha is equal to L by 3 I. And again delta B A which is displacement at B is equal to ML square by 16 EI again we can write this M into f BA where f BA is equal to this. So, because due to the action of this moment M at A we have displacement at A and we have deflection at delta, and this rotation and the deflection is related to M as this.

Now, the next consider the second part, second part is another load this. So, if we apply a load P here again displacement is rotation at hey is theta A and displacement is theta BB. Now please a please note the notation used theta AA means, it is the rotation at A due to the moment or due to the force at A. theta B A means it is the displacement at B delta BA, displacement at B due to the force at A. Now here theta AB means, it is displacement or rotation at A due to the force at B and delta BB means it is the displacement at B due to the force at B.

Now again from structural analysis one, we know the relation and the this is the relation how this theta AB and delta BB related to B and then these are coefficients these coefficients are if we look at carefully these coefficients are flexibility coefficients why it is flexibility coefficient? Because alpha AA is essentially the displacement for unit force when M becomes 1 then alpha A becomes theta A. Similarly, when M becomes 1, delta BA becomes f BB. So, these are the displacement, these are the flexibility coefficients. Now again coming to the that point I am repeating that point again and again because it is very important to understand or or put that in mind that whenever we talk flexibility and stiffness, it is always associated with a pair force and the displacement. And which force we take and which displacement we take depending on that we have different flexibility and stiffness coefficients.

For instance when we take force M for moment at a and rotation at a, flexibility coefficient becomes this, and in a reverse way stiffness coefficient becomes that the opposite of this. When we take moment at A and displacement at B your flexibility coefficient become this or the stiffness becomes one by of one by this. Similarly for this and this so, now if we go back to our previous problem, the actual problem actual problem was if we recall the actual problem was this.

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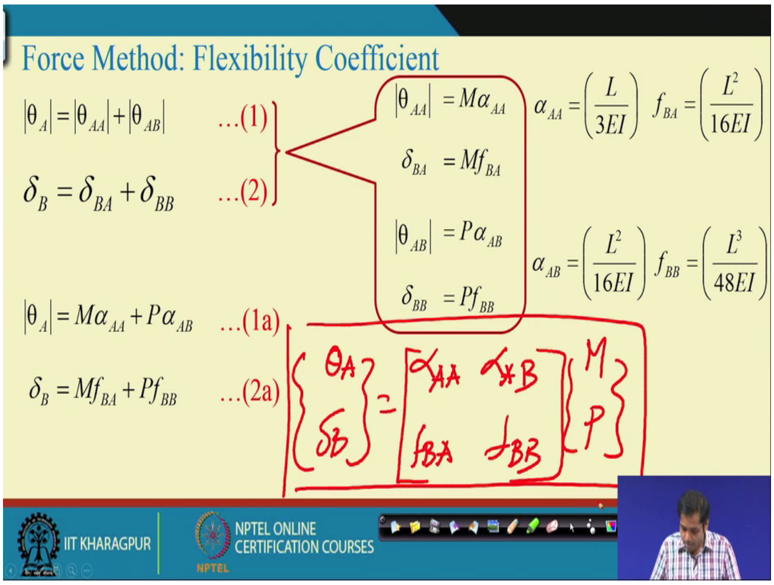

So, this problem is divided into two part, these are the two parts; so in actual problem. So, these are the effects of moment at A, these are the effects of displacement force at B. So, total fx will be. So, total displacement total rotation at A will be this plus this and then total displacement at B will be this plus this and that is what we can do. So, total rotation will be this and total displacement will be this and if we substitute that; so this are the total displacement will be this plus this plus this and total displacement will be this plus this, and total rotation will be this plus this.

Now, if we write that if we write that in a matrix form, you see what we may get is you see. So, total will be total is this and total will be this, now can we if we write that in a matrix form, where we have say theta A, theta A and delta B the entire thing we can write as something and then we have a we have M and P, and this becomes alpha AA and then alpha AB and this is f BA and f BB is not it.

So, this relation this relation gives me the total theta A and total delta B. Now this is also this is also force displacement relation you see, it is very similar to spring where we had just one force and one displacement component, here we have just two forces and four displacement components. And these matrix relate the forces which is M and P and the displacements which is theta A and theta B. So, this is again a constitutive relation now so, if we see this is written in matrix form. So, this is written in matrix form. Now one thing is very important here in this matrix form I will just in the next slide I will substitute the value of alpha AB AB all these values, but you will see when we substitute that, that these matrix is a symmetric matrix means the off diagonal terms are same.

So, in this case f AB is equal to f BA what does it mean f AB is essentially your f AB is the rotation at A due to force at B, and f BA is equal to displacement at B due to the force at A. So, these are same. So, this is if you recall this is known as Maxwell Betti laws of law of reciprocal displacement that you studied in and this is very important because when we this is very important this property is very important because when we when we when we derive equations for matrix method of structural analysis and see the properties of matrices, then we will see the symmetry is one of the very important property that those matrices satisfy and we will also see what is the physical significance of those matrices being symmetric ok.

Now next slide we just substitute those values.

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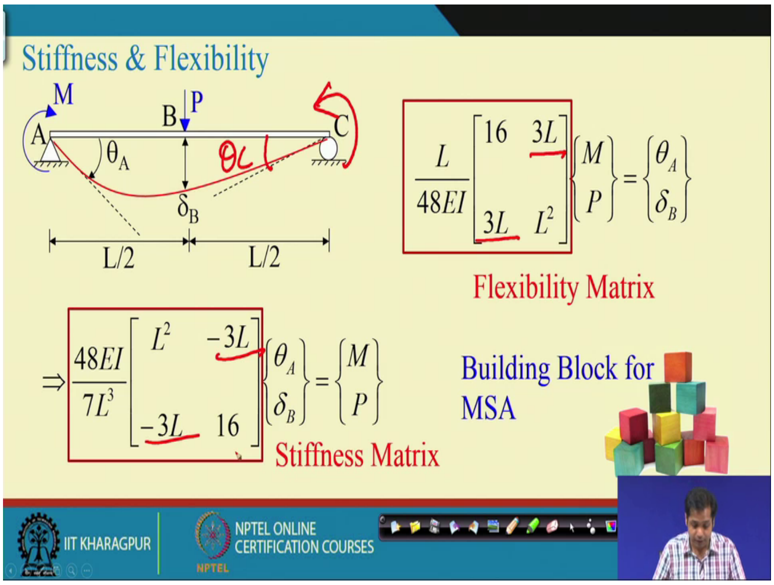

If we substitute those values if we see, these 3 L and 3 L this off diagonal terms are same, this is flexibility matrix and this is stiffness matrix you see why it is flexibility matrix? Because if we make unit force then this gives you the displacement; so displacement for unit force that is the flexibility and in this case a unit when displacement becomes unit and then for force for the unit displacement is essentially the stiffness. So therefore, this is called stiffness matrix and this is called flexibility matrix. This is very important and we can this is the symmetry can be observed here symmetry can be observed here ok.

This is very important because this is the building block of matrix method of structural analysis, we will discuss these matrices flexibility and stiffness matrix in detail as we as we as we progress. But in this case say, but another important thing you see you have only two forces, two forces and four displacement components that is why you have two by two matrices.

But then you if you apply another force here, if you apply another force here and then you then we if we use if we consider the displacement at this point also, then we have three displacement then the matrix the stiffness and flexibility matrix they become three by three matrix.

So, size of the matrix it also depends on how many forces or how many displacement components are we at we are talking about or innovate or how many degrees of freedom that the object has, depending on that we have different sizes of the these matrices.

Now, these matrices are very important, this is called flexibility matrix stiffness matrix. So, if you go back to the go back to the spring problem, where we had just one force and one displacement component, we can write the stiffness and flexibility those are scalar because they relate one first component to one displacement component. But then here when you had when we talk about various components or the forces and the various components of the displacements at various locations, and then we instead of a scalar, these become the relation becomes a matrix relation. So, they are the flexibly dense stiffness mat stiffness are they become matrices.

Now, and see. So, we stop here now, the flexibility as I said this relation the matrix stiffness matrix place the crucial row that is the building block of matrix method of structural analysis. So, we will discuss these stiffness matrix in a more detailed way for different kinds of structure with different scale or different scale means whereas, one dimension, two dimension, three dimension what happens to these matrices we will discuss those in details.

But here the for today's class the idea has been to review the stiffness the concept of stiffness and flexibility and the concept of stiffness and flexibility as a English word or the meaning of stiffness and flexibility, the meaning of stiffness or flexibility in matrix in structural analysis concept context they are also very similar.

So we will stop here today, the next week what we do is we will see. Now again we will see briefly review the different methods for solving statically indeterminate structure, we have just now discussed two concepts flexibility and stiffness, then which concept to use to solve a problem depending on that we have two class of methods one is force method and displacement method next week we review briefly the basic concept of force and displacement methods.

And then once we do that our review of structural analysis will be done. And then, in the third week we will be having brief review of the matrix algebra. And from fourth week onwards we will start formulating matrix method of structural analysis for different kinds of structures. Then, I stop here today. See you in the next week.

Thank you.