Matrix Method of Structural Analysis
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Lecture - 41
Introduction to Finite Element Method (Contd.)

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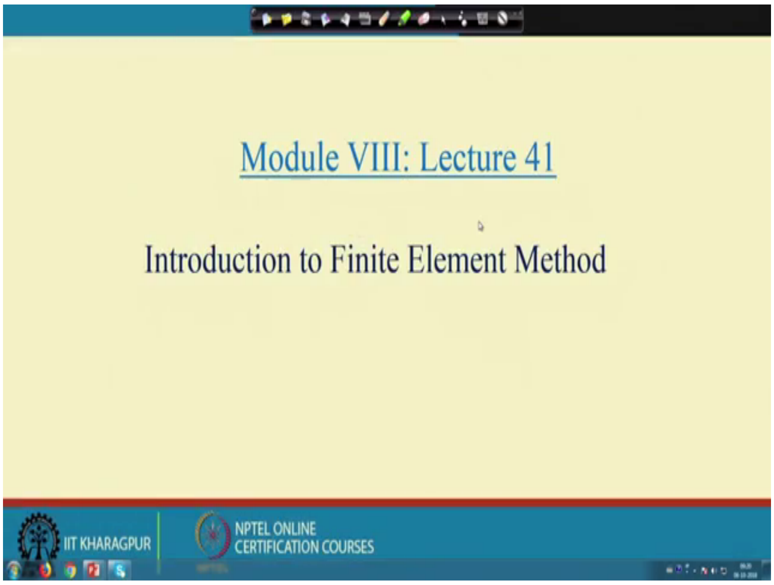

Hello everyone, so we are almost towards the end of this course. What we have been discussing where we have been discussing salient features of finite element method.

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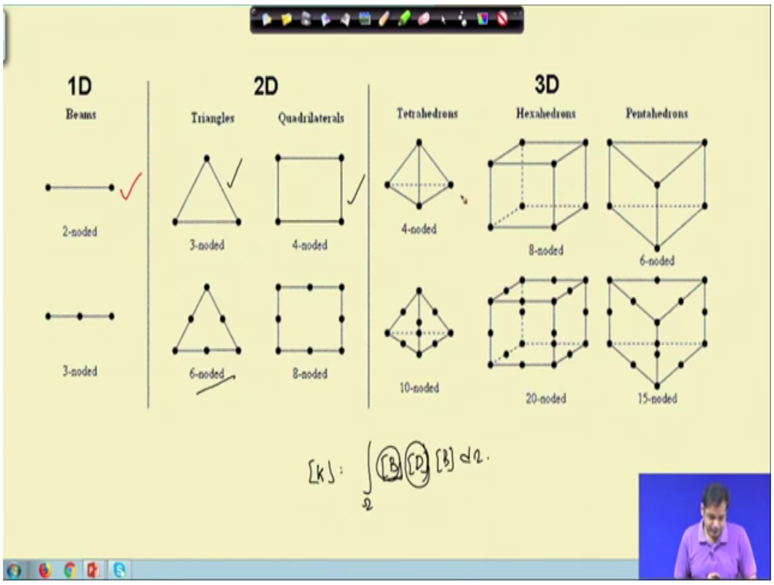

Now, you see if you recall these are the different kinds of elements we may have different depending on the depending on the dimensions of the problem and what kind of accuracy we need. We already discuss, we already discussed the formulation for this element, then formulation in the sense we have a general formulation if you recall that your stiffness matrix is equal to B transpose integration over domain B transpose dB.

Now, way what is B? This integration over the entire domain now depending on whether it the problem is one dimension, two dimension, three dimension this integration will be over the over the entire domain. And similarly depending on what kind of problem you are doing whether it is a truss problem, beam problem or some other problems, these D will be different.

And then B depends on what are the degrees of freedom you have at every nodes. And how these degrees of freedoms are interpolated are how those degrees are freedom are represented through a functional through a functional form to a function that depends that gives you b. So, B essentially the strain displacement relation, not necessarily strain displacement relation, but the examples that we have been discussing in that context B strain displacement relation right depends. We will see towards the end of this lecture that depending on if your problem is different degrees of freedoms are different, then the interpretation of B will be different, but essentially the B is you have some primary variable, you have some degrees of freedom that degrees of freedom in our case is displacement.

So, you have degrees of freedom and their degrees of freedoms at every nodes are interpolated or a that is represented through a function. And then when we calculate the derivative of that functions that gives you B. So, this is the general form of stiffness matrix. Similarly, we can have a general form of load vector as well we discussed that.

Now we have seen how to for a triangular elements, for a quadrilateral elements, we will briefly see if the elements is not. If you have a 6 noded triangle and if you have a 4 noded set it right down in three dimension then what how to how to get those shape functions. The process exactly will be same, we will see that shortly. Now, one important point ok let us wait for that.

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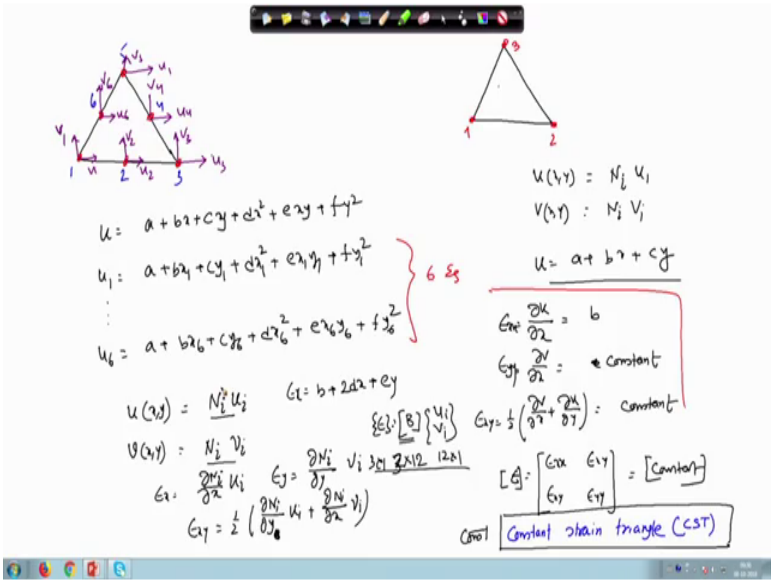

Now, let us start with let us see if our element is a 6 noded element, then what will happen. Now, if it is a 6 noded element then draw a 6 noded triangle, then we have these are the nodes. The process will exactly be saying I am repeating this process just because. So, again you have a node number for instance, suppose this is 1, 2, 3, and then 4, 5, 6. We already have seen then how you number this node in the context of one-dimensional problem how we number the nodes depending on that you get the stiffness matrix the population of the stiffness matrix right.

Yeah, you can be a number in you can number those elements in such a way that your stiffness matrix you get a bandage stiffness matrix. And bandage stiffness matrix always better to have from the computation of perspective. Now, so these are the node number similarly you can have say these are the degrees of freedom at every node. So, you can have degrees of freedom say here it is u 1, u 2, u 1, u 2, u 1 v 1, then here it is u 2 u 2 then v 2 v 2, u 3 u 3 v 3 and so on. And then u 4 u 4 v 4, u 5 then v 5 then we have u 6 u 6 and then v 6.

Now, the first the generic approach for all the; whatever elements you are using general approach will be same. Now, you have two degrees of freedom here u and v, though are two degrees are independent degrees of freedom right independence displacement quantity and u will be approximated as. Now, since we have 6 nodes yes we have information from 6 nodes, so we can u say a plus b x plus c x c y plus d x square plus e x y plus f y square.

We will shortly see what when we approximate a function through a polynomial then what are the terms to be considered. For instance see we can have just 6 nodes only, so information from 6 nodes are available. So, in the approximation we can have we can have 6 unknown.

Now, the question is how you we can have a several as some other approximation where your polynomials are different, the monomials are different instead of say x we do not take the term x and we take a different term. But keeping the number of unknown same is that enough is that is that right. We will see that shortly.

So, similarly u have this, and similarly v also have this. So, then what will recall what is the approach we did, we the conditions that we have is u 1 is equal to a plus b x 1 plus c y 1 plus d x 1 square plus e x 1 y 1 plus f y 1 square.

Similarly, we have u 6 is equal to a plus b x 6 plus c y 6 plus d x 6 square plus e x 6 y 6 plus f y 6 square right. So, this gives you 6 equations, this gives you 6 equations, and then we have 6 unknown we can obtain those 6 unknowns and when we have the 6 unknown, finally u which is a function of x and y can be represented as N i U i. So, N i is the shape function it is a function of x and y. Similarly V at any point x and y can be represented as N i and V i. U i and V i are the nodal values and N i is a function of x and y.

Now, naturally we can look at by looking at this now let us compare this with these three noded triangle. Three noded triangle also we already derived the expressions for n right this is node number 1, node number 2 and node number 3.

And there also we had u is equal to U x y is equal to N i N i U i ok. This expression is same. And V also V x y is equal to N i V i now, but then only difference is here is here you have summation over only three nodes, here you have summation over only 6 nodes.

Now, there is another major difference. The difference is if you recall when we when we when we constructed the shape function for this element, how what was the initial approximation of field variable, approximation one u is equal to a plus b x plus c y. So, your u, u that time similarly v. So, u was essentially a linear function of x and linear function of y right.

But now here we have u is that much we could have afford for this three noded triangle, because we had only three nodes, therefore, the informations are available only for three nodes which will help us to calculate only three unknown that was the reason why we had just only linear approximation.

But whereas, here we have 6 nodes, so we can approximate this field variable with a quadratic approximation like this. Then what is the consequence of that consequence of that is other consequence we will see when we talk about convergence. But immediate consequence is you see in this case if we approximate say if we calculate strain say epsilon; epsilon x, epsilon x, if you if you recall epsilon x is the strain in x direction right longitudinal strain normal strain in x direction that is equal to. So, if you have d u d x then this becomes b.

Similarly, epsilon y becomes d v d x this becomes similar a constant like this that may not be view or some other constant some constant. And similarly if we have say epsilon x y, which is if you recall half of del v del x plus del u del y, this will be also because some constant because both u and v are the linear function of x and y.

So, what this means this means that your entire strain field, now the strain field is represented as if you have a strain like this in two dimension. So, this is epsilon x if write epsilon x epsilon x x epsilon x y epsilon x y epsilon y y or epsilon x y and y x are same.

This is the entire strain field. This entire strain field is equal to constant right. Constant means not this constant, this constant.

Now, so you have a triangle you have an element within that element your strain field is constant and that is the reason this element is called constant cons let us use different color this element is you will called constant strain triangle or CST. If you see any finite element book probably the initial phases the elements will come across is constant strain triangle.

Now, this element has some disadvantages, advantage or disadvantage, those aspects we will not discuss here. You yourself apply you yourself write a code for finite element method the way we had written a code for 1D bar and beam element you can do that exercise for higher dimension as well. And then use constant same triangle and then see yourself the results what you are getting how the results is closer to the actual results ok.

So, there it says the strain is constant along the within the element. Then therefore, naturally if your strain is constant stress is constant everything is constant right. Now, strain and stress are constant. Now, the thing is which I mean by intuition you can say if in if you have body, then you discretize the body into small elements several set of elements.

And then within every element you are assuming this if you use this element it gives you strange constant strain within the element. And therefore, if you use a larger size of the a element, then what happens the strain is constant over a larger area which may not be possible which may not be consistent with the loading field on the structure.

Therefore, this element can give you a results, but the condition is at the very limiting case when this element size is very, very small. Now, even with smaller elements whether you get closer results or not and what are the difficulties you face, well solving a problem is in this element that will not discuss here. I leave it to you find that yourself.

Now, but when we have a these triangle, where the u is essentially a quadratic function of x and y and then if you calculate epsilon x here, epsilon x for in this case, so epsilon x will be B plus 2 d x plus e y. So, it is a linear function of x and y. So, this is not constant. Similarly, you can have epsilon y and epsilon x y. So, here you that is restriction the strain is constant that restriction is not applicable here. So, you get varying strain field even within an element.

Now, once you have this N i U i, then next is next is again the same approach calculate strain epsilon x will be your del N i del del N i del x into U i. And epsilon y will be del N i del y into V i. And epsilon x y will be half of half of del N i del N i del y i del, del y into U i plus del N i del N i del x into V i.

So, this gives you a strain displacement relation. So, it is these thing you can write as epsilon is equal to is equal to some B matrix into u and v right U i and C i. So, this gives you the B matrix strain displacement relation, where B matrix is essentially what will be the size of the B matrix in this case, the size of the B matrix will be you see here you have here total degrees of freedom is 12. So, size of the B matrix will be 2 into 12 right. This is size of the B matrix. And size of this will be 12 cross 1, and this will be.

So, this is how you can this 3 into 3 into 3 into 12, and this will be 3 into 1 this strain will be 3 into 1. So, this is how you can have this. Once you have B matrix, D is the constitutive relation strain displacement relation substitute that in the stiffness expression and a integrate it over the entire domain, you get this stiffness matrix for this element.

Now, a similar exercise you can do if your element is in three dimensions. So, and another thing you have to you must do is once you get these shape functions and then the properties of that shape function we discussed a. First thing you have to plot this shape functions and then see whether the shape functions satisfy the Kronecker delta property or not whether the shape function satisfy partition of unity property of not. You take any in any arbitrary point and sum all the shape functions whether they are they are giving you one or not. So, these are the some property that shape function must satisfy.

And this is a check see the by construction it should satisfy. But when you write a code, then these are the some checks that you should perform to check whether your computation the codes are written properly or not. Now, similarly if we have suppose if we have a element in three dimension, three dimension for instance if we have a this kind of element if you have a this kind of elements a tetrahedral, we have four three elements three nodes and one another total four nodes we have. So, if we have this kind of elements then what happened.

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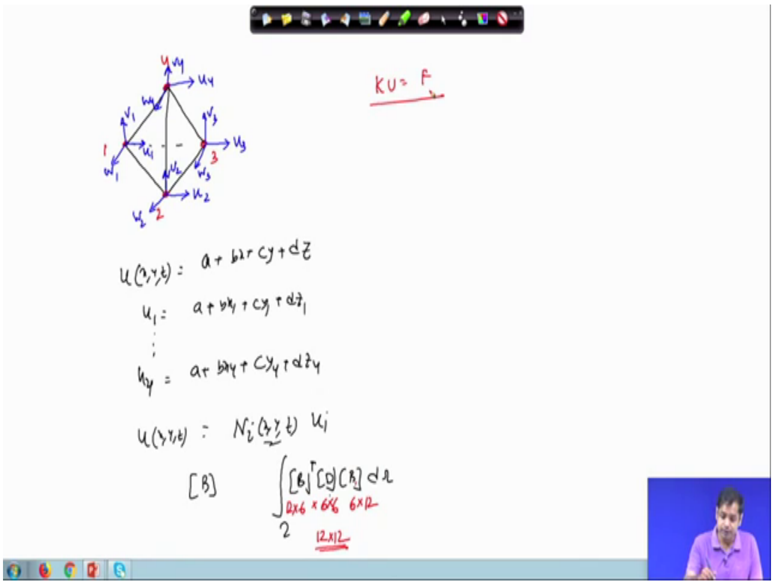

So, what we have is we have we have we have say then and then another element is this, another node is this. So, we have total one node is here and one another node another node is here. So, our elements is essentially this right elements is this.

Now, similarly we have to first give the numbering of the nodes say 1, 2, 3, 4. We will not discuss [FL] now that every nodes, every points you have now since it is in three dimension. So, every points you have 3 3 degrees of freedom right.

So, this is your u 1 u 2, then u u 2, then you have can have v 2, and then you can have w 2. So, u u 1, v 1, and then w 1, and so on similarly, here you have u 3, then v 3, then w 3 and then u 4, v 4, and then similarly w 4. So, at every points you have 3 degrees of freedom.

Then next is we have to construct, these are the values of the nodal points. Now, we have to get a value at the intermediate point within the element for that we have to represent them all this nodal value through a function. And for that assume u, which is a function of x, y, z both say x, y, z both. You have to approximate it. Say it a plus b x plus c y plus d z ok.

And then you have you have 4 nodes. So, essentially we are getting information from 4 nodes, so 4 equations we can have for every degree of freedom, so that is why, we have to restrict it a, b, c, d. Now, then substitute u u u 1 is equal to a plus b x 1 plus c y 1 plus d z 1 and so on.

Get this get u 4 is equal to a plus b x 4 plus c y 4 plus d z 4. So, get this solve it, substitute a, b, c, d in this expression. And finally, get u x, y, z is a function of N i, which is a function of x, y, z and then u i, and similarly for v i and w i.

So, once we have this displacement approximated like this, again you can check all the properties of these shape functions. Then we have to differentiate it, you to get this strain. And that gives you strain displacement relation B. And again we have to substitute that in this equation, B transpose D B, and then we get integrate over it, get the stiffness ok.

Now, in this case what would be, this is a three dimensional problem. So, what would be the size of D, size of D will be 6 by 6, size of D will be if you recall in a three dimensional space for plane strain, and plane stress idealization the size of view was 3 by 3, because you have earlier 3 stresses or 3 strain.

Now, but in this case we have all 6 components of stresses, and 6 components of strength. So, constitutive relation will be 6 by 6 ok. And then what is the how many degrees of freedom total how many degrees of freedom we have, at every node you have 3 degrees of freedom, total 4 node. So, we have total 12 degrees of freedom. So, B matrix will be B matrix will be 6 cross 12. And then B transpose naturally will be 12 cross 6.

So, essentially this gives you a matrix, which is 12 cross 12 the stiffness matrix size of the stiffness matrix ok. So, once we have the stiffness matrix, the again similar exercise you can do for loading. And then we get this get the expression K U is equal to F. And rest of the things at same, you have to solve it ok.

So, this is this is how we can do it for any other any degrees any other nodes as well, but there are some efficient ways also to write the 2 x to construct the shape functions. Now, all the shape functions the common thing, if you see whether it is for 1D, 2D or 3D three dimension, when you construct the shape function, we assume that displacement the degrees of freedoms that the field variables is approximated through a polynomial.

But, again depending on the more that is the most of the time that is a general thing common thing, we take there are reason for that; we are not going into that theory. Here, but just to inform you there are it is there are cases, you can you can think of not having polynomial expression for your field variables, you can have a similar different kinds of a expression for your field variables ok.

Now, so you can have in three dimension, you can have any other degrees of freedoms like this, but many the essence will remain same. Once you have the degrees of freedom, approximate them give at function representation of those degrees of freedom through which we can get the values at any intermediate point substitute that in the expression. And get the corresponding stiffness matrix ok.

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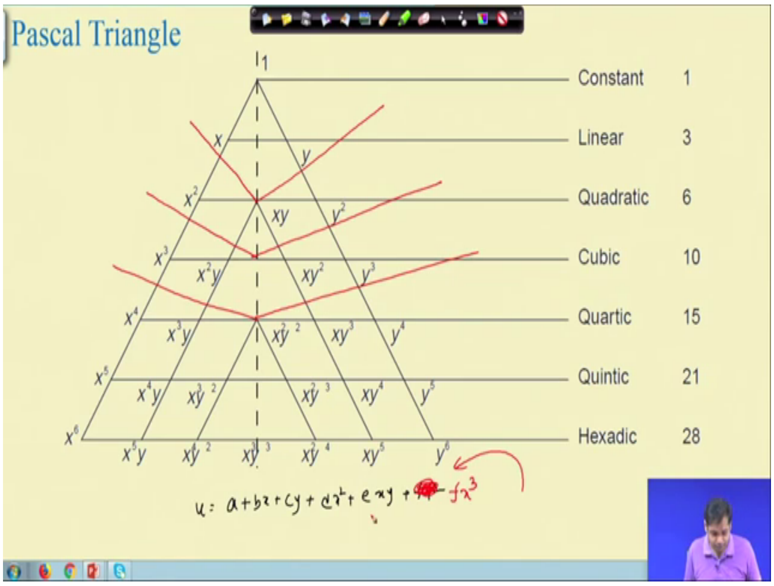

Now, then yes, this is important. You see just now I was I was telling you what when you approximate the this field variables, then what are the elements you should take. One thing is the number of elements in the approximation, the number of number of unknown in the approximation that depends on how many nodes we have, how many informations we have from the element right. This is one thing.

But, the second thing is then keep can we can we can we take any arbitrary combination of x and y, keeping the keeping the total number of unknown same, actually not. So, this gives you a guideline what are the points you have to take. This is a Pascal triangle. It it tells you that it tells you that if your if it is given in two dimension, now with the similar thing can have in six dimension three dimension.

Now, it tells you that if it is constant, then of course, you have just one term constant. If it is linear approximation, then you have to use all this term you have to use. You have to use one term associated with constant, then one term for dependency x dependent, and one term associated to y.

And then if it is quadratic, so here how many terms you have, you have y term. Now if it is quadratic, then you have to take all the terms up to this. Means you have to take if you recall for quadratic for quadratic, we assumes when we have when we assume that y is a quadratic, we took a plus b x plus c y plus d x square plus e e e e e x y plus f y square. So, total six terms are taken, all these terms are taken.

Similarly, if you have if you if you have a cubic polynomial, then you have to take all this ten term to make it cubic, in the in a way that the you should not have any biasness in the in x direction or y direction. For instance, if I do not take this instead of that, if I take plus f plus f say x cube, still we have only six unknown. And we have six equations, we can get these unknowns. But, the thing is we have by doing so, we have introduced a biasness in the direction. So, this is not a complete polynomial, completeness in with respect to both the coordinate axis.

So, when you take this these if you follow this rule, then you have a polynomial, you have an expression, which is complete, and that is very important for getting convergence. We will discuss the convergence shortly, what is convergence, so that is the reason why when you take this approximation, you have to be careful what are the term you take. Now, this is for a Pascal triangle.

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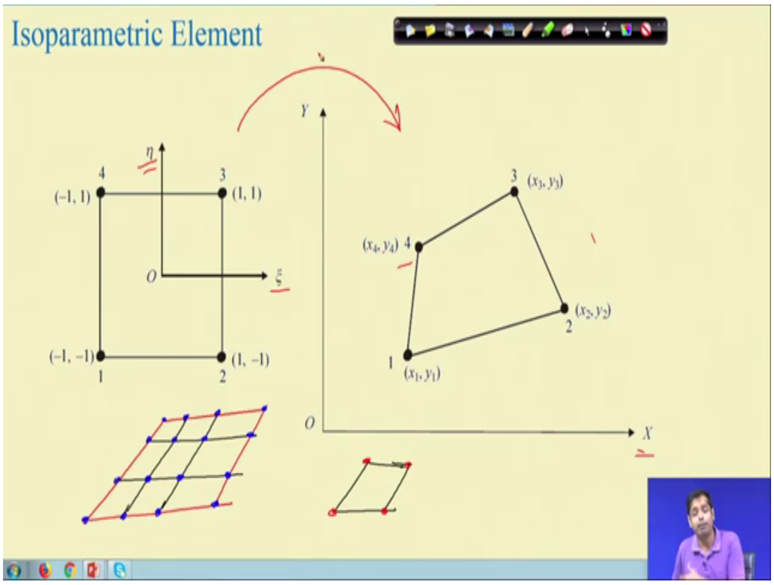

And the thing is isoparametric formulation. This is important you know you see all the all the cases, where we discussed so far. What we did is we know what is the values of coordinates, we assume it is a 1D element, we assume it is a triangular element, we assume a is a four-noded element.

But, you know the what happens many time that for instance for instance if you have if you if you have a domain like this say anything if you have a domain like this say for instance say for instance if you have a domain like this a domains any arbitrary domain like this, and you discretize this domain into say set of elements. These are your elements four-noded element ok. For every node, these are your for this is the boundary, this led is the boundary. And for every these are the nodes.

Now, you recall, when you talk about this quadrilateral elements four-noded elements, then we assume if you recall, there again if I go back to yes again go back to four-noded quadrilateral, we assume that this length is l and this length is h right. So, it is a perfect rectangle. At least that was the thing that we assumed while deriving these shape functions, but it may not be possible every time right.

So, what we have to do is suppose in this case your elements are like this elements are essentially, then if you draw one element, your element will be like this, your element will be like this, which has four-nodes. These are the nodes we have, but they are not rectangular.

Then what happens to this case, how do you calculate this stiffness matrix for this. And this is just an example I am giving, you it happens almost all the time, your elements are not perfect square or perfect rectangle how depending on the what size you are, what shape you are taking, it is not perfect in that sense.

So, what we can do is suppose, we can define two space, one is parametric space, where your element is a in this case, if you look at is a perfect square, which is it these coordinate is sees it is 2 by 2 square, you can have 1 by 1 square. Now, then what happens, this is your parametric this is your parametric space, and then you then you have an actual element like this, whose coordinates are x 1, y 1, x 2, y 2 and so on.

If you recall in the previous cases, we wrote the shape function in terms of x 1, y 1 stiffness matrix (Refer Time: 27:38) x 1, y 1 and so on. But now, now this is directly not possible, because the shape this shape is shape of the element is not perfectly rectangle.

So, what we can do is we can we can define a parametric space like this, and then we have we have an actual element like this, and then we can a map we can have a map between this space and this space. And get and get all the then approximate your field variables over this with respect to the parametric coordinate psi and eta, and then find a relation between psi eta and x y through a map like this. And then do all these approximation integration on this on this element, and preserve this map to get the stiffness matrix final stiffness matrix of this element. So, this is called isoparametric formulation.

Why it is call isoparametric, let us not bother about right now ok, isoparametric is essentially we will come to this point shortly. Now, this is then isoparametric formulation, now or the concepts. So, there is there exists a formulation through which even your elements are not perfectly square, perfectly rectangle, you can map these two an element in a parametric space, and do all the exercise, and get final approximate final stiffness matrix of this element. Now, this kind of formulation is also exist and play, and these are very common.

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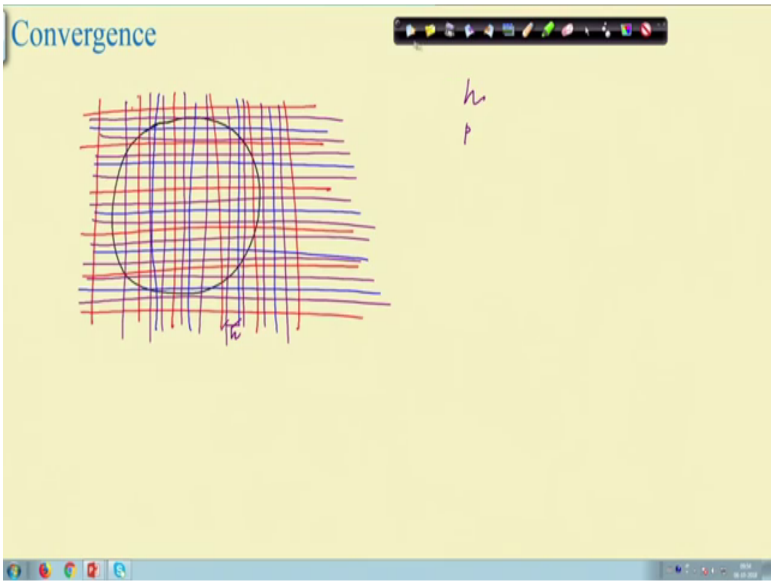

Now, this is important convergence. Now, before we talk about convergence, let us talk about some of the important term. You see essentially what we do, we do we have essentially we are solving some phenomena some physical process like.

Now, what happens you for instance, if I give you an example. Suppose, I have an I want to I have an arbitrary area like this arbitrary area like this, now I want to determine what is the area of this shape. Numerically, it is not a square, it is not a rectangle, so we do not know the closed forms close form relation of the area and the its geometric parameter.

So, what we can do is one the common thing we can do is we can divide it into some squares some squares like this ok, and then what we do is we calculate the number of squares. Suppose, for this it is full square, we take the full area, and this is how much percentage of square depending on that we take the area, and then we sum them, we get the area of this entire shape right.

Now, one common sense tells you that if you take smaller squares, if you take smaller square, then you get your area computation will be better. So, if you take further smaller square, for instance if you take if you take further smaller square, and then you compute the area, so your computation will be more accurate more closer to the actual value.

So, in a sense what we say is when we take further smaller square means what, when we are taking if we just bring the analogy that every square is essentially an element. So, when we take larger square, the element sizes are larger, but the number of elements are small. So, if we keep on reducing the number of squares reducing the size of the squares, then what will happen, your number of elements increases and also you get better accuracy right in the context of this problem.

Now, you if you keep on doing that, we will and at some point what will happen that this if we if we plot it, it will what will happen that, if we increase the if we plot and cut we with x-axis the number of elements, and y-axis is the actual value, then we will see as the number of elements become more becomes more in the your actual the computed area is approaches to the exact value right. So, this is good.

Then when we say that these approaches to exact value means, it is converging to the exact value. So, an every computation and this is just to bring an analogy, and finite element method exactly we do that. We enter divide the entire into small small elements, and then we then we write the equation for each element, and then assemble them to get the solution of the entire system.

Now, naturally very similar to that, if we increase the number of elements, the intuition says that your predictions the computations form computation will be closer to your actual value. And these entire thing (Refer Time: 32:37) this if you and this is called convergence. Means your if you keep on increasing the elements, your prediction will converge to the actual result ok, will be closer to the actual results; or in other way we can say that if we keep on increasing the number of elements or reducing the size of the elements, the error between the predictions from your finite element simulation and the actual results, so this error will decrease right. So, this is called convergence.

Now, now the convergence generally I can be achieved in two way, one convergence is called h convergence. For instance, in this case what we have done is we have just increase the number of elements right or decreases, we have decrease the size of the element. Say size of the element is if we h is the characteristic size of the element, then we say that if we this convergence is achieved in this case, if h is the size of the square each square. So, this convergence is h is by reducing the h that is it.

Keeping all are squares only, but larger square and smaller squares. So, this convergence is achieve the error is being reduced by reducing the size of the elements. This convergence is called h convergence, this convergence mean the convergence achieved by reducing the size of the element.

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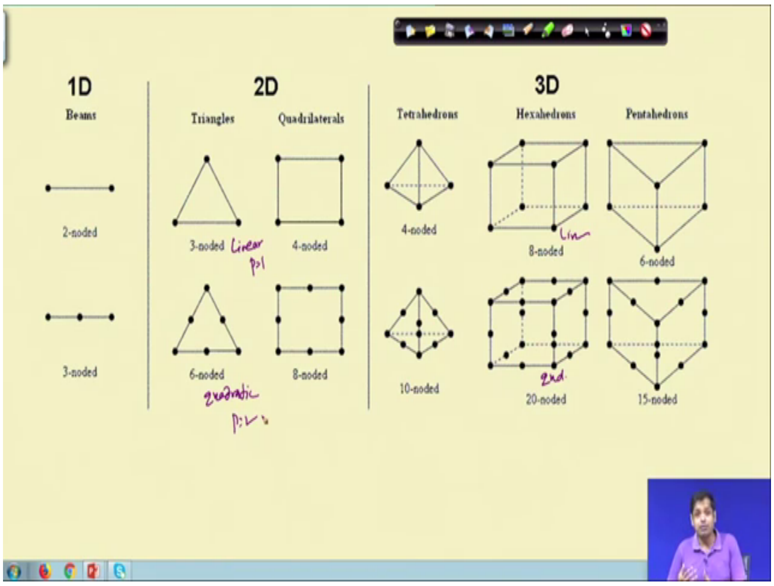

Now, you can have another convergence, which is called p convergence. What is it, the p convergence is p convergence is now you go back to this set of elements, you see what happen in these case. If you recall, in this case your displacement field was linear. In this case, your displacement fields was quadratic quadratic.

Similarly, here also you will see in this case, your displacement will be linear; in this case displacement will be quadratic. So, you can have an element, where displacement field is a displacement field is displacement field is cubic. You can have element, where displacement field is fourth order and fifth order and so on.

So, if we increase the order of this displacement field, means the order of the approximation, then what happened, then also you can get convergence. And this convergence is called p convergent; p stands for the order of the approximation. So, here linear your p is equal to 1, here quadratic p is equal 2 and so on, you can have a cubic with p is equal to 3.

So, if we increase the higher order elements, then also you get a results, which is closer to the actual results. Then this is called the p convergence. So, convergence can be achieved in both way either you reduce the number of elements or increase the order of the approximation. Now, with this, this is called convergence.

Now, convergence is a very important part in finite element method. And if you take any book, there is a chapter given and convergence, different kinds of convergence, what are the estimates of convergence, how to how to see whether the convergence can be achieved or not, what are the test, what are the error analysis is everything is given there. Now, here our objective what we have been doing in this week is to give you the information that these are the things exist, and you have to go through the books to read what exactly it says.

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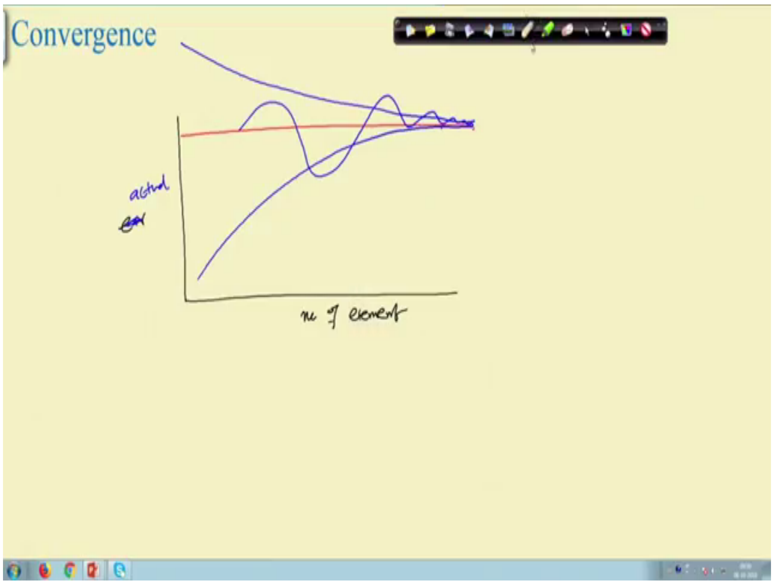

Now, now the question is you see here another important thing is convergence. Suppose, for instance you have you have you have here number of elements number of elements elements, and this is error. Now, suppose this is your actual results, this is your actual results. Now, essence is you increase the number of elements, you can get your error should be reduce. This is not the actual results; suppose this is the, it is actual result actual.

And now, the convergence essence is if you increase the number of elements, you the results should converge to the prediction should converge to the actual results. Now, you see it can converge to the actual result like this; this is also converging to the actual results. Now, this is also converging to the actual results, then this is also converging to this is also converging to the actual results ok. There are many way your prediction can converge to the actual results.

Then the question is whether all these convergence are acceptable convergence or not, or infinite element framework what convergence do we expect. Now, before I discuss that before I say that, you see what exactly we are doing. If we take an any infinite any system, which is a continuous system, and if you might have (Refer Time: 37:31) you know dynamics course that or any others any structural mechanics course that if you take an infinite if you take a continuous system, continuous system is essentially a collection of infinite number of points right.

Now, if you take a point in three-dimensional space, every point has 6 degrees of freedom. In two-dimensional space, it has 3 degrees of freedom right. Now, when you have a continuous system, which is a collection of infinite number of points and every points having 6 degrees of freedom, naturally a continuous system having infinite degrees of freedom right; so, infinite way in there is infinite in there infinite possibilities that the structure can deform right a continuous system can deformed.

But then, what you are doing, when you discretize a system, then you are essentially what we are doing. When we discretize a system, then your actual continuous system is essencia[lly is a collection of infinite number of points, but we are representing that through a finite number of points discretization is essentially that.

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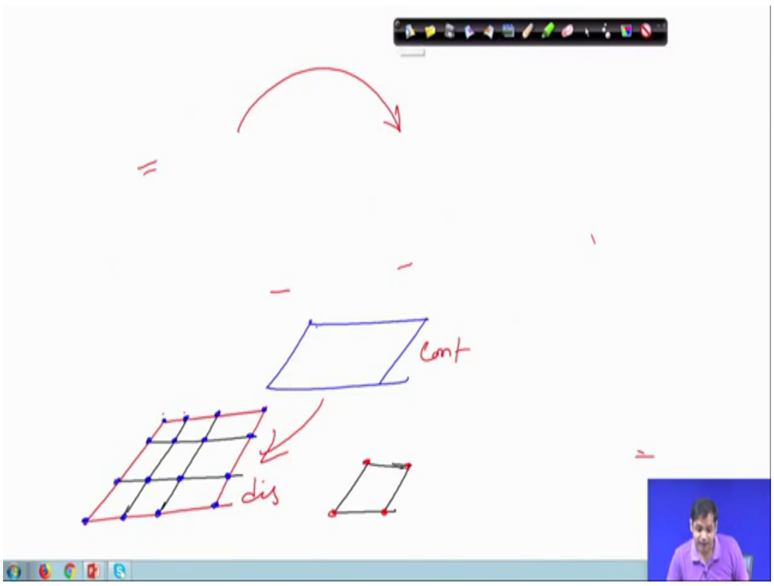

When we say that when we say that an element is for instance if we take this case in this case, your actual points actual in area was this, which is an continuous system having infinite number of points, thereby having infinite number of degrees of freedom.

But, now this is represented by just set of points, the collection of collection of these points 1, 2, 3, 4 only sixteen points. So, when you discretize it, this is a continuous system, this is this is a continuous system, and this is a discrete system. In the discrete system, essentially we have a finite number of point. When you have a finite number of points, then what is essentially we are doing, we are we are reducing the degrees of freedom right.

So, every points has 6 degrees of freedom 16 every points are 3 degrees of freedom, 16 points 48 degrees of freedom, but actual system has infinite degrees of freedom. When you discretize it, we are allowing only 48 degrees of freedom. When we reduce the degrees of freedom say essentially what we are doing, we are restricting the freedom of this object to deform.

The way the way it may deform, it can deform in a continuous system, we are restricting that deformation. Means we are making our system stiffer compared to the continuous system. When you make your system stiffer, then when you solve for displacement, what do we expect, your displacement is more than the actual system or the less than the actual system? Obviously, less than the actual system, because always we are dealing with a stiff system if it is the case, then if you plot if we plot sorry.

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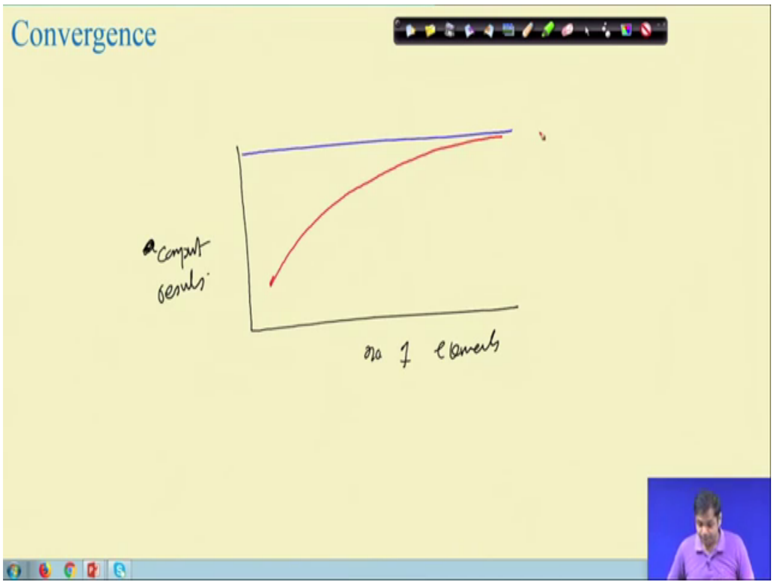

Now, if this is the case, then if you plot, it is number of number of elements number of elements, and this is actual this is your computed results. And this is your actual results, then always the results we get less than the actual results. We are talking about displacement less than the actual value, because we are always dealing with a stiff system.

And if we increase the number of elements, you are slowly increasing the degrees of freedom, you are making this structure from stiffer to stiff, and then more flexible than more flexible. So, these we will converge to this, this is a typical characteristic in convergence in this finite element method. Say it is always bounded from top. So, this is the way we get the convergence for (Refer Time: 41:11). But again, so we would not get the converges any opposite way. So, this is the convergence.

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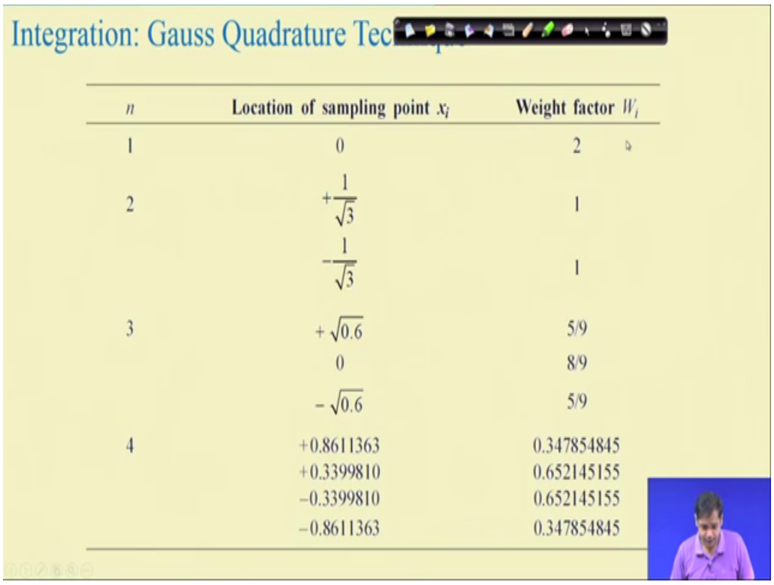

Now, then quickly one important thing is still left that is that is integration. We will (Refer Time: 41:31) we will discuss that integration.

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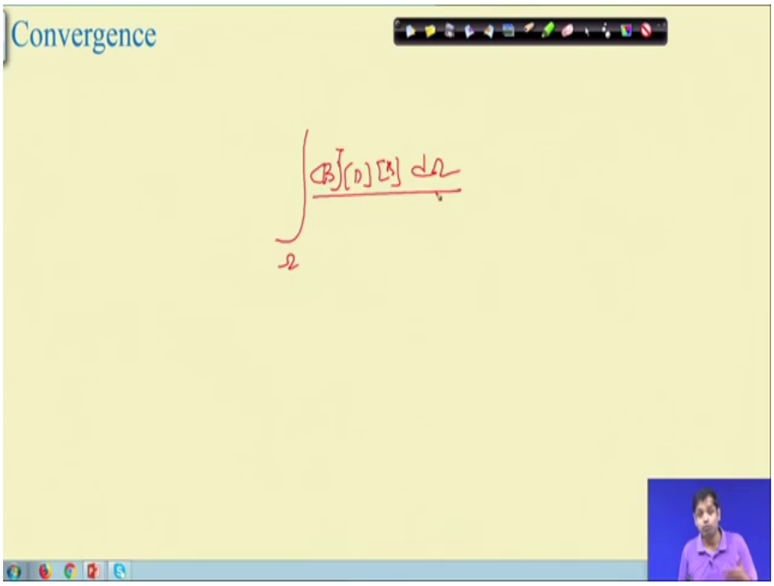

You see when we talk about when we say that in your stiffness matrix is integration B transpose D B D transpose D B d omega, so this has to be integrated over the domain. Now, the domain is it can have a different kinds of problem, so this integration may not be possible always most of the time in a close form. So, we have to do it in a numerical following a numerical technique and that is an important aspects infinite element method. Will that we will discuss that integration in the last class in the next class. So, I stop here today; see you in the next class.

Thank you.