Matrix Method of Structural Analysis
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Lecture - 04
Review of Structural Analysis - I (Contd.)

Hello everyone, this is the fourth lecture of module 1.

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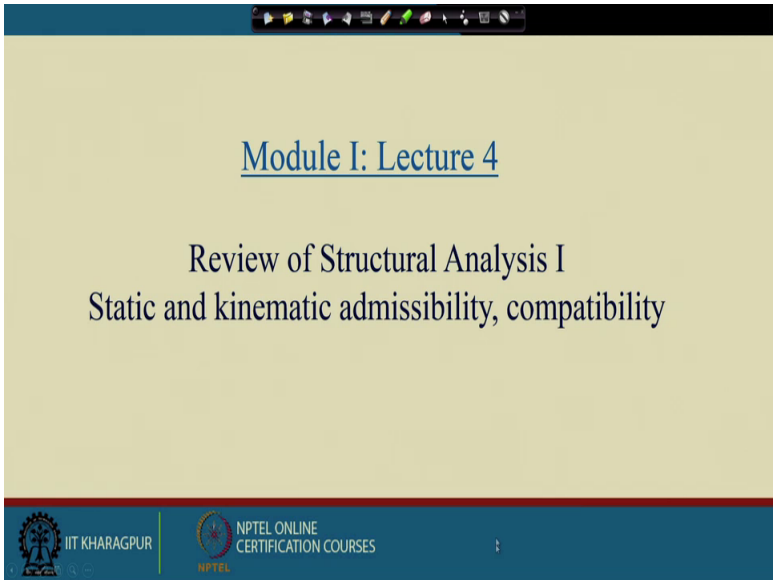

Today what we are going to see is, what we are we had been doing is, we have been reviewing the reviewing some of the concepts in structural analysis one. Today we will see what is static and kinematic admissibility and then we introduce what is compatibility ok. You see the term admissibility, what is the meaning of term admissibility? Admissibility is we say something is admissible means some things is varied or something which is acceptable right, then we say that this is admissible. Now before coming into the point what the admissibility means in structural mechanics, it has tried to give like to bring on analogy so, that you can we can put the definition of admissibility in a proper prospective ok.

For instance, take one example suppose you want to buy one shoe right you want to buy one say running shoe or to be specific, you want to buy a running shoe which is blue color now this is your condition suppose. Now what you do is you go to any footwear

store, now when you enter a store then you have a variety of collections of different kinds of shoes slippers sneakers different kinds of footwear.

Now, what is your job; your job starts from where is you have to find out a shoe which you want or rather you have to find out a shoe which is which is consistent with your requirement ok.

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Now what you do there first for instance when you go to a shoes any footwear shoe footwear store you get, this is just for a representative figures you can have say thousands hundreds of collections, but it is just to just to make the point I took some variety.

Now, suppose this is the entire collection you have. So, so this is the entire space, now your job is to from the entire space select the one which satisfies your requirement and what are your requirements? Now first requirement is here it should be shoe we do not want any slipper, you do not want [FL], you want a shoe. So, from this entire collection so, you take only those which are shoes; so, these are all shoes right. So, these out of all eights all seven these four satisfy your this condition

Now, then your next requirement is, it should be running shoe. Now your next requirement is a running shoe now out of this four all are not running shoe, now only these two are running shoe. So, these two satisfy your this condition as well as this

condition. Now then your requirement is furthermore, you are not satisfied with only the running shoe you want a shoe, which color whose color is blue. So, out of this 2 only if the next condition is then these satisfy this condition.

Now, then we say this is admissible with respect to your requirement ok. If this is admissible because this is the shoe, this is the running shoe and this is the color of the shoe is blue. So, it satisfy all the condition all the requirement that you have. So, now, if we say the second one this if we say this collection these two are admissible with respect to this condition and this condition, but this is not admissible when we put another condition that the color has to be blue. Similarly these four conditions the this one, this one is admissible only with respect to only with respect to this condition only with respect to this condition.

But when we these are not admissible when we put this as condition and this as condition right. So, when we say the admissibility, then it is not an absolute word admissibility always comes with a condition with respect to which condition something is admissible right. Now in this example with respect to all this condition this one is admissible.

Now, you see when we what is the ultimate objective of structure analysis? Structure analysis ultimate objective is to find out a solution. Find out a solution means say for instance if you want to find out the deflection of the structure, internal forces support reactions of the structure, what are these internal forces support reactions or say or say deflection, they are the solution of mathematical model representing the structure right. If you remember we discussed what are the steps you follow in structural analysis. So, this these bending moment shear force all this internal forces support reaction deflection; they are the solution of some mathematical model right

Now, when you solve it solve an equation essentially what we do? We search for a solution; it is very similar to when you got to a footwear shop and search for a shoe. Now when you search for a shoe you search you try to identify that identify the one, which satisfies your requirement. Similarly when we search for a solution, we have to find out the solution which satisfies our requirement or requirement of a structure right now what are the requirements a structure can have?

Now, as I said whenever we talk about admissibility, there are always it is always associated with a condition some conditions, admissibly has to be with respect to certain condition.

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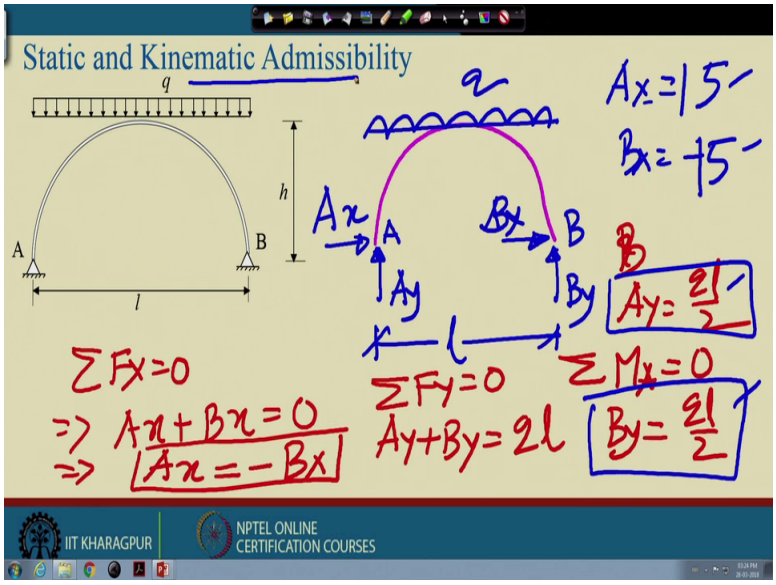

In structure analysis we have such 2 conditions; one is static admissibility and one is kinematic admissibility. Static admissibility a condition which is which is with respect to the equilibrium of this entire system, and the kinematic admissibility something which is related to the compatibility of the entire system.

Now let us, now if in order to demonstrate that, let us take one example here. This is we already now know what is indeterminate determinate structure. So, this is a statically indeterminate structure right. Why it is statically indeterminate structure? Because if we draw the free body of the free body diagram of the structure, then we have 2 forces here, we have we have we have two forces here, those are hinged these are 2 forces right 2 forces and then we have this is the externally applied load like this.

Now, suppose this is A y the vertical reaction and this is B y the vertical reaction at B, and this is a this is by B y the vertical react B x the verti horizontal reaction at B and this is A x the horizontal reaction at A. So, this point is A and this point is B right. Now if we and then this is q and this entire length is this length is l right. Now you see if we draw the if we write the free body, if we write the equilibrium equation the equilibrium equation is equilibrium equation is summation of fx is equal to 0. Now summation of fx

is equal to 0 means what? If this gives me that A x plus B x is equal to 0 right or we can say that A x is equal to minus B x right this is the condition this is how A x and B x are related.

Now, if you take another condition that summation of summation of F y is equal to 0 summation of F y is equal to 0, this gives us A y plus B y is equal to q into l right total external load. Now we have summation of summation of summation of M say at M A is equal to 0, if you apply you can apply this if you have if we take summation of M A is equal to 0, this gives us B y is equal to ql by 2 right. If B y is equal to ql by 2 if I substitute this then we have B n then we have A y is equal to ql by 2.

So, we have determined what is now how many unknowns we have here? We have one unknown A x, A y, B x by total four unknown. And out of this four unknowns we have determined what is B y and we have determined what is B x and we have used already all few all equilibrium equations. And this still we need to find out A x and B y. Now we have one equation that A x plus B y is equal to 0, but still we have we have shot with another equation and that is obvious because he had the static indeterminacy of the problem is F a problem is 1. So, we need one more additional equation.

Now, you see here if I consider a four system, which now A y and by is ql by 2, which is we have already obtained right. So, forget about that, but A x and B x satisfy this equation. So, as long as A x and B x satisfy this equation, then we say this entire force is means A y is equal to ql by 2 by is equal to ql by 2, and A x and by B x such that A x is equal to minus B x if it happens for some A x and B x, then we say this force is this these reactions are admissible. Admissible it is called statically admissible why it is statically admissible? Because for instance if I have say A y is equal to ql by 2 B y is equal to q A l by 2 and if I say that A x is equal to say 5 and bu a B x is qual to minus 5.

Suppose I have a solution like this then all this A x is equal to 5, B x is equal to minus 5 A y is equal to ql by 2 B y is equal to ql by 2, all this four forces are statically admissible because as far as equilibrium equations are concerned, all this forces satisfy the equilibrium equation. Similarly if I if I do not have A x is equal to 5, B x is equal to 15 if I take af is equal to 15 and B x is equal to 15 minus 15, then also A x this and this satisfy the equilibrium right.

So, even that also statically admissible; so statically admissible and static admissibility here is the all these forces say internal forces and support reactions, they satisfy the equilibrium right. If they satisfy the equilibrium, then we say that it is statically admissible. Now write static admissibility is not the only thing that we want our solution to satisfy. For instance in the previous example the analogy that we that we brought shoe having just because it is shoe something is shoe is not enough to satisfy our requirement. In addition to that the shoe has to be running shoe, in addition to that a shoe has to be has to be of blue color.

Similarly, here only satisfying the static admissibility only satisfying the equilibrium condition, suppose you have a state of solution which satisfy the equilibrium condition a state of support reactions internal forces, which satisfy the equilibrium conditions yes they satisfy the equilibrium conditions, but only satisfying the equilibrium condition will not qualify something to be a solution of the system. When is something else some other requirement also need to be satisfied and that requirement is kinematic admissibility what is kinematic admissibility? See kinematic admiss now when you any structure if you take the structure has some support condition. Now the structure if we apply a load the structure undergoes deflection undergo deformation.

Now, a deformation of the structure should be such that it is kinematically admissible means, the deformation of the structure should be such that it satisfy all the boundary conditions and the continuity requirement in the deflection for instance take.

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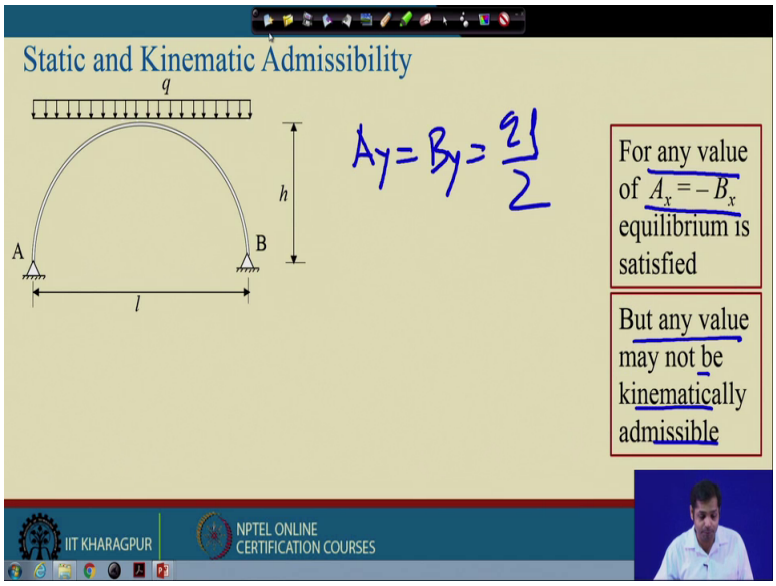

So, this case is this one is in this case, it is statically it is it is this problem is for any value for any value of A x minus B x, this is statically admissible because equilibrium is satisfy. Always we have A y is equal to B y we have already determined ql by 2. So, for any value of A x is equal to minus B x, this is equilibrium is satisfy.

But any value may not be kinematically admissible, let us see with the same example what does this mean.

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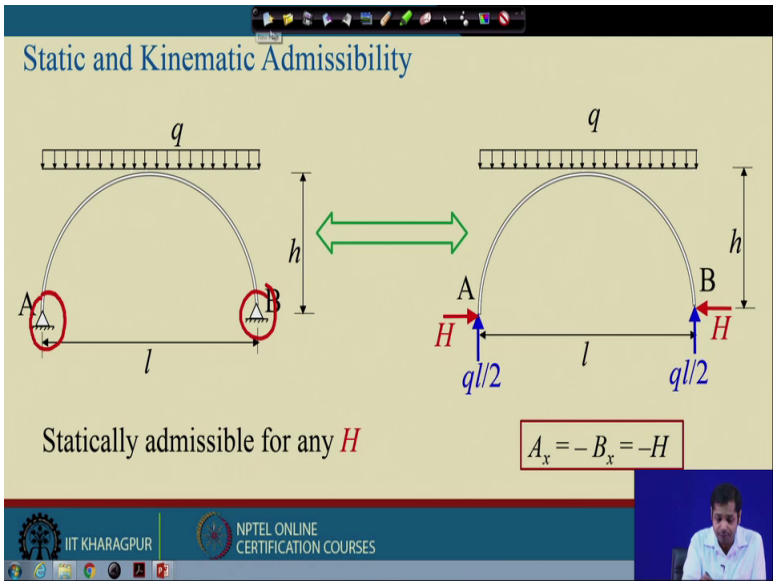

Now, as I say it for any value of H any value of H any value if you take any value of H, then this is this is kinematic this is kinematically if this is statically admissible. Now let us see what happens for different what happens to the deformation of the structure for different values of H.

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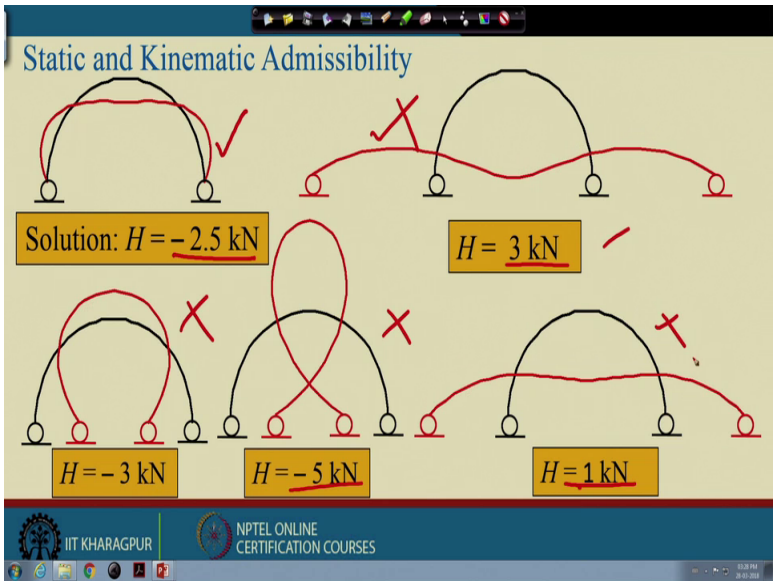

You see we have just changed H, with H and these are the deformation deform shape for different values of H. Now what is the kinematic condition we have here? Now this support and this support, this is this is this is a roller support and it our previous if we go previous page, then our support condition was support condition was point A and point B they have their hinged support. So, they cannot there will be no translation takes place at point A and B.

Now, let us see if we change value H, when H is equal to when H is equal to minus 2.5 kilonewton, the deform shape is like this. When H is equal to 3 kilonewton, the deform shape is like this similarly when H is equal to 1 kilonewton deform shape is like this. So, minus 1 kilonewton deform shape is like this. Now you see for all h minus 2.5, 3 1 pa 1 kilonewton, 5 minus 5 kilonewton, minus 3 kilonewton they all satisfy the equilibrium condition. Because all this for H x is equal to H and B x is equal to minus H. So, all this H satisfy the equilibrium condition, but a deformation that we obtained for different H just by looking at the deformation we can easily say that this deformation all deformations are not consistent with the boundary conditions given here.

And the boundary condition says that at point A and B, there will be no translation whereas, for this H for this all this H we have translation at the boundary. So, among all this H and we have an infinite set like this, we have infinite choices of H we have infinite such deformation. So, among all this H, the H which satisfy the kinematic condition the boundary condition will be our solution.

So, in this case the solution will be this one not this, not this, not this, not this ok. So, when your deformation satisfies this condition, this boundary condition and the continuity requirement, then we say that this deformation is kinematically admissible. Now when we search a solution, search for a solution of a system that solution should be such that we can have kinematic and static admissibility both. So, the solution should be statically admissible as well as kinematically admissible.

this is a very important point because you see in the third week you are going to have review of matrix theory and there you will be studying rank of matrix, the requirement for having an unique solution for a system and then when we actually analyze the structure using matrix method, then we will see those matrices has those matrices will have certain properties and then we need to do something that is what that something let us not bother right now, we have to do something so, that the uniqueness in the solution can be ensured.

But in this case the uniqueness is ensured when we say the solution is unique means it satisfy the both static and kinematic admissibility now great. So, once we understand; what is static and kinematic admissibility. So, what is so among all this set among all this possible solutions not the possible solution among all these solution which satisfy static indeterminacy, we need to enforce one more condition so that kinematic admissibility can also be satisfied.

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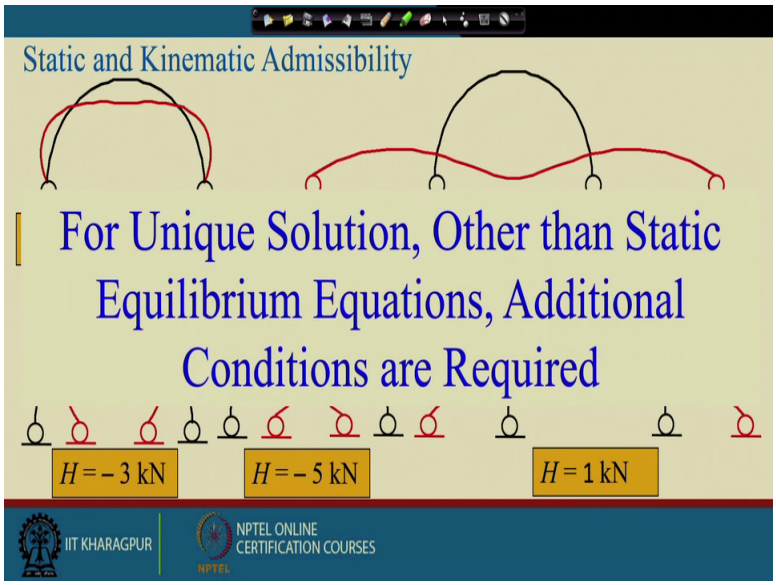

So, we need to have an unique solution other than the static equilibrium condition, we need additional conditions we need additional condition; the way we had additional condition when we bought when we went to a footwear store to buy a running shoe.

Similarly, here also we have to have an additional condition and what is that additional condition? The additional condition is compatibility condition right.

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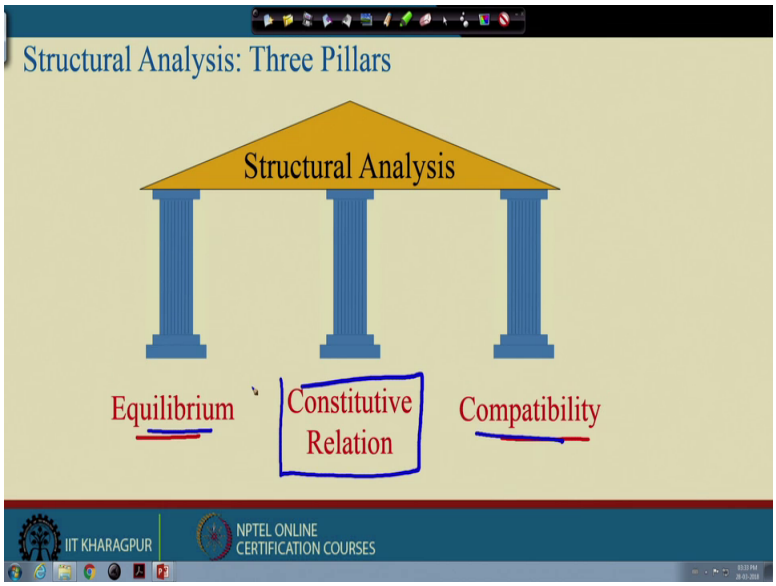

Now, you see. So, what we have discussed is, we have discussed what is static equilibrium and then static equilibrium for many structures, static equilibrium along

cannot calon; a calon not sufficient to find out the solution to solve the entire structure and those structures are called indeterminate structure, and then we saw what is static and kinematic admissibility, essentially our objective is to find a solution to search for a solution and the solution will be such that it stratify the static and kinematic admissibility both. Static admissibility tells you how the equilibrium is maintained in the system and the kinematic admissibility tells you how the compatibility maintains in the system.

So, now the structure analysis is based on a structure analysis has three important pillars one is one is equilibrium. So, we have to have equilibrium satisfied and then next is compatibility means the boundary conditions and the continuity in the displacement they both satisfy. So, this gives us this gives us static admissibility, this gives us kinematic admissibility. Now we have one more important pillar which connects these 2 requirement, which connects the requirement you see the equilibrium is how the forces are in the system and compatibility is how the displacements are in the system.

Now, the middle pillar the another pillar which combine these 2 how the forces the equilibrium and compatibility combine the third pillar, which is called constitutive relation means it tells you how force this equilibrium is something related to forces compatibility is something related to displacement and then constitutive relation, then the another pillar tells you how this 2 important parameters the force and the displacements are related to each other, and that is called compatibility conditions. I am sorry that is called constitutive relation.

So, what we do in the next class is we will reveal we will revisit the compatibility condition. So, what is constitutive we will revisit the constitutive relation not the compatibility. So, constitutive relation there we come across two term one is stiffness and flexibility which are very important, because they are the building blocks stiffness and the flexibility are the building blocks of matrix method of structural analysis, you already have an idea about stiffness and flexibility.

Next class we will just briefly review what is the concept, as a concept what is stiffness and flexibility. We will stop here today. See you in the next class.

Thank you.