Matrix Method of Structural Analysis
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Lecture - 35
Analysis of Beam

Hello everyone; is the fifth class of this week. The last first 4 classes we discussed how to analyse truss 2D truss and then 3D truss, and now will be doing the similar exercise for beam this week ok.

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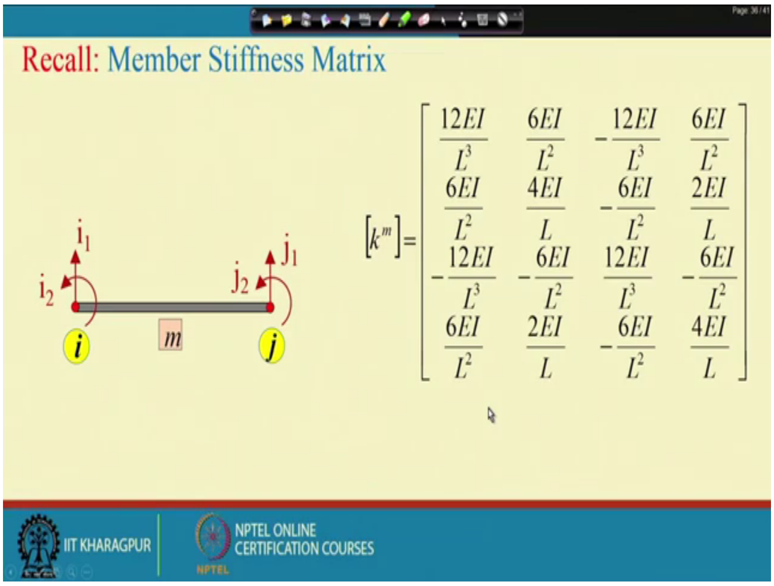

So, the first example that we take is; but before that recall this is the most important part in a beam. If you have a beam which is named like this, we have 2 degrees of freedom per node. One is the translation and another one is rotation of the joint. There is no axial deformation, because we do have axial deformation in the case of beam in a case of frame, but for beam we (Refer Time: 00:58) the axial deformation, ok.

Now, this is the general form of the global stiffness matrix. You have change the EI for different members and get the different values of these stiffness matrix, ok. Now this is important.

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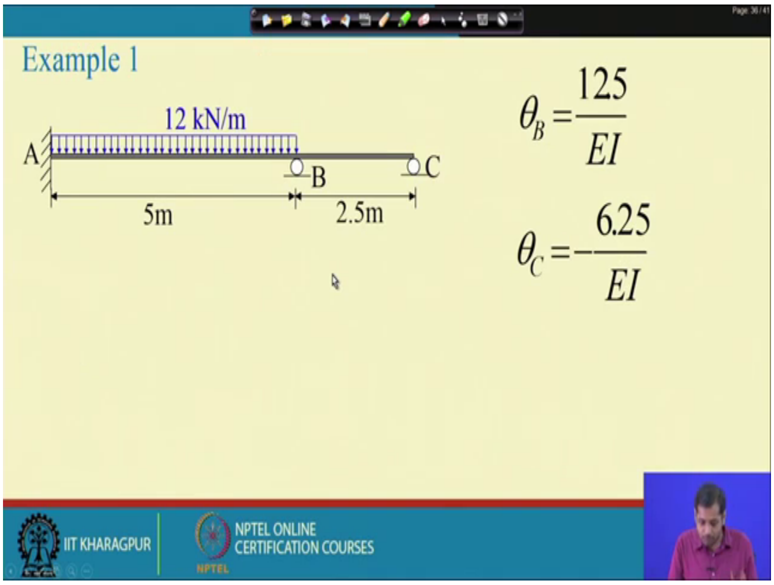

Now next is, but this was the example we consider when we discuss design of we discussed the formulation of design of beam. And this is the solution we already have, we have already solved it and this is the solution for that. So, to start with we demonstrate the code through example because for this example you have done exercise manually. So, we will do this for this example, and then will have another example which is in terms of number of members it is like a more than this ok.

So, let us start with that. So, now this is the code we have for beam, ok.

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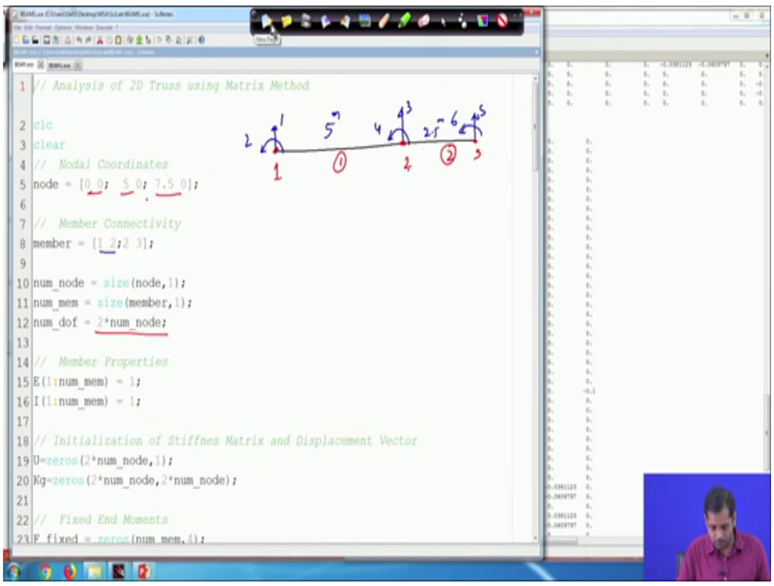

Now, for this beam, this beam is now. So, this was the beam, and this is first this was suppose this is node number 1. This is one node this is another node ok. So, this is say node number 1, this is number 2. And this is element number 1, this is element number 2. And the degrees of freedom, degrees of freedom is this is these degrees of freedom is this is 1 and this is 2, 1 2, this is 3 and 4 and then finally, this is 5 and then 6, ok.

While we did this did manual calculation probably our degrees of freedom, the numbering of the degrees of freedom were different. But here, the reason why we took differently at that time the degrees of freedom because we wanted, because we saw that if we take the degrees of freedom numbering in such a way the partitioning of the stiffness matrix will be easier.

But here since were doing it using a computer code. So, you really do not have to bother right now. So, these are the coordinates of joints join number 1, join number 2, join number 3. This not this one, this is the core ok, this one. So, this is join number 1, join number 2 and then this is join number 3, ok. Least was 5 meter if you recall, this was 5 meter and this was 2.5 meter, ok. And these element connectivity, element number 1 is connected between 1 and 2, element number 2 is connected between 2, and this is 3, this is 3, ok.

Now, the size of the total degrees of freedom would be 2 into number of node per it each per node we have 2 degrees of freedom. This is total, and now this is the young E E and I, Young's modulus and the second moment of area. Again for demonstration purpose, we are taking it one, but you can change the values ok. This is a vector you can change the corresponding values. This is again very similar to the previous one, the in terms of structure of the code it is actually same, this similar. Now similar means similar to cross.

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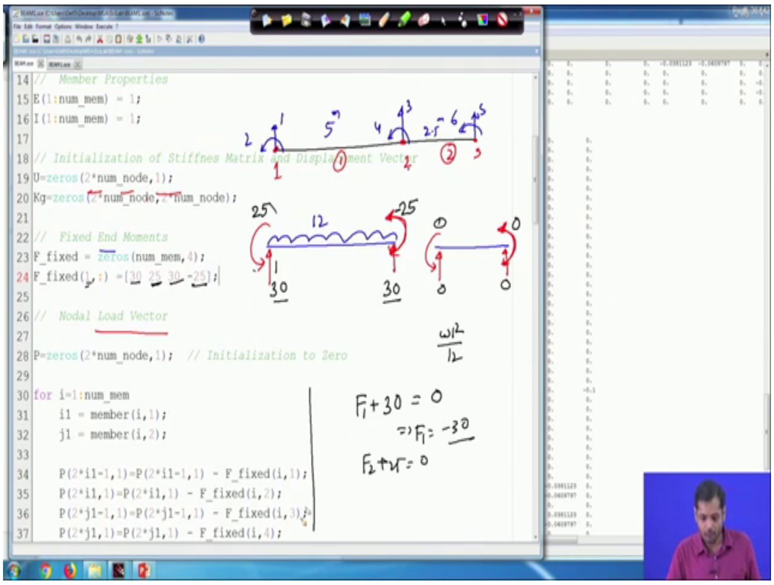

Now, this is the initialization of these difference matrix, global stiffness matrix and the load vector.

Now, then fixed end moments; fixed end moments we did not have in the case of truss, because in the truss if you know the whatever load acting on the structure immediately from there, we can calculate the equivalent joint load. But here in order to get the equivalent joint load, if you recall this is one of the very important difference. In a class all the forces are acting on them on the joints. Here when you solve it, we solve degrees of freedom, solve for unknown and the joints, not at the member right. But then in truss which it was very easy because, he was solving for the joints and the loads are acting on the joint. But where is it beam, we want solution at the joint, but the loads are acting on the member.

So, what we have to do is, we have to transfer that member, that load on the member to the joint. And that is why this equivalent joint force that concept came. So, the joint force which will have similar effect, equivalent effect of the actual structure, ok when the forces are acting on the member ok. In order to get the equivalent joint load, first we have to get the fixed end moments, fixed end moments is if we assume all the members are fixed all the beam elements are fixed, and then calculate the corresponding reactions. Those are called fixed actions, fixed end moments and actions. Now if we take the first case, the first beam will be in first case it is this, and it is subjected to load 12 kilo Newton per meter.

So, this is 12 kilo Newton per meter. 12 and then it is the fixed end moments will be, this is fixed end moment. This is fixed end moment and this is fixed end reaction, and then fixed this is fixed end reaction. Similarly, for the other one, one this is the truss, this is the member. There is no load acting on it. So, this is fixed end moment, fixed end moment, fixed end forces and fixed end forces. So, since there is no load acting and it acting on it this will be 0, this will be 0, this is 0, and this is 0. We discussed that in one of the classes, but here for the completeness of entire discussion let us do this exercise.

Now, if this length is 5, and this is 12, then this becomes 30, and this become 30. And for a beam with uniformly distributed load, fixed end moments will be wl square by 12, and w is equal to 12 and l is equal to 5, and this becomes 25, and this become 25, ok. But recall as per our sign convention, the sign convention is always clock anticlockwise taken as positive. Anticlockwise taken as positive and that is why this will become minus 25 ok. So, these are fixed end moments. So, these fixed end moments we have to give information here.

Now, what is the first? First degrees of these degrees of freedom degrees of freedom associated with translation, vertical translation. This is 30 and then 25; 25 which is clockwise, that is why it is 25 and then again 30, 30 upward reaction 30. It is a fixed end moment was it is anticlockwise, but our as for a sign convention anticlockwise is positive. So, this will be minus 25. This is for member number 1. For member number 2 there is no fixed end moment.

So, already we initialize fixed end moments are 0, we do not have to explicitly write it again, ok. This was fixed end moments. Now once we have the fixed end moments next is we have to calculate the equivalent joint load or the nodal load vector. And what is the nodal load vector? Nodal load vector is we get by substituting equilibrium at different joints, ok.

So, if we substitute the equilibrium and this joint what would be the suppose if nodal force this is F 1 say nodal force will be F 1; F 1 plus 30, these 30 that will give me equal to 0 for the equilibrium. So, F 1 will be minus 30, right. Similarly, F 2, F 2 plus 25 will be 0. So, F 2 will be minus 25. These exercise is been done here the equivalent joint load, this is equivalent joint load or the nodal load vector, ok.

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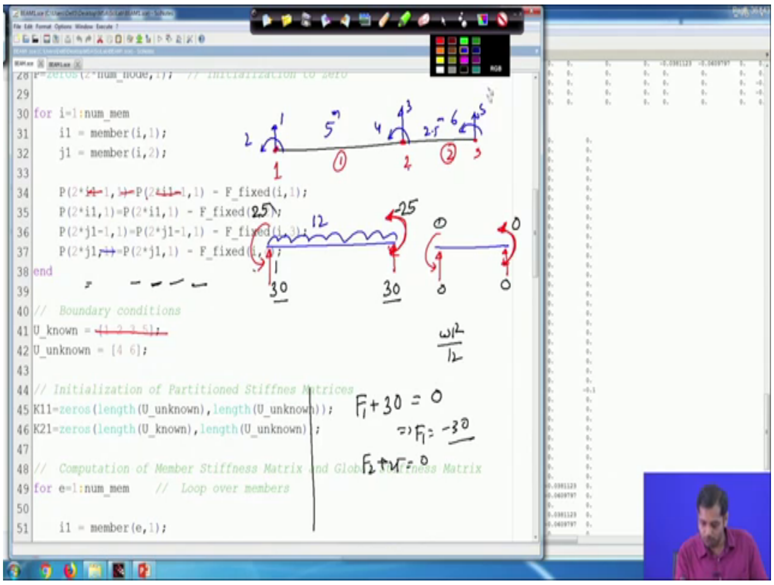

This is done these exercises performed here ok. So, once we have the equivalent joint load, then what next is? So, we have now known an unknown displacement. So, known displacement is if you look at your these end was fixed the node this end was fixed. So, u 1 and u 2 is are known.

It is roller support at this joint at joint number 2, had joined number 2. So, u 3 also 0 and it is also roller support here. So, u 5 is also 0. So, known displacement will be 1 2 3 and 5. So, 1 2 3 and 5 were the known displacement. And then unknown displacements are 4 means which is rotation at node number 2, and rotation at node number 3 which is 6. So, 4 and 6 are the unknown displacement, ok. Now next is the globalization of this is the partitioning of the stiffness matrix.

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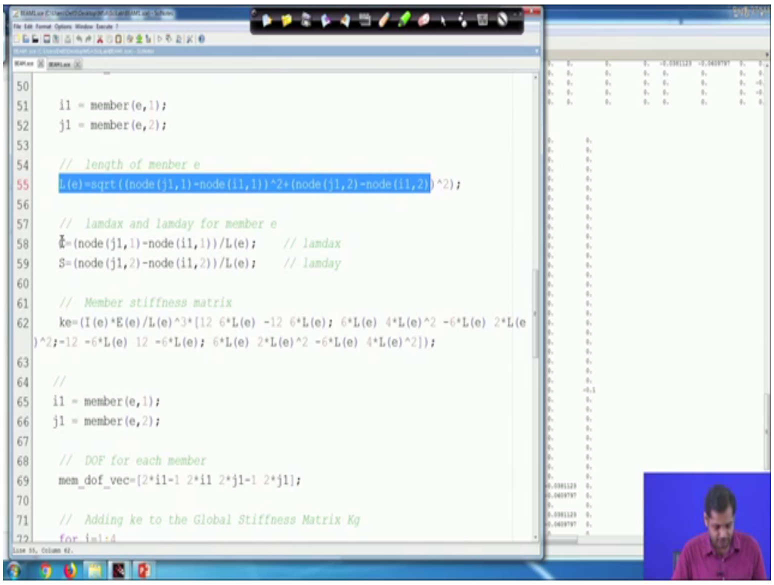

Now this is this is the length of the member, and this is the cos theta and sine theta. Now if you which is not required for in case of beam.

Because in the beam always the members the orientation of the members are with respect to a local coordinate system. But when we come to frame, then we will see that there we need to transform from local coordinate to global coordinate system, and there we need this lambda x and lambda y.

Now, this is these expression this is the member stiffness matrix which is essentially this. This is the expression for member stiffness matrix right. Now there we get the member stiffness matrix.

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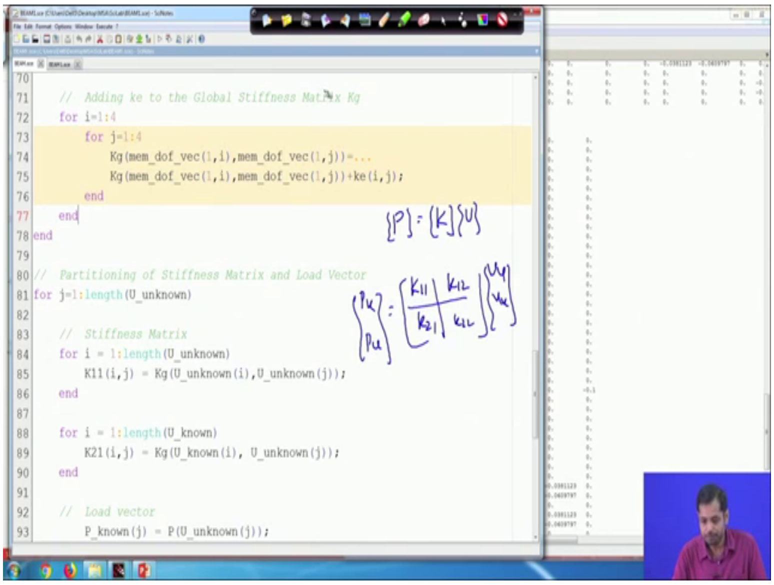

And then one-way member stiffness matrix, we then we have to take the corresponding element in the member stiffness matrix to get the to get the global stiffness matrix right. And this will give you the global stiffness matrix; this is the global stiffness matrix.

Now, next is once we have the global stiffness matrix, and then we have and the load vector also we have, then we have to do the same exercise, that P is equal to K into U. And then these we have to partition it, k 1 1, k 1 2, k 2 1, k 2 2 and then this will be u unknown u unknown, and u known and this will be P known and P unknown right. And this P are essentially equivalent joint load, equivalent nodal load vector equivalent joint load.

So, this is the partition of the matrix, and this partitioning is done here in this loop ok.

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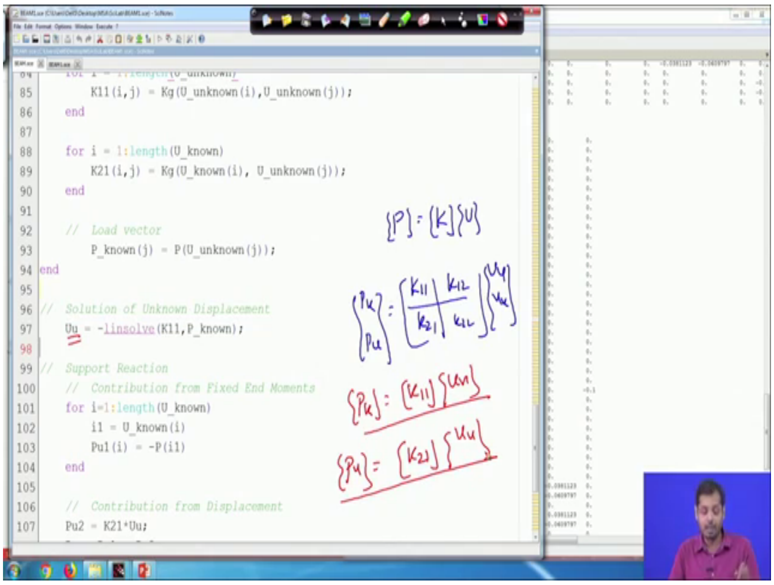

Now, once we have this, but then we solve for this solve for k 1 1. So, if we rise substitute this that P k, P is equal to k 1 1 into u unknown. So, you solve this for u unknown. So, we get solution of u unknown, u unknown we get. Now next is calculation of support reactions. Now calculation of support reaction in the case of truss, truss what happened? Truss we calculated support reaction from the second equation; means P u which is unknown forces that will be k 2 1 into u u. That expression gave us support reactions ok.

But now you see if in the case of beam this will not be support reaction. This is one part of the support reactions. What we have decomposed we have divided the structure into 2 part. One part is where there is no degrees of freedom all joints are constraint ok; which is kinematically determinate structure and subjected to only external load.

And the second is there is no external load, all and the structure is only structural have the deformation. Now so, total reactions will be the reactions from this kinematically determining structure, and the reactions from the structure which has no force, but which is only displacement. So, this is contribution the second contribution, this is the contribution the forces generated at the joint because of the displacement. And then total reactions the reactions will be this contribution, plus the reactions because of reaction from the kinematically determinate structure which are essentially fixed (Refer Time: 13:57).

So, total reactions will be, total this is the P u 1. So, P u 2 is the contribution from the displacement, this contribution and P u 1 it is the contribution from the fixed end action. And the total reaction will be P u is equal to the first one and the second one. That will be the total reactions and then finally, it is the globalization of the stiffness globalization of the load vector ok.

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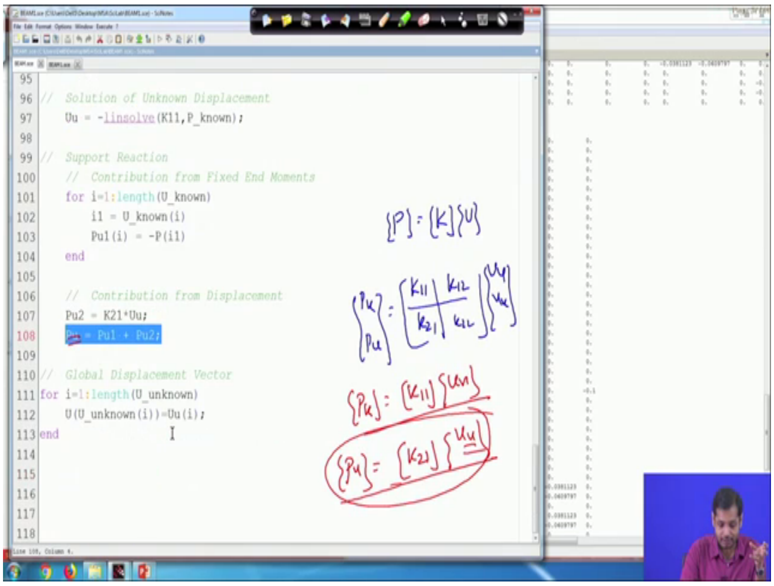

Now for this problem we already have the solution. Let us solve it here once again and then see what is the solution.

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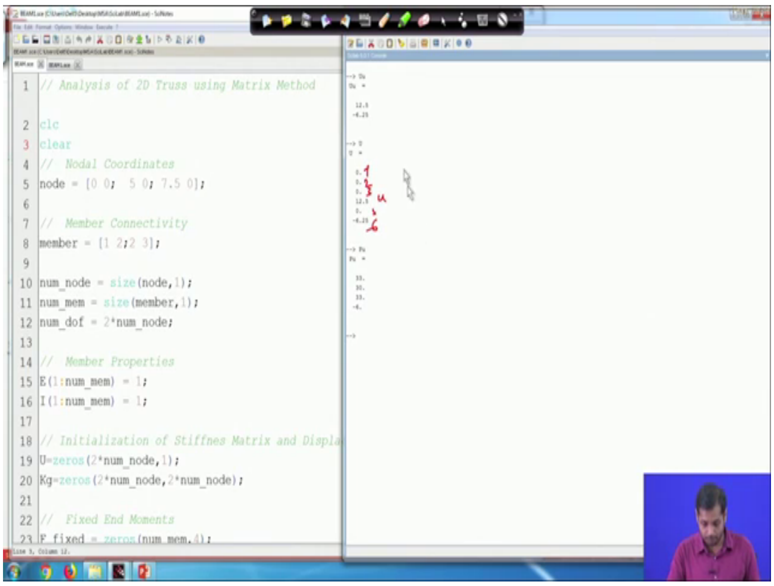

So, the solution is u u is equal to 12.5 if you recall the solution was 12.5 by I, and minus 6.25 by I here is the solution. And then let us find out support reactions. So, total support, first let see the total displacement. So, degrees of freedom 1 2 3 are constraint. 4 is it is theta b, and this is 5 is constraint and the 6 is theta c.

And then support reactions, support reactions are this is the support reactions ok. These are support, if you can check the solution that we have with the solution we already obtained so, this is for this problem. Now so now, let us see the second one, second problem it is these are the solutions already we have. Let us see one more example, this example has suppose take this these example.

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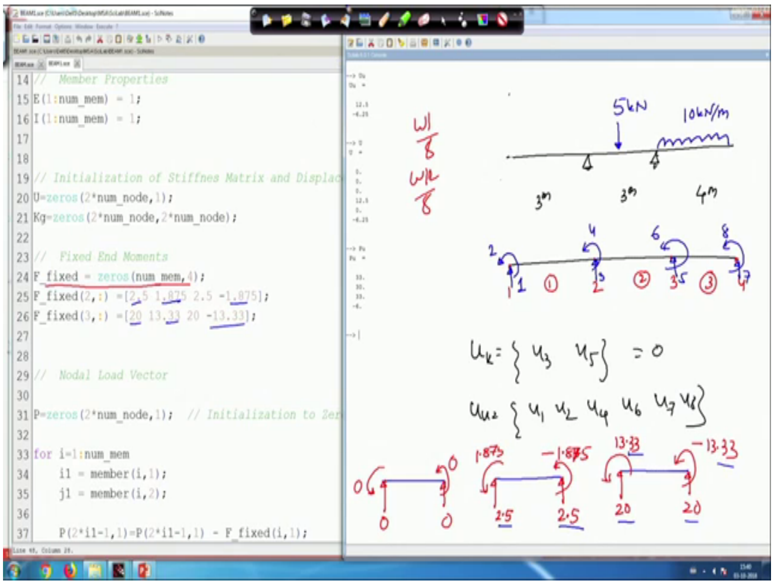

Take an example like this, where we have suppose we have a 3 span beam. We have a hinge support here, and we have a hinge support here. Suppose this is 3 meter, this is 3 meter and this is 4 meter, ok.

Now, suppose this is subjected to uniformly distributed load as 10 kilo Newton per meter. And then suppose this is a concentrated load; concentrated loads is say 5 kilo Newton at the middle ok. And EI is constant for all the member, and let us solve this using this. So, first what you have to do is, first is the numbering. So, numbering will be we have this is the beam, and these are these are different nodes. This is one node another node, another node. So, this is node number 1, node number 2, 3 and 4. And this is element number 1, element number 2 and element number 3.

Corresponding degrees of freedom; suppose this is degrees of freedom one, this is 1 2, then this is 3 4. This is then 5, this is 6, this is 7 and this is 8. So, total 8 degrees of freedom it has. Now in 8 degrees of freedom what are the information we have? The information we have is now see at now joint number u 1 and u 2 these both are unknown joint number 7 both are unknown join number 3 only u 3 is known which is 0, and u join number 4 join number here it is u 5 is only unknown. So, u 5 is known.

So, then what are the information we have about this joint? Let us take this is also. So, u 1 u 2 is also your known now so, this is the ok. So, now let us let us remove it, make it like this ok. So, then u unknown will be u known will be your only u 3, u 3 and u 5, because this is anyway 0. And u unknown this will be u 1, u 2 and then u 4, then u 6 and then u 7 u 8; u 7 u 8, these are unknown ok. Now let us see this expression these are the nodal coordinate, this is the member connectivity, and then this is EI defined. This is now this is your u known is 3 and 5 u unknown is these are the u unknown.

Now, next is we have to give the fixed end moments, ok. Now fixed end moments let us find out fixed end moments. Now in this case fixed end moments will be for the first one draw the first one, say the first one is this. This is first one, this is second one, this is third one. Now first one is fixed end moment, this is moment and these reactions, this is reactions and this is moment. Now reactions anticlockwise moment, reactions and anticlockwise, reactions moment reactions and moment ok.

Now, for the first case there is no load. So, this is 0 0 0 and 0 right. For the second case it is beam fixed beam which has concentrated load at the mid span, and in that case your these value will be 2.5, and these value will be 2.5, right reactions will be 2.5, 2.5. And the moment will be w l by 8.

So, this w l by 8 is, this is 1.875, this is 1.875 and this is 1.875, this is minus 1.875. Similarly, here it is w l square where this load will be 20, this is 20, this is 20. Total span length is 4 meter, 20 kilo Newton meter per intensity. And this will be w l square by 8, and if we substitute w l square by 8, it will be 30.33, and this will be minus 13.33, ok.

So, these are the fixed end moments ok. Now you look at the fixed end moments first is initialized like this, and the second thing is for member number 2, member number 2 fixed end moments will be fixed and actions will be 2.5, 8.8, 1.875, 2.5, 1.875, 2.5 and then minus 1.875.

Similarly, for member number 3, it is 20 13.33, 20 minus 13.33 these are the fixed end moments. Now once we have this fixed end moments, next is then we do not need to give any other information, this is the joint load equivalent load vector. And then boundary condition we have already specified. This is the stiffness matrix and then globalization of the stiffness matrix, solution for unknown displacement once again and then solution of unknown displacement here. And then finally, this is the calculation of support reactions, again for support reactions we have 2 component, one component for the displacement, and another component for the coming from the fixed end action.

Now, if I do this exercise so, let us run it.

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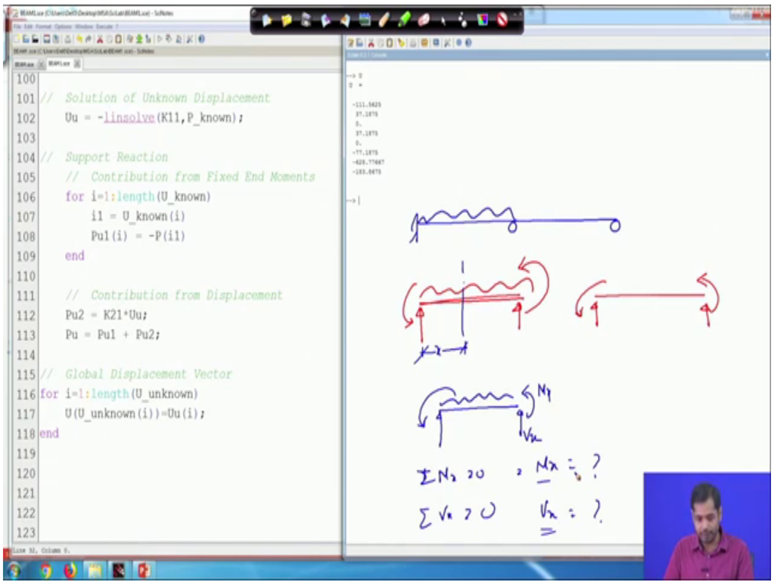

And if you run it then suppose, let us first see the displacement. Displacements are this the very large value you can see here, that is because of your EI value is taken very small, EI value is taken very small. Will do one exercise where the will just change this EI value and then see what happens what happens here. Now you see so, these are the displacement and then corresponding support reactions are these we have just 2 corresponding.

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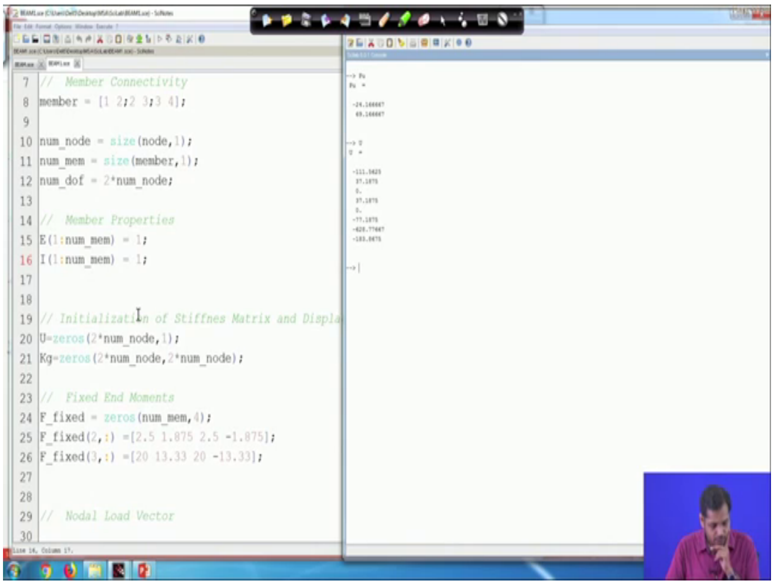

And then P u to support reactions, we have just 2 support here, these this is the corresponding reactions. And you can check the equilibrium; whether the equilibrium is being satisfied or not. The total load acting on the structure was 10 into 40 and then 5. Total 45 kilo Newton, and this is the total reaction 45 kilo Newton. And then final displacement, these has the final displacement, these are the displacement.

Now if you change E, suppose if you change E 10 times then what happens? Your then or say 100 times if we change, then the displacement will automatically reduce. You see the displacement. Now one thing suppose support reactions if we change the support reaction, support reactions support reactions are this. Next what we do is, next let us make something like this. U 1 is this and then E 2 is E 2 you make E 2 is 1. And then E 3 is again 1.

And then is m, and again if we solve it, and then if we calculate the u calculate u, you will see that the difference you can see the difference between these values. Now this is how you can do this exercise and then check what is the effect of different geometric and material parameter, on the final deformation ok.

So, see then finally, once we have the once we have these analysis is not yet complete, because we have calculated only the displacement, at the deform displacement at the node and the support reactions other than displacement at the node and support reactions very similar to truss. We also in the truss we also determine what is the member forces.

Here also you have to determine what is the member forces, what is the moment distribution, moment diagram, bending moment diagram or the shear force diagram in a member.

Now, that exercise you can do, but all the in order to find out bending moment and shear force for a given member, whatever information you need that information that is already determined right. For instance, f if we know this for instance, if we beam on this 4th; take the first example because it was in the test in determinant beam right. This was a beam. Now once we know what is the support what is this if we draw the free body diagram of this, then on this beam we know what are the loads on it, what are the forces and what is the these forces. So, we know these all these information we know about this beam. We know the support reactions everything. For this beam, for this member also we know what are the forces and we also know what are these reactions.

So now you take if so now, this is essentially every you are dealing with now mainly statically determinate structure; where external load is known, and both ends moments and share is known. Now in order to get the you take a section, section at this suppose section from x, then you take the bending moment diagram at this section. This is the bending moment diagram at this section, this is the shear force and this is the bending moment. It is M x and this is V x and then summation of summation of M x is equal to 0, summation of V x is equal to 0 will give you M x is equal to something and V x is equal to something. So, you get an expression for V x and M x as a function of x that you can do it for different members ok.

So, you get the entire bending moment and shear force at a given point ok. So, this is how we can have a complete solution of the problem. So now, what we have discussed so far in this week is, the idea here has been as I said many times and before closing the week, let me repeat that once again, the idea of the entire week has been to tell you that to demonstrate that this method it can be translated into a code very easily, it has that ability; and another important thing the scalability, for demonstration purpose we might have taken very smaller example.

But you can do this do this exercise for a very large problem and then see your own that the solution becomes very easy. And now you compare the compareamount time, amount you are spending to solve a structure or the difficulty you are facing to solve a structure.

And if we have to solve the same structure with the methods that you learnt in your structural analysis 1, then how whether that those methods can be applicable, the way we can apply these methods. So, that was the motivation of the entire course. We need that was the reason we why we had to move beyond structural analysis 1, and this is the reason for that. And this is how we can this is how we have a method which is more us which is implementable, translatable and then which can be translated and then which is scalable.

Now, one thing is still left; that is we already discussed frame, once thing one thing is still left that is 3D analysis of frame. But as I said, the same approach you can follow to translate to write the code for code for frame, and then again translate that for 3D cases, ok. So, this is for the week. Now next week if you recall the very first class with this we mention, the there is the objective of this course was to one is of course just now, we said moving beyond the structural analysis one.

And also another objective is to prepare our self for a method which is more general, more widespread method which is called finite element method; and when we talk about finite element method in the next week a very, very brief introduction of on finite element method, not the finite element method, the essence of the method.

Then we will see that these steps the sdd the concept behind this method is very similar to the matrix method of structural analysis matrix method or structural analysis. So, that has also been our objective to set the platform for a more advanced course that is on finite element method, for which a very, very brief introduction will have in the next week ok. So, I stop here today, see you next week.

Thank you.