Matrix Method of Structural Analysis
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Lecture – 30
Matrix Method of Analysis: Frame (2D) (Contd.)

Welcome. So, this is the last lecture for the plane frame. So, we are discussing here the problem 2.

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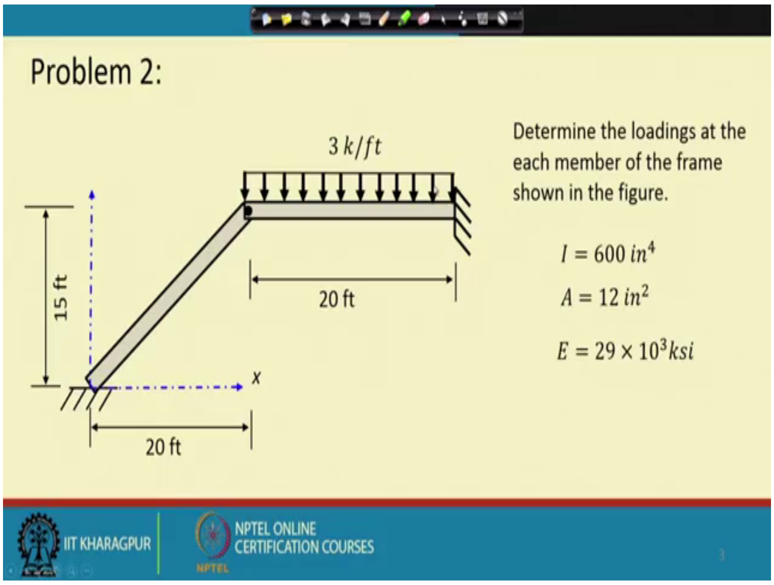

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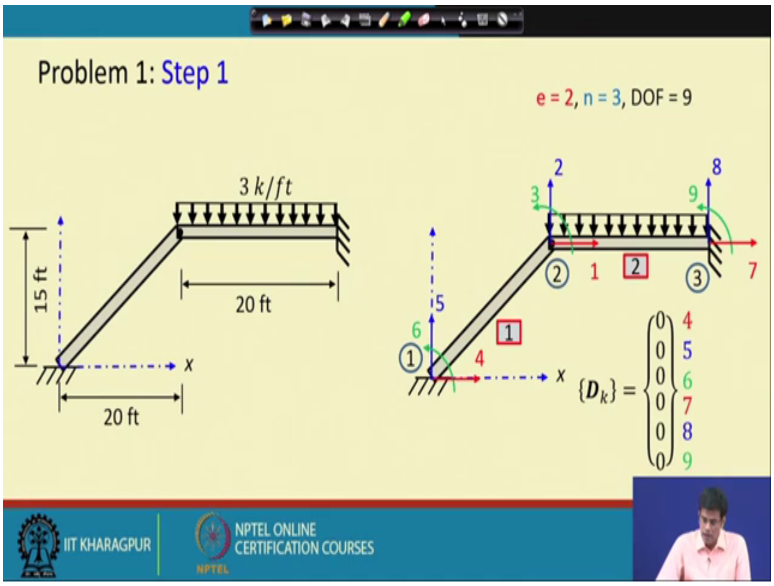

Which is a 2-member frame, plane frame, now in this frame we have chosen such distribution of kinetic degrees of freedom, where node 1 corresponds to 4, 5, 6, node 2 corresponds to 1, 2, 3 and node 3 corresponds to 7, 8, 9 degrees of freedom. Now, in this frame, we have found out what the stiffness matrices for each element with the standard computation procedure that we have seen in the last class.

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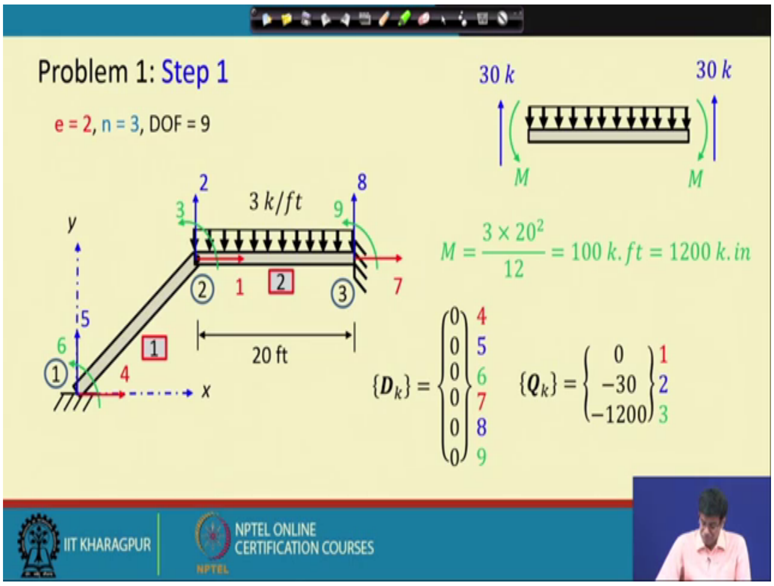

Now, this is the load vectors and the known degrees of freedom.

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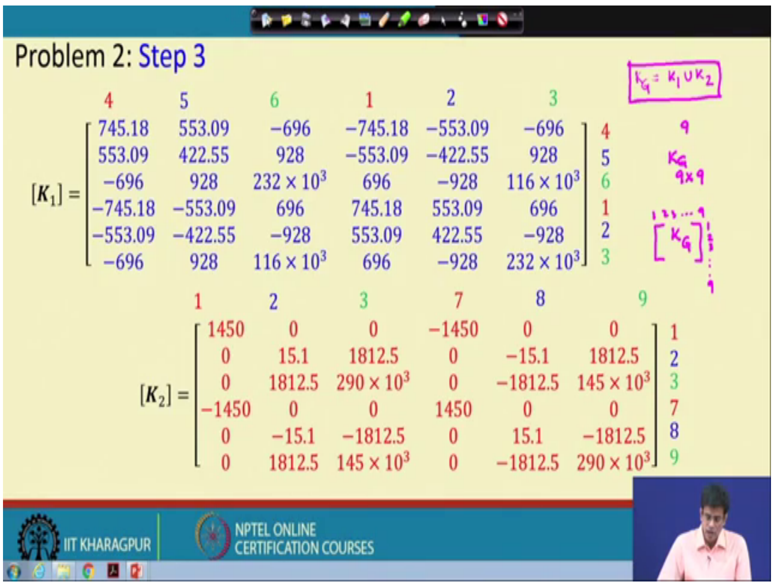

So, this is my corresponding to stiffness matrix. Now here, my final objective is to assemble the stiffness matrix K 1 and K 2, which is essentially to give KG, which is essentially my K 1 union of K 2. So, this K 1 and K 2 means, the elemental stiffness matrix. So, these elemental stiffness matrix, we have to put into a global stiffness matrix in a proper manner or the degrees of freedom corresponds to K 1 belongs and degrees the freedom corresponding to K 2 belongs. So, K G will be there are total 9 degrees of freedom. So, K G size of K G will be your 9 cross 9, matrix K G will be 9 cross 9 matrix.

Now, so how to assemble that? To understand this assembling procedure or is your, this K G will be this. So, here I will go through 1, 2, 3 and so on to 9 and then similarly, 1, 2, 3 to 9, now with this let us see, how we can assemble this. So, this if you understand you can easily code it, this assembling part of the code.

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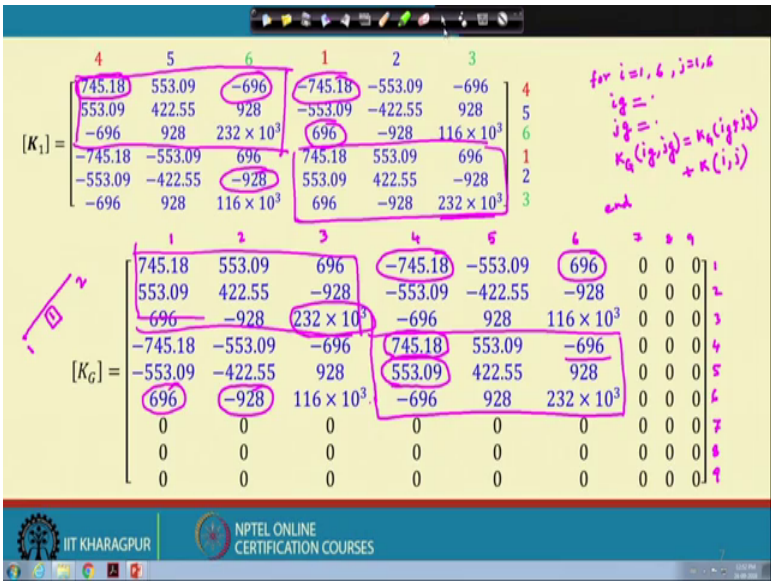

Now, let us see how we assemble it. So, this is my the first element now, see here this row of this global stiffness matrix corresponds to 1, 2, 3, 4, 5, 6. So, I am writing corresponding degrees of freedom and then 7, 8 and 9. Similarly these columns 1, 2, 3, 4, 5, 6, 7, 8, 9, now here the first element say the first element corresponds to fourth degrees of freedom, this represents the row number, this first entry and here it represents the column number. So, the first entry here, this entry will be fourth row, fourth column of the global stiffness matrix; that means, this first entry will be your fourth row and fourth column of the global stiffness matrix; that means, the fourth row means, this is my fourth row or and fourth column means this. So, this is my entry here, you see.

Similarly here, this is the fifth row and fourth column. So, fifth row is my, this is my fifth row and this is my fourth column. So, this is my entry and similarly, suppose I want to find out this entry, which one is this entry? So, this is the first row and sixth column. So, this is my first row 6 column. So, this is my entry. So, in this way you can easily assemble these stiffness matrices because, and this is this degrees of freedom can be directly coded. For instance, if you want to write a single code so, you have to for instance, in a MATLAB.

So, for i equals to 1 to 9, you have to want to 6, you have to find out the corresponding global degrees of freedom or the ig and jg is the global degrees of freedom and then once, you know and this is actually the connectivity these relates the connectivity of the element and then K G ig comma jg you can write it K G ig plus jg comma jg plus K i comma j form. So, that will be j is 1 comma 6 also. So, this is the simple 0 code and this ig and jg you have to find out from these connectivity matrix.

So, what is this connectivity matrix? Essentially, connectivity matrix means this element for instance, the first element is connected with what are the degrees of freedom? 1, 2. So, this 1, 2 degrees of 1, 2 nodes are connected and what is the degrees of freedom corresponds to node 1, 4, 5, 6, corresponding to node 2 is your 1, 2, 3. So, this information is getting here, these 2 thing and this goes to the column and row number of the global stiffness matrix. So, similarly all those elements can be assembled here now. If you look carefully that this element is not the third element here. So, this is not the; suppose, if we want to find out this element, this is the sixth row and fourth column.

So, let us go to sixth row and fourth column. So, this is no, this is the yes, this is the sixth row and fourth column. So, let us see yeah this is here so, this first column. So, this is 696. So here, it is interesting to know that here that all this thing are arranging in this proper manner. Now if you look carefully that there is probably let us see, this element. So, this is first row and fourth column this is this. So, this is this element.

Now, let us see another element. So, this is suppose this, so, this is my sixth row and second column. Yes this is sixth row second column. So, this is my this element, now similarly other elements can also be dance. For instance, this element, this is third row third column. So, this is simply this. So, here you see this block is essentially transferred into these and this part of the block and similarly, this part of this block; this part of this block is essentially transferred to here. So, this is due to the shift of the degrees of freedom. So, this you have to write it properly and this becomes here and this becomes here.

So now yes, I think this will be here. So this finally, gives you the global positioning of the global stiffness matrix K 1.

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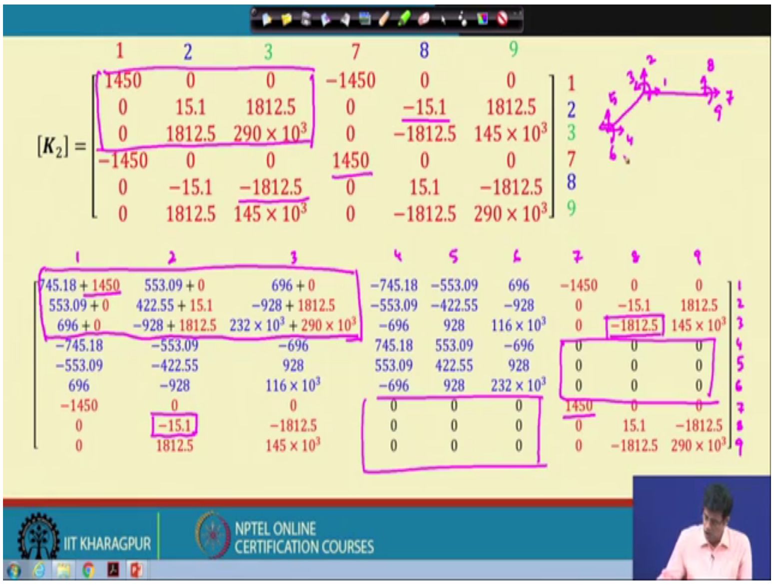

Now similarly, I can go for the element number 2, where once I fill the element number 1 then I can go for element number 2.

Now, if for an element number 2 if you see now here the first element. So, this will be pretty similar because, the 1 1. So, this will be plus 1450. Now suppose this element, this is what is this? So, this is seventh row seventh column. So, this will be seventh row seventh column. So, if I write it here 1, 2, 3, 4, 5, 6, 7, 8, 9. So, this is again 1, 2, 3, 4, 5, 6, 7, 8, 9.

Now similarly, suppose this element. So, let us consider this element. So, this is third row eighth column. So, third row eighth column. So, what is this third row eighth column? So, third row eighth column. So, this is my third row eighth column, now suppose this. So, this is eighth column eighth row second column. So, this is my eighth row second column.

So, this is my entry like this, you see only this portion is having the addition of the there is a addition of the additional addition part that is K 1 and K 2 rest of the part and; that means, the this part of the stiffness matrix of K 1 1 is added with the K 1 part of the or the part of the member stiffness matrix because, you see the degrees of freedom which is these are only the common thing 1, 2 and 3. This part does not have any common. So, and this part also does not have any common thing because, this is 4, 5 and 6 and this is 7, 8 and 9. So, only this part 3 cross 3 block will be added and rest of the part will be same.

So, this assembling is slightly different compared to the previous case, where we have just followed here, 1, 2, 3, 4, 5, 6 and 7, 8, 9, but here the structure of the stiffness matrix is such that it does not preserve the band structure. You see that band structure format it does not preserve, if you look in the previous case we obtained a band structure, but here you see there is a 0, 0, 0 here. So this 0, there is a block here and there is a block here. So, this does not preserve a simple band structure, what we have followed in the, what we have seen in the previous case. So, this band structure we cannot this there so, for a large degrees of freedom or large scale structures, where the number of members is very large.

We generally do not follow, this kind of assignment of degrees of freedom because, we lose the band structure of the matrix, which results in a more solution time or the solution try time or the solver time essentially increases, if we cannot preserve the band structure or rather, I would like to say that if we can have a banded matrix, the solution time will be faster for large-scale degrees of freedom because, we will have the less factorization time.

So, this procedure or this assignment of kinetic degrees of freedom is not generally, followed in a usual coding structure and reason is this, but why did we follow that? Because, these will results in non or the global stiffness matrix, which need not to be partitioned and this is specifically for the problems, which we are going to solve by hand. So, this is one way of solving this problem, where we can essentially, explore the assignment of the degrees of freedom in this way.

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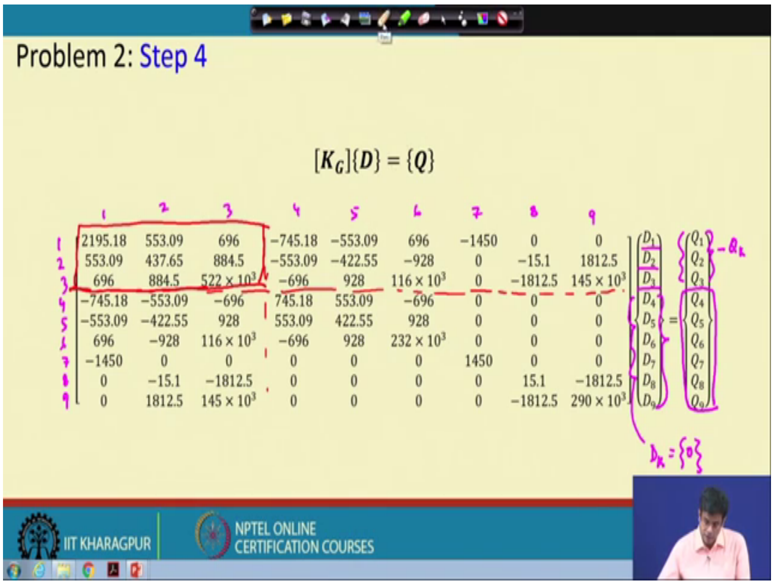

So, if we now see the stiffness matrix. So, you see this is 1, 2, 3, 4, 5, 6, 7, 8 and 9. So, and this is corresponds to 1, 2, 3, 4, 5, 6, 7, 8 and 9. Now see you have D 1, D 2 and D 3, these 3 are unknown and these all those three is D 4 to D 9, these degrees of freedom is known to you. So, these are my D known and which is a 0 vector. So, correspondingly you have the Q 1, Q 2 and Q 3 is known vector, which is Q known and these are my unknown degrees of freedom. So, you see this is my stiffness matrix.

Now naturally, to partition that matrix it is very simple because, you have to just partition in this form. So, that you do not have to do the row transformation. So essentially, this is my partitioning. So this 3 crossed 3 block is my the essentially K 1 1 part of my matrix, which I can just multiply with D 1, D 2, D 3 to get Q 1, Q 2, Q 3 because, this counted is a 0. So, multiplying with this equals 2 will give me the 0 vector.

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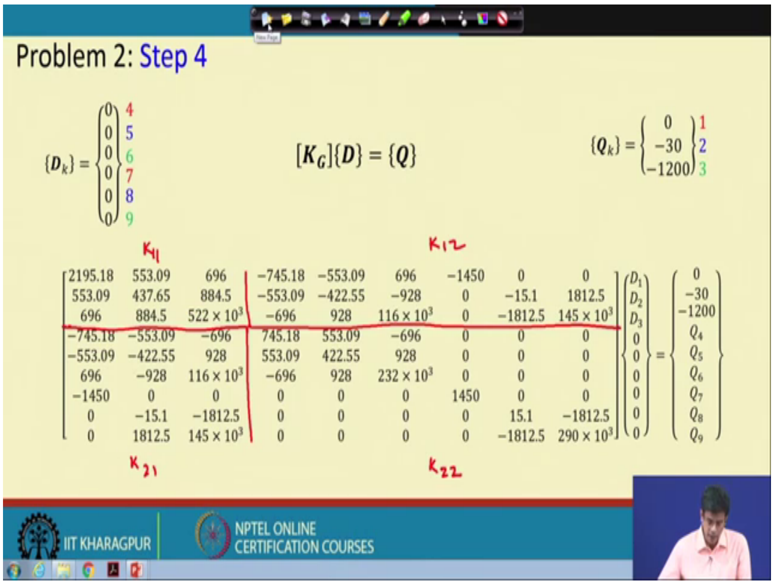

So here, If I see now, what are these degrees of freedom. So, if I write it. So, this is my D 1, D 2, D 3 and so Q 1, Q known and D unknown are also we know.

So, my partitioning, my matrix partitioning records only this part and. so this part will be essentially, my the K 1 2, this is my K 1 1, this is my K 1 1, this is my K 1 2, this is my K 2 1 and this is my K 2 2. So now, we know the details of the partitioning equation and then we can find out D 1, D 2, D 3 and then we can find out Q 4, Q 5, Q 6 and Q 7, Q 8, Q 9.

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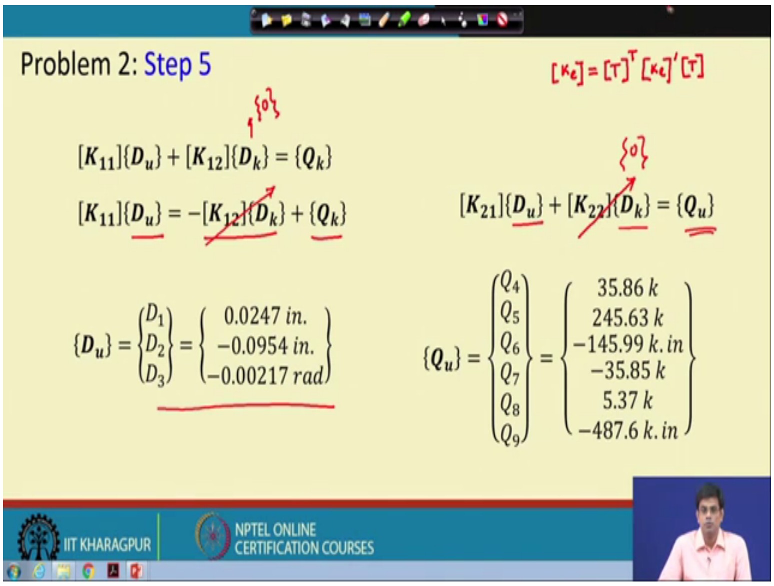

So, all those things we can find it now by just solving the equation and this turns, out to be this. So, D 1, D 2, D 3 is this and Q 4 to Q 9 is this. So, the first part is computed through this equation. So, in this equation D k is a 0 vector. So essentially, this is all known degrees of freedoms are 0. So, this quantity goes to 0. So, Q k we know which is 0 minus 30 and minus 1200.

So, the unknown we can find out in this way and then once, we know the D unknown we can substitute D unknown here and then D known again it is a 0 vector. So, this term cancels. So, similarly these terms cancels. So, this is D unknown multiplied by K 2 1 gives us Q unknown. So, this completes the problem.

Now, I wanted to recap, what we have learned here. So, in this module we have learned what is the way of finding out the frame stiffness matrix. So, the frame stiffness matrix is already composed of truss and beep because, truss we have considered only axial forces in case of a beam, we did not consider axial forces rather, we called considered bending moment and shear forces. So, we combine these, truss and beam. So, where a member can have axial force, shear force and bending moment and then, we find out the stiffness matrix or the elemental stiffness matrix for the frame, which is we did through the principle of superposition.

And principle of superposition is valid for the linear problems, because we are assume that material is not largely deformed or it is not a large deformation problem. So, that is why principle of superposition valent. So, with the help of principle of superposition, we obtain the frame stiffness matrix.

Now, after that what we need, we found out, what is the stiffness matrix if there is a rotated frame element or an inclined frame element with the global structural axis with the local or the member axis. So, that also we have found out with the transformations, where we have seen that there is a transformation matrix and this transformation matrix were relates the local to global or the structural axis with the elemental stiffness matrix with this formula. So, that also we have derived.

So, this also we have derived with the help of local elemental equilibrium equation and global equilibrium equation or the structural axis equilibrium equation. So once, we derive that we solved then 2 simple problems of 2 member plane frame and then we have in the first case, we have seen the we could obtain a band banded stiffness matrix, then format of the stiffness matrix with the declaration of kinematic degrees of freedom.

But in case of a problem 2, we have also seen that if we change or tweak this to avoid the row operation and column operations in the matrix, we lose the banded property of the stiffness matrix and in the global stiffness matrix; obviously, and then, but we have an advantage there, which really is not an advantage, when you code it in a computer. So, this we have seen and then we have also calculated the internal forces and the reactive forces in the main frame.

Now, this procedure certainly, these methods are not for solving 2 or 3 member frames, it is naturally for solving large degrees of freedom. So, and naturally you would not be doing it by hand. So, you finally, will be doing it in with the help of computers. So, a coding is always required. So, in the next module will show you how to implement these metric stiffness methods in the any computer language. Or how to give instructions to the computer so that computer calculates these steps, in a proper manner and solves a large scale problem for you. So, I stop here. So, we will follow in the next module, the implementation of stiffness methods.

Thank you.