Matrix Method of Structural Analysis
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Lecture – 03
Review of Structural Analysis – I (Contd.)

Hello everyone, welcome to the third lecture on Matrix Method of Structural Analysis, what we have been doing is we have been revisiting some of the basic concept in structural analysis that, you already had in previous semester last class, we discussed what is indeterminate structure what is determinate structure, when see at the end of the day we have to solve certain equations right.

And if that equations number of equations available from equilibrium equation, if their equilibrium equations are not enough not sufficient to solve completely, the entire structure, then we say the structure is indeterminate structure.

Now, now what we are going to do today is we are going to see how that indeterminacy or indeterminacy of the structure is quantified, what are the different kinds of indeterminacy, we can have and how to determine those indeterminacy for different kinds of structure. So, today's topic is static and kinematic indeterminacy ok. Now, you see the static degree of static indeterminacy.

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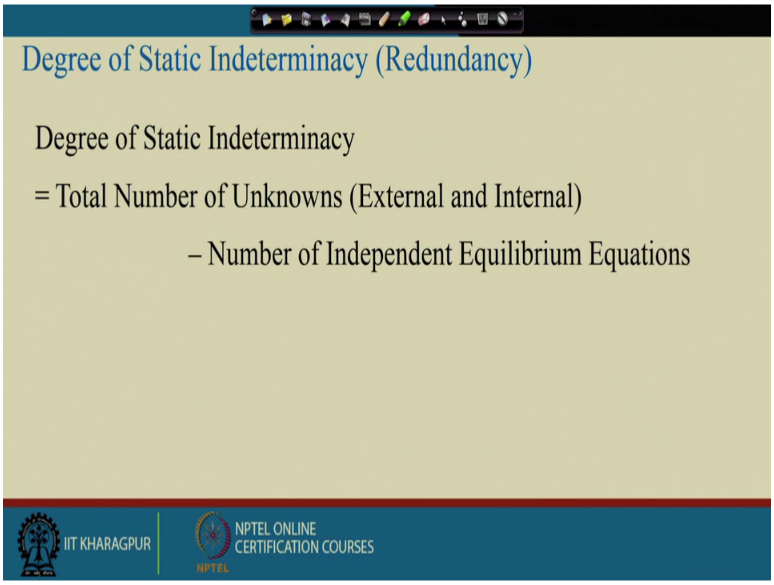

if you recall from your structural analysis 1, the degree of static indeterminacy is total number of unknown, that unknown could be external unknown it could be internal unknown, we are just going to see what is external unknown and what is internal unknown. These are the total number of unknowns and, then we have number of independent equilibrium equation.

You see it is very important it is thus not equilibrium equation, it is independent equilibrium equation on the 3rd week you will be having a brief review on matrix met on matrix theory and, there it will discussed what is what does it mean by independent equation. So, number of equations should be independent and, then this and then this is called degree of static indeterminacy.

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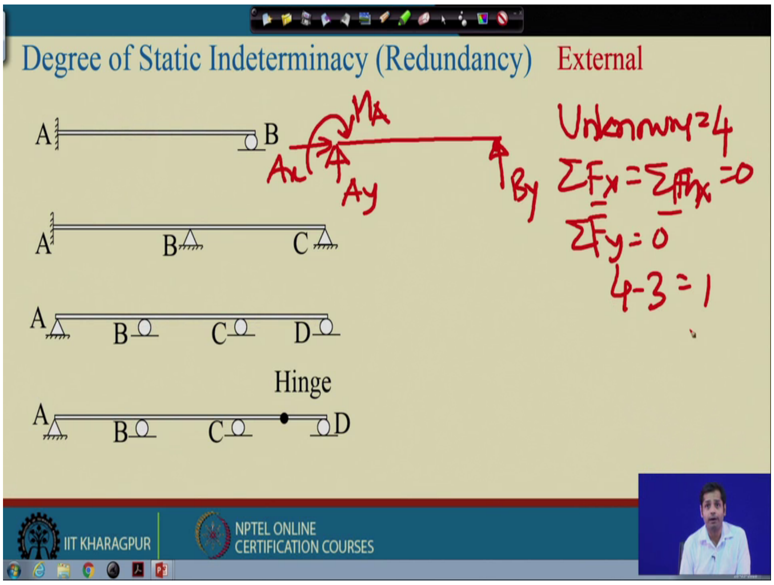

Let us see through some example, if you for instance there are some beams here given, if you have to find out the static indeterminacy of the structure. Now for instance consider the first case, the umm it is a propped cantilever beam.

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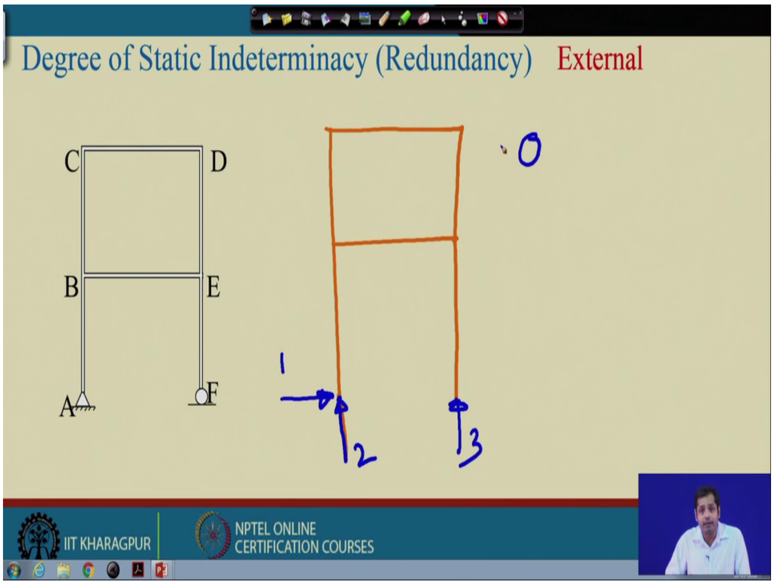

Now, if I draw the free body diagram of the beam the free body diagram the free body diagram will be this, free body diagram of the beam will be this, this is a fixed support. So, we have 3 reactions here and, this is A x A y this is A x and, this is M a and this is roller support, we have one reaction that is B y.

So, number of unknown is 4, 1 2 3 4. So, number of unknown 4 and, then how many equations are available, we have 3 equilibrium equation summation of F x is equal to 0 summation of M x is equal to 0 and, summation of F y is equal to 0. These are the 3 equation we have so, a degree of indeterminacy static indeterminacy is 4 minus 3 is equal to 1.

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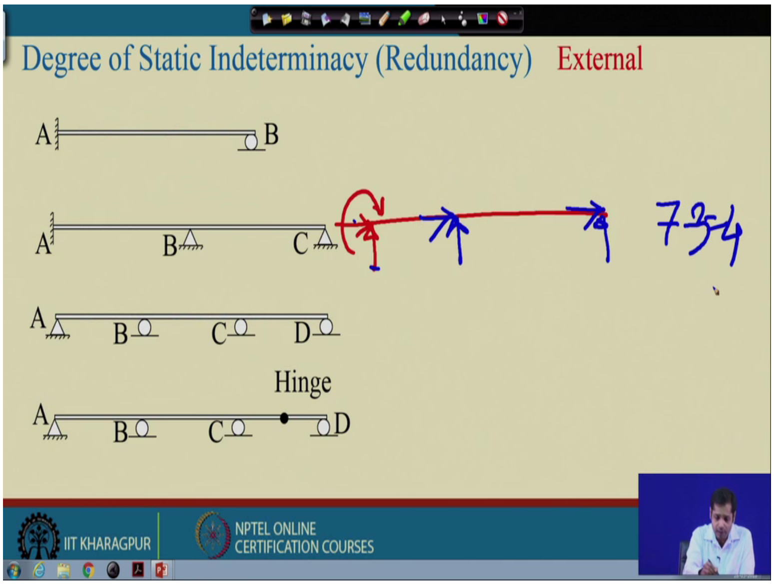

Now, if we take the second case, the second one second one the free body diagram, if I draw then again you have three reactions here and, then we have it is a hinged support. So, again we have two reaction say let us use different colors, we have two reactions here one is this and another one is this and, we have another two reactions here one is this another one is this. So, we have total unknown is 1 2 3 4 5 6 7. So, number of unknown is 7 and number of equations available is 3. So, total static indeterminacy becomes 4 right.

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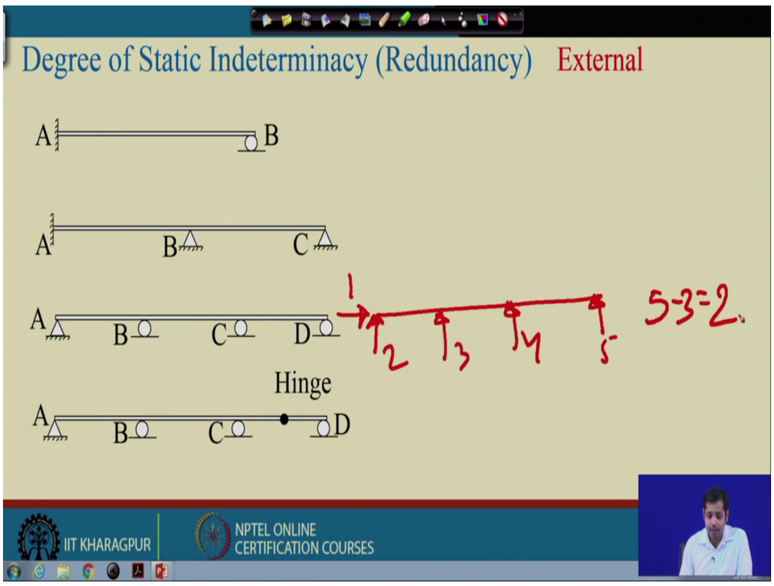

Similarly, if we take these examples here, this is here we have two reactions, these are two reactions and, then you have one reactions at the roller support, another reactions at the roller support, one more reactions at the roller support. So, total reactions are 1 2 3 4 5 to get 5 unknowns and the number of equations available is 3. So, 5 minus 3 is equal to 2 so, in this case of static indeterminacy is 2.

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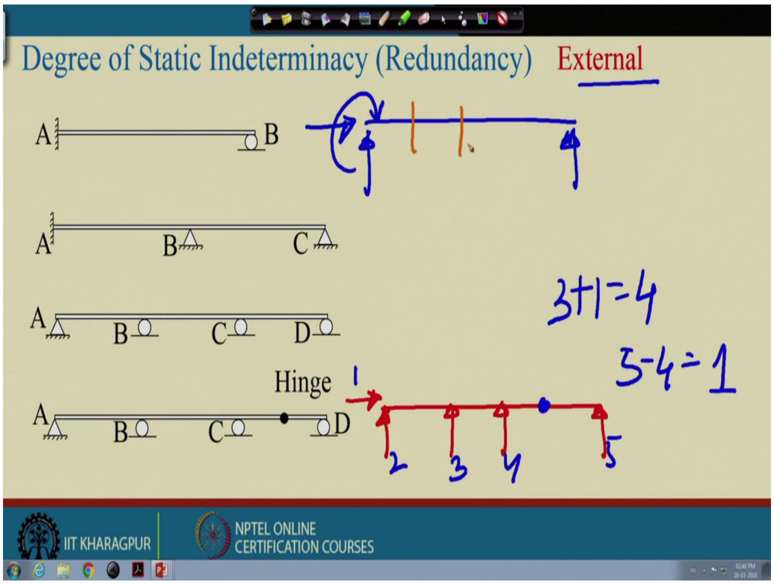

Now, consider with the last example last example you see, if you draw the free body diagram of this is two reactions here, because it is hinged support, if a roller support another reaction, another roller support. And then another roller support another reaction and, then in addition to that we have one hinge here, we have one hinge here.

So, how many equations are available, we have 2 equi 1 2 3 4 5 there are 5 unknowns and the equations are available, equilibrium equations available is 3 and then in addition to that you since it is hinged, hinge cannot take any moment. So, addition equation available at the point of hinge that umm moment at this point is equal to 0. So, total number of equations available is 4 and total unknowns are 5 so, 5 minus 4 that is so in this case the degree of static indeterminacy is 1.

Now, you see the in this case the degree of why it is called external because, somehow if we determine the determine the support reaction, suppose for the first case, for the first case these support reactions are somehow. If we can calculate these 3 support reactions, then at any intent an, any intent at any point at any point, we can determine what is the bending moment shear force and axial forces. So, internal forces can be determined easily, if we know the external if we know the support reactions. So, the indeterminacy here is we cannot determine the support reaction based on the equilibrium equation.

Now, if you remove on support reactions, then the structure become determinate and we can calculate the support reaction. So, here the indeterminacy is due to the additional number of supports, which is not required for stability of the structure. And that is the reason why this, these indeterminacy is called external indeterminacy.

Now, if we see one more it umm if we see the French structure, you see here if you draw the free body diagram of the entire structure. This is the free body diagram and, then we have this is this is one force, this is hinged support two forces and, this is roller support we have one more forces. So, here support reactions are three, 1 2 and 3 and the equilibrium equations available is 3 so, in this case the external there is no indeterminacy due to the support additional support of the a additional support provided in the structure.

So, in this case external indeterminacy, external indeterminacy external static indeterminacy is 0 but you see when they say that we have to analyze the structure, it is not just a support reaction we have to find out the member forces as well.

Now, let us see we have just now seen this is externally it is determinate structure, which we can calculate the support reactions. Now, if we know the support reaction can we calculate the all the member forces in the structure. If we can and the structure is internally determinate as well, but if we cannot the mem if we cannot calculate the member forces, even with the knowledge of a support reactions, then the structure is called internal in internally indeterminate.

Now, for instance now let us see this structure. Now, suppose now we know the calc we know the, what is the support reactions right.

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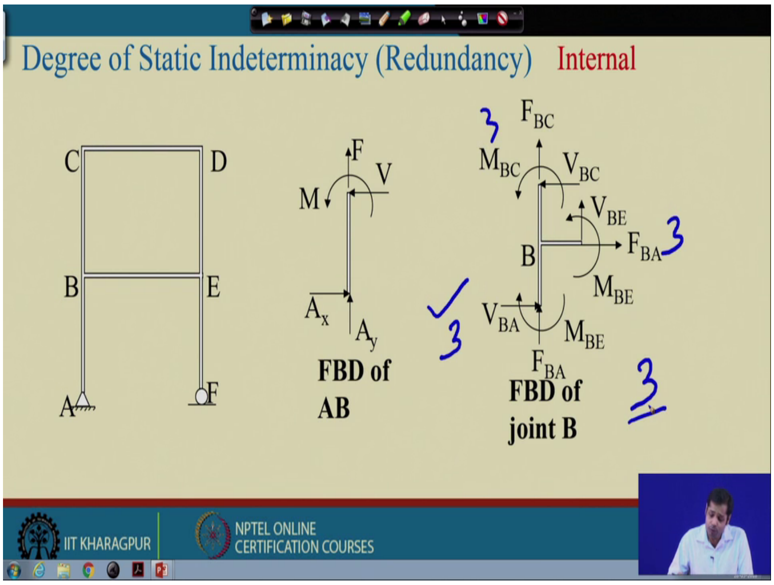

Now, suppose take the free body diagram of A B, if we draw the free body diagram of A B, then free body diagram is free body diagram is like this, we have two reactions A x and A y and since this is a rigid frame, then at any point we have three forces at the axial force, shear force and moment these are 3 forces. Now, for this A x and A y Ax and A y these two are known because, we have already A x is known and A y is known because, you have already calculated the support reaction because, it is not externally indeterminate. So, these two values are now.

Now, if these two values are known then in this free body diagram only unknown are in F and V, then we can we can we have 3 equation summations of F x is equal to 0 and, summation of F y is equal to 0 and, summation of F a summation of M is equal to so, that any point. So, this gives us V is equal to A x V is equal to V is equal to A x, this gives us say F is equal to F is equal to A y. And similarly this gives us M is equal to M is equal to 0. So, for this as well as A B segment is concerned. So, we can determine this three forces.

But again as I said, it is when the structure is when we say the structure is determinate it means that every member, we can calculate the member forces.

Now, let us see whether for other members as well we can determine the internal forces are not, consider now these joint B. If we take joint B, then the free body diagram of joint B becomes this, we have we have umm we have three forces here, three forces three forces here and, again another three forces and another three forces.

Now, from this you see already these are known MFV are known right, what is this one this one is actually, if it was the, if it is for member AB this is for member AB. So, this gives you MBA. So, MBA is known MBA is known this is known, then F is known F in BA is known and, then shear force in BA that is also known. So, this 3 are known now in this free body diagram total unknown total unknown are here, it is 3 here it is 3 and, here it is 3 total 9 unknowns. And out of these 9 unknowns 3 unknowns are already known from the free body diagram of AB.

Now, but still we are left with the 6 unknowns we have 3 equilibrium equations and with this 3 equilibrium equation, we cannot determine these 3 unknown. So, indeterminacy for this structure is 3 right. Now, these now you comp you come try to compare this example with the previous example, in the previous example the indeterminacy is caused, due to the additional support. So, that is why the indeterminacy was called external indeterminacy.

In this case the indeterminacy, externally the structure is indeterminate, but the indeterminacy is due to the additional member of the structure right. So, therefore, that member force we cannot determine with the knowledge of with the information of equilibrium only and, that is the reason why this indeterminacy is called internal indeterminacy, when we say the indeterminacy of a structure, it is total indeterminacy, external indeterminacy plus internal indeterminacy.

Now, let us say let us if you if you recall from, if you recall from your from your structural analysis one that.

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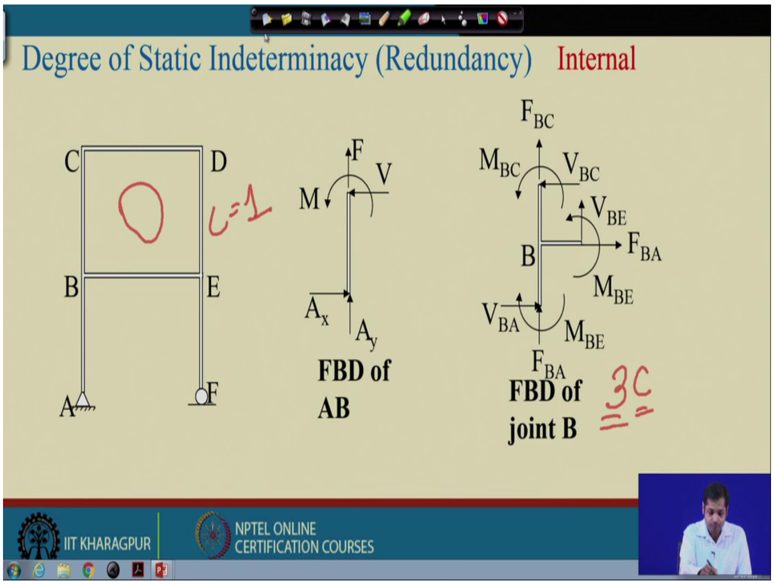

If the if this is the internal indeterminacy of this rigid frame can be obtained by a formula 3 C when C is the number of closed loop for in for instance, this here it is one closed loop, this loop is not closed. So, here C is equal to C is for this example C is equal to 1 so, internal indeterminacy is 3 into C means it is 3.

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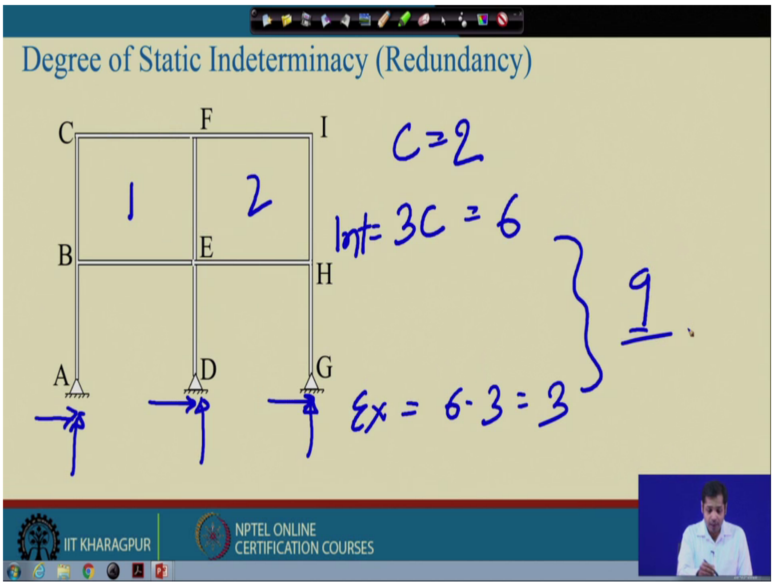

Now, take few more example for instance, if we apply the same concept here, in this case what is the external indeterminacy, you see here we have two forces, here also we have two forces and, here also we have two forces. So, there are total 6 unknown and we have only we have only 3 equilibrium equations. So, a internal indeterminacy of this is externally in external indeterminacy is 6 minus 3 is equal to 3.

And what about internal indeterminacy say C is equal to 1, if we have 1 closed loop here, we have 2 closed loop here. So, total closed loop is C is equal to 2. So, in detailed indeterminacy is C 3 C it is internal indeterminacy which is equal to 6. So, total indeterminacy is 6 plus 3 total indeterminacy is 9. So, for this in determine for this indeterminacy is 9.

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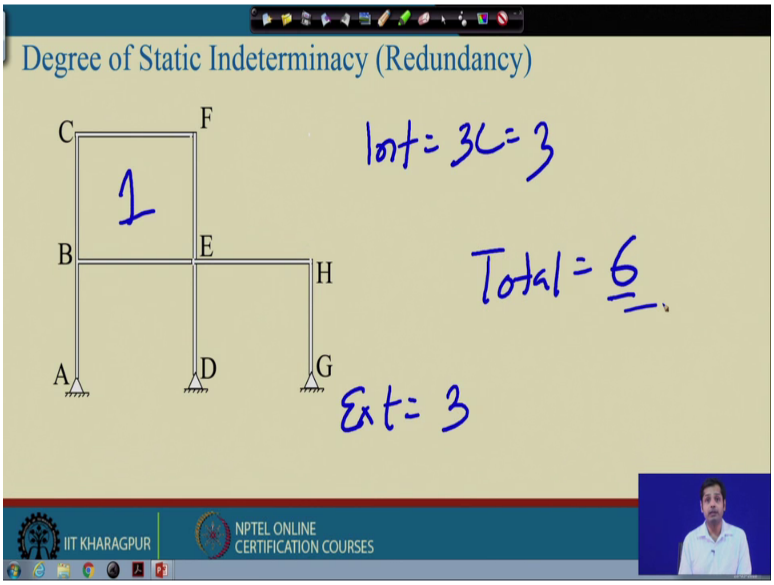

Take one more example. So, in this case here it is also same example. So, your external indeterminacy will still remain same, external indeterminacy is external indeterminacy is 3 and, internal indeterminacy we have just 1 closed loop. So, internal indeterminacy is 3 C which is equal to 3. So, total indeterminacy is total indeterminacy is 6 3 plus 3 total indeterminacy is 6.

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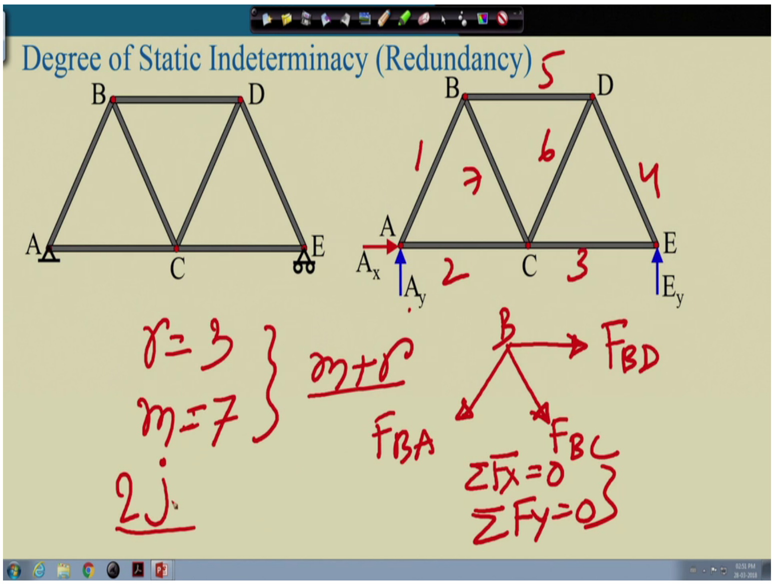

So, this is static indeterminacy. Now, these are all for front structure beam and front structure and, what about for truss problem. If you remember the truss problem was for truss problem ok, for truss problem say first consider the free body diagram of this truss, if we take the free body diagram of this, the free body diagram is this and, we have two we have two support reactions here, we have one support reactions here.

Now, suppose r is equal to suppose r is equal to total number of reactions required, for instance in this as this truss is concerned r is equal to 3, 1 2 at this point and at joint E it is 1 So, total 3 and suppose in this in this truss how many members, we have 1 2 3 4 5 6 and 7 suppose m is equal to total number of members. Now, then total number of unknown becomes what r is the support reaction. So, r many support reactions we have to determine and, then in truss all members are the two force member.

Therefore the internal forces we have only one which is the axial force in the axial force in the truss member and therefore, the internal forces number of internal forces is same as the number of members. So, the total unknown here is total unknown here is m plus r right

Now, how many equations we have available. Now, if we take the free body diagram of any joint, say for instance joint B for I could draw the free body diagram. So, free body diagram of joint B will be something like this. And this is FBD this is FBC if you recall and, this is FBA this is joint B. Now, here we do not have 3 3 equilibrium equation because, the truss we cannot have truss member all the joints are connected through pin. So, there is no moment resistance. So, only resistance we have in the in translation x and y translation. So, only mem the equilibrium equations available here is summation of F x summation of F x is equal to 0 and summation of F y is equal to 0.

So, this is the only two equation we are available at every joints. So, if we have total number of joint is j total number of joint is j. So, total number of equations available at each joint is two equations. So, total number of equations available is 2 j. So, this is the number of unknown number of unknowns and this is the number of equations available.

So, we say that the structure is determinate, if this 2 j is equal to m plus r and if say the ta structure indeterminacy again plus r is greater than 2 j right.

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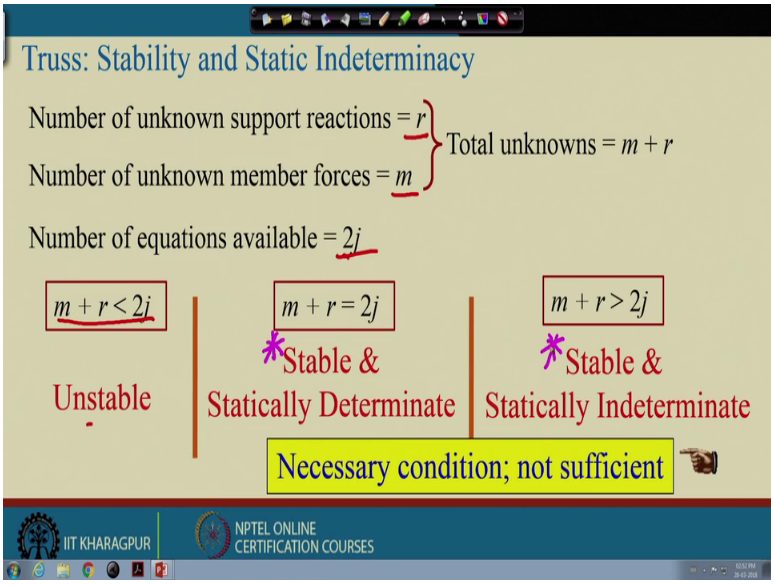

Now, therefore, for truss we have we have the following, if the if the number of this r is the number of reactions total reactions, m is the number of member two j is the number of total equations available, then we have 3 cases very similar to the m a wa umm depending on whether m plus r is less than equal to or greater than 2 j, if m plus r is less than 2 j means your structure is your number of number of number of unknowns is less than the number of equations available.

The structure become unstable and, when this is exactly the same the structure becomes stable and, and statically in determinates. Now, I would try to emphasize this points stable here, you have just put a star mark here

And then again if the m plus r is greater than 2 j, we say the structure is indeterminate again put a star mark here stable, you see these conditions say m plus r is equal to 2 j or m plus r greater than 2 j, these conditions are necessary condition for a structure to be to be stable and, oh or the or the structure to be to be determinate or indeterminate, but these conditions are not sufficient conditions just look just based on these condition we cannot comment, or we should not comment on the stability of the structure.

The stability of the structure something that you have to you have to at this point, you have to physically verify look at the different orientations of the member look at the joints and, then see by where is the structure the entire configuration their assembly is stable, or unstable. Just to make it in a proper umm proper prospective let me give you two examples, for instance these examp ok.

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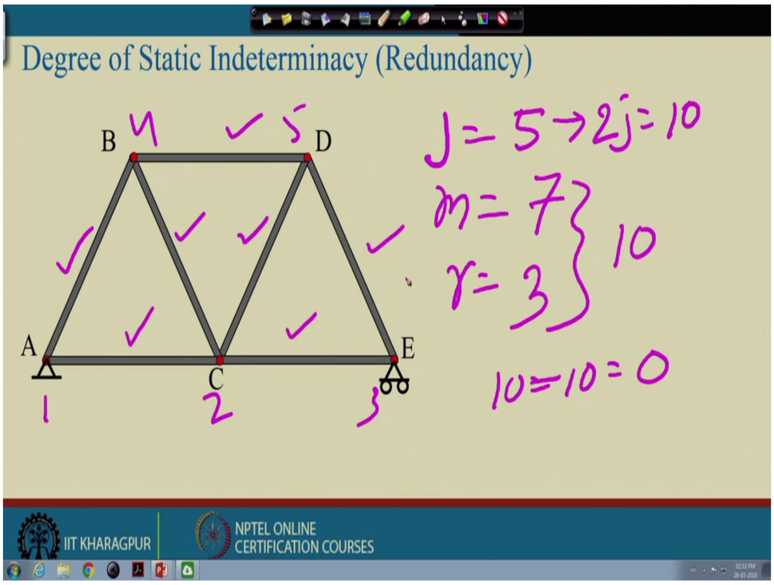

So, these just now before than before we give you the example, let us say few a few pro umm few examples, to calculate the static indeterminacy of structure of truss member.

Now, you say here number of joints j is equal to 1 2 3 4 and 5. So, number of joint is 5 number of members m is equal to 1 2 3 4 5 6 7 this is 7. And number of support reactions, we have two support reactions here we have one support reaction at E. So, r is equal to 3. So, total number of is equal to 10 total number of unknown is equal to 10 and number of equations available is 2 j which is 10. So, statistical indeterminacy is 10 minus 10 which is equal to 0.

So, this truss is statically determinate right. Now, let us t let take one more one more example.

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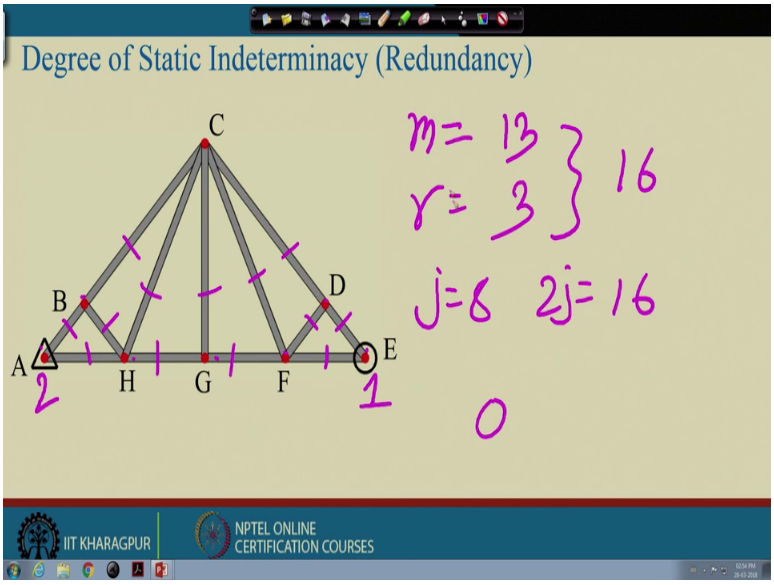

Now, here also if you see your number of member m is equal to 1 2 3 4 5 6 and then 7 8 9 10 11 12 13. So, number of member is 13 and, then the number of joints is here 2 here, it is 1 number of support reactions r is equal to 3. So, total number of unknown is 16 and then number of joints we have is 1 2 3 4 5 6 7 8. So, number of joint is equal to 8. So, number of equations available 2 j is equal to 60 though static indeterminacy is 0. So, structure is statically determinate structure.

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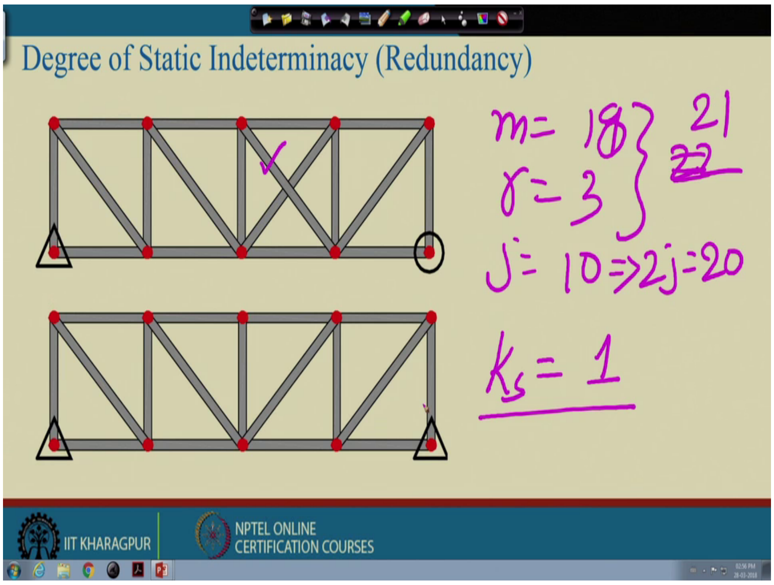

So, we know that now take these example. Now, the two example the first one is say number of a number of member is number of member is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19 number of member is 19, number of reactions are number of reactions are 3. So, total unknown is 22 m plus r and the number of joints 1, 2, 3, 4, 5, 6, 7, 8, 9, 10.

So, number of joints number of joints is j is equal to 10 number of member is 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18 please correct it this is 18, this is 18. So, this is this is 21 right. So, number of equations available is 2 j, which is equal to 20. So, indeterminacy is say K s static indeterminacy is 21 minus 1 this is static indeterminacy. Now, consider the same example once again, but here this member is removed and this roller support is replaced by a hinged support.

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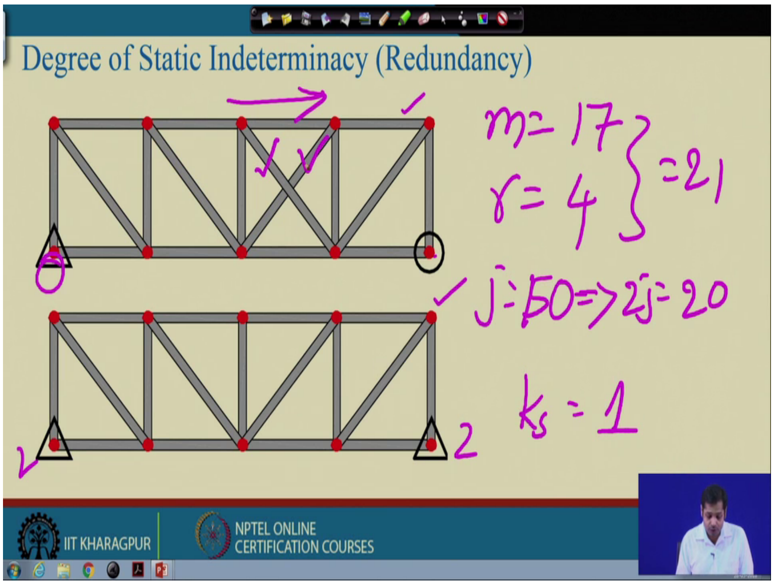

Now, in this case what happens, in this case your number of member, now becomes 17 and number of number of reactions are 4, 2 here and 2 here. So, total becomes 21 and number of joints is again same 5. So, number of joints is 10. So, your 2 j is equal to 20. So, static indeterminacy is 1.

Now, cons now try to try to understand the difference between two to this two example, this is also indeterminate, this is also indeterminate, this is indeterminate because here the number of supports are additional support, in extra support is provided. If we make this support roller, then the struc this structure becomes determinate.

But in this case if we make if we remove if we make this step support roller, if we make this support roller the structure will not become determinate, rather the structure will become unstable because, in this case the sta entire assembly can move in this direction. So, here just making by with by changing the support by reducing the support constrain we cannot make the structure determinate right.

Here that indeterminacy is due to the additional member additional and additional member. So, if you remove one member and the structure becomes determinate. If you remove 1 member the m become 16 and, then umm in a m is less than m become 17 and number of number of reaction is 3 total unknown is 20. So, in this case it is indeterminate because of the, because, the additional internal forces you have to compute and, in this case it is indeterminate because of additional support reaction you have to consider. So, you have to compute.

So, in this case it is an external and, in this case it is internal indeterminacy ok. Now, these are some example of a calculating static indeterminacy umm.

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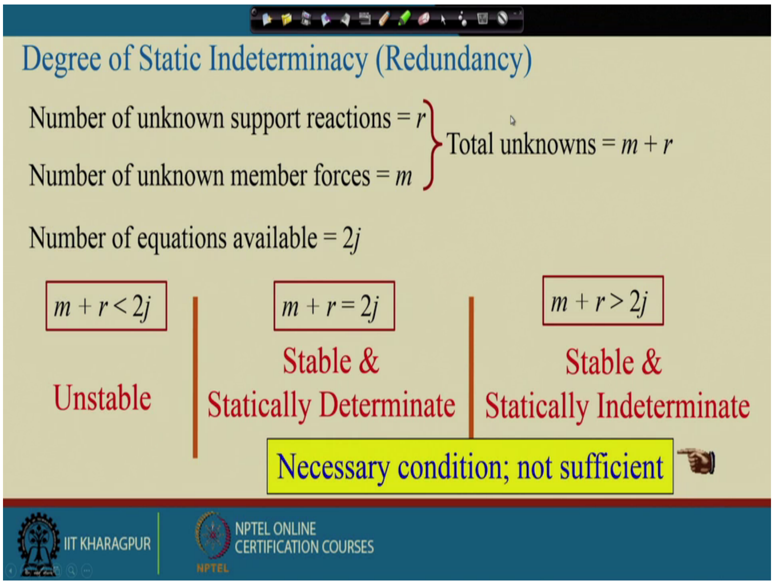

Now, as I said this is this when we say that this is stable, this is stable we cannot take this condition for granted, it is just a necessary condition, but may not be the sufficient there are some examples which I am going to show you in next two slides, where your structure is still statically determines such statically determinate and indeterminate determinate, but the structure is structure is not stable ok, we will see that now.

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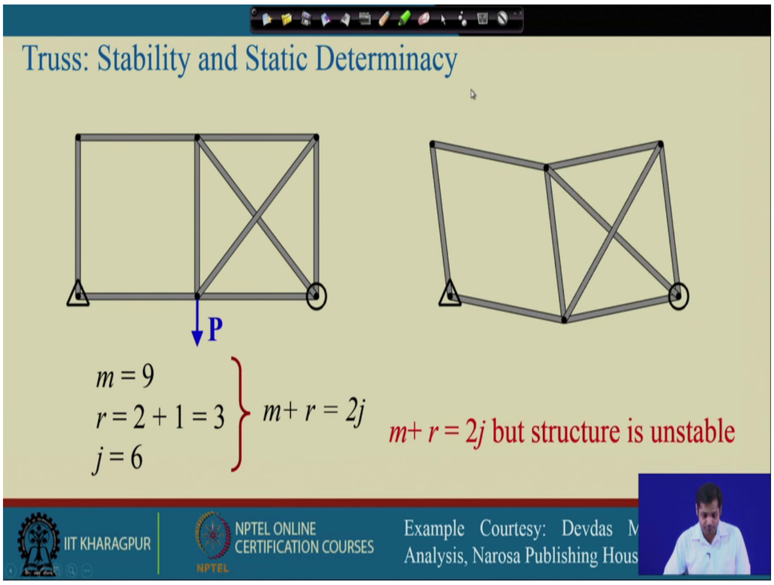

Next example next slide for instance you consider these example, now you see here, number of this is a statically determinate truss, as far as these equations are concerned, these equations are concerned, it is statically determinate truss. Now, in this truss if you apply a load like this, if you apply a load like this, then the structure may deform like this. So, entire assembly may be entire assembly may collapse like this. So, the structure is statically determinate, but again not stable right.

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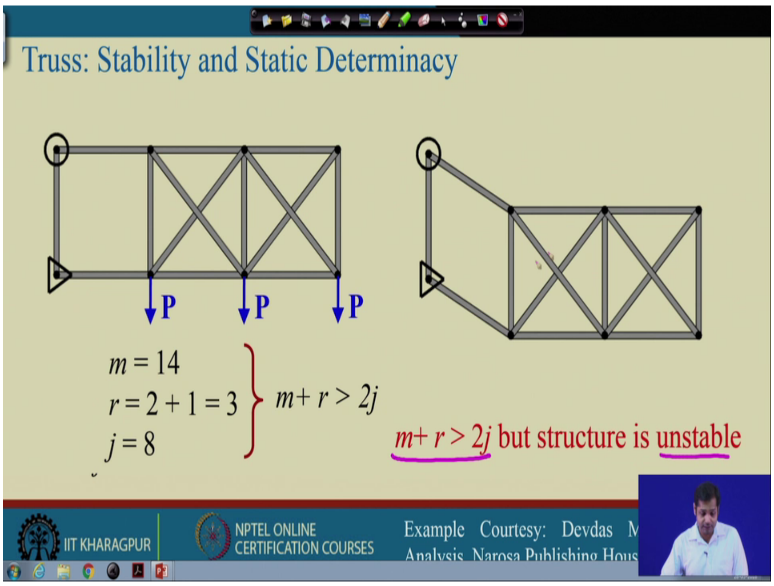

Similarly, if you take wa another example next example, it is again your which is m plus r greater than 2 j. So, as per as per as the previous umm definition was concerned is this is indeterminate and stable, but again if you have lap a load like this. The structure the entire assembly may collapse like this, in this case the structure is structure is indeterminate this, this m plus r greater than 2 j, but still the structure is unstable.

So, as far as the stability of a structure is concerned please do not take these equation for granted, it is just a necessary condition, but not the sufficient ok.

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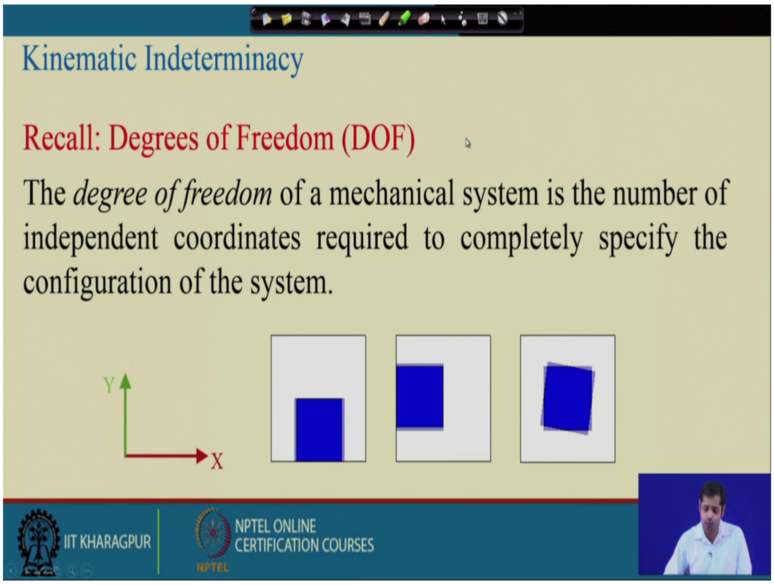

So, this was the, a static indeterminacy. Now, quickly review what is kinematic indeterminacy, now if you already know the definition of degrees of freedom, the degrees of freedom with mechanical system is number of independent coordinates required to completely specify, the configuration of the system in two dimension, where for any object in two dimension, we have 3 degrees of freedom two translation and one rotation, we have seen this in the last class right

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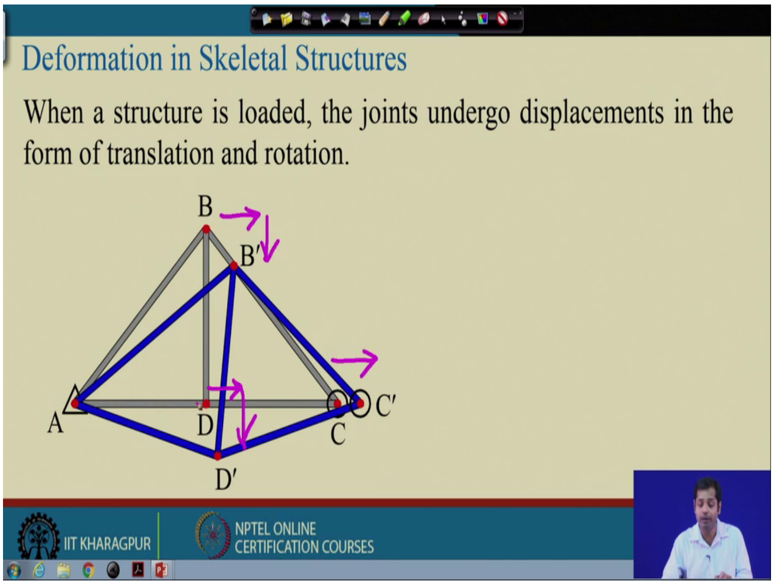

Now, the kinematic before we define kinematic indeterminacy, let us see what happens when you, when you take a structure and apply load on the structure, when the structure is loaded, then the joints undergoes displacement right and in the form of translation and the rotation.

And now for instance if we have if we take a truss like this and, if it is subjected to some kind of load the entire truss may deform like this. So, in this case what happens the B po point B goes here, it first goes here and then goes here, in the form of translation point C goes here and, the D is a support it remain same and point D what happens point D go goes here and then goes here.

So, every joints in the (Refer Time: 27:30) they move from their original configuration, which has free onto move, or if from the original configuration in the form of translation and rotation ok. If the rotation you do not have here in the truss because, only translation degrees of freedom we have in the truss member.

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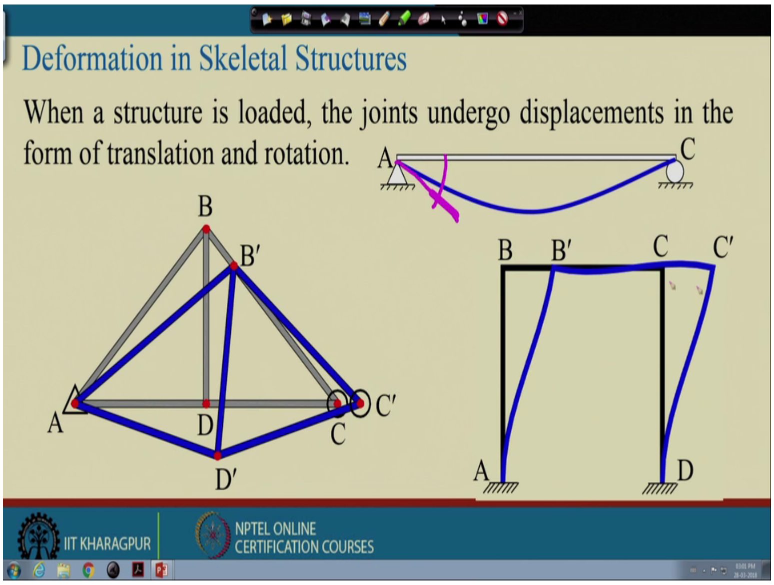

Now, now the static the kinematic indeterminacy, for instance for a beam these are the rotations. If you these are the rotations this is the rotation right for a beam and, for a truss also for a frame also, this point goes here and this point rotate like this point depending on the support condition and the constraint you have right.

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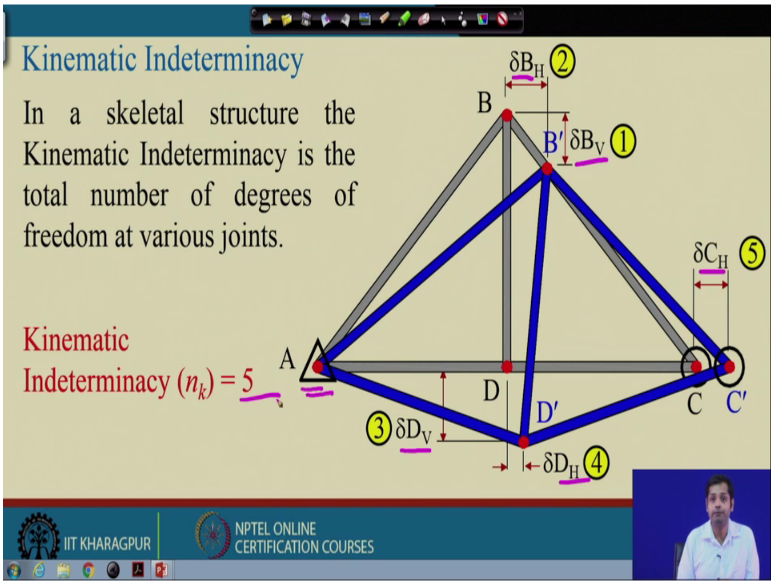

Now, the kinematic indeterminacy is defined as for a skeletal structure, the that the indeterminacy is the total number of degrees of freedom at various joints. Just to give you an example what does it mean, this is the example just now we say. Now, point A is point A is fixed here so, point A is hinged here so, point A cannot move only B point B goes to B dash point C goes to C dash and point D goes to D dash.

Now, let us see what are the possible wa how this entire assembly from A B C D to A dash B dash C dash and D dash, how these how these 2 configurations the deform configurations form, from the original configuration Now, what happens you see these are the these are the different displacement, these are different displacement this is the horizontal displacement, horizontal displacement, horizontal and vertical displacement.

Now, then how many displacement we have here is we have 1, then 2 then 3 or at 4 and then 5. So, total 5 joints movement has taken place right. Now, the indeterminacy of this structure the kinematic indeterminacy of the structure is then 5 right.

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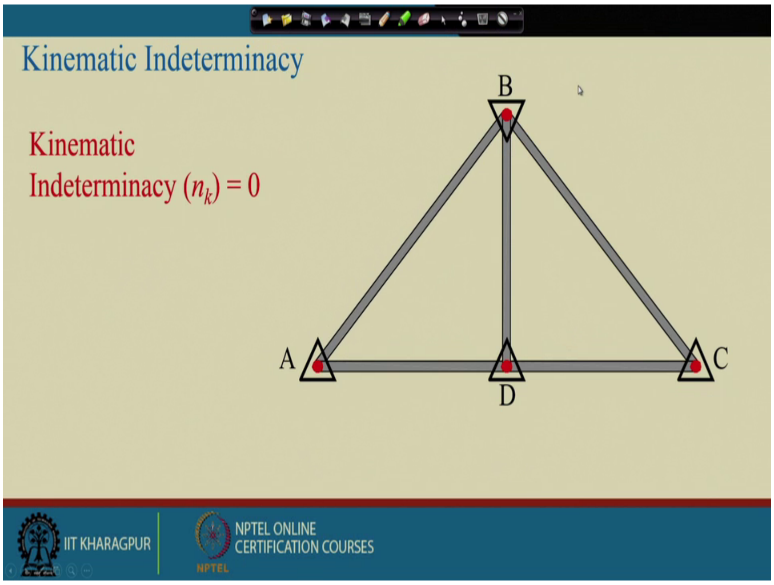

Now, umm take few more example for instance in this case all the joints are all the joints all the joints are hinged here. So, all this joints cannot move in space, all the joints none of the joints have any degrees of freedom. So, in this case the total number of joint movements has taken place in this structure is 0. So, in this case the kinematic indeterminacy is 0 right. Now, quickly get see the another example so, n k n k is 0.

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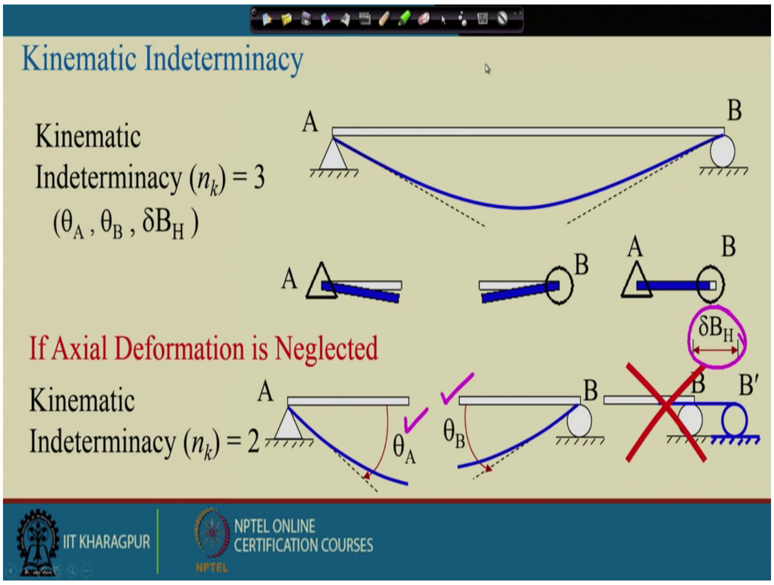

So, now for instance in this case you have a simply supported beam and the blue one is the deformed configuration. And then you see these are the possible deformation given, the animation shows the how that, if point A B and B, they can they can they can move in space, you see point A is hinge support. So, only degrees of freedom this point A has his rotation. So, point A can rotate.

As point B has roller support so, it has 2 degrees of freedom one is rotation like this and, another one is the because of this roller translation in this direction is also allowed. So, this is this is another degrees of freedom that point B has. So, then in this structure the total degrees of freedom 1 degrees of freedom is theta A which is the rotation at A and, then theta B rotation at B and then translation at B. So, total degrees of freedom total joint moments can have we have is n k is equal to n k is equal to 3. So, kinematic indeterminacy for this for this structure is 3.

Now, if we restrict if we restrict this if we if we if we make this structure, suppose so in this case these are the 3 degrees of 3 degrees of freedom. Now, if we assume that this the actual deformation of this of this member is neglected, why this actual deformation takes place in this case point A is fixed in space and, then may it cannot move or it cannot, now translation takes place at point B point A, but point A can move in this way point B can move in this way. So, there is a stretching takes place at a A in member A B. So, the member A B the length of A member A B changes.

Ah, but if we assume the length remain same and there is no actual deformation allowed in a in member A B, then this is not allowed. So, these degrees of freedom we do not have. So, these degrees of freedom we do not have. So, only degrees of freedom we have is theta A and theta B. So, in this case the kinematic indeterminacy of this structure is 2 right.

So, you see so now we stop here today, what we have done what we have done or reviewed here is what is what is static and kinematic indeterminacy, how for different structure static and kinematic indeterminacy can be determined. Now, once we know what is what is determinate structure indeterminate structure and how to quantify indeterminacy, the next thing we need to understand is that what is static and kinematic as admissibility and compatibility.

Now, we will review the static and kinematic admissibility and compatibility in the next lecture. We will stop here today I will see you with the next lecture.