**Matrix Method of Structural Analysis**
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**Lecture – 29**
**Matrix Method of Analysis: Frame (2D) (Contd.)**

Welcome so, we are discussing frame plane frame structures.

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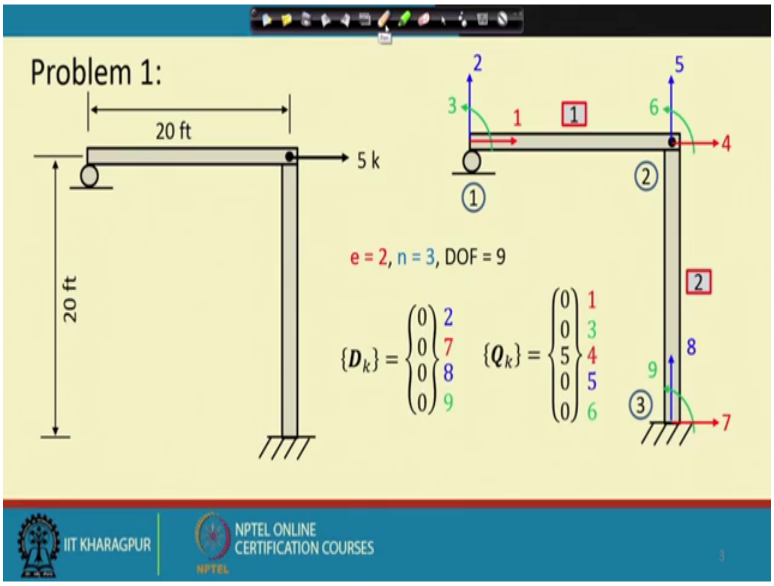


So, in the last class we have solved one example, this example is essentially a two-member plane frame. So, in this frame there is a horizontal load of 5 kilo pound and the one end is fixed and another in this ruler supported. So, we solve this frame by matrix method or specifically the stiffness method and this is our degrees of freedom.

So, we have started with node 1, 2 and 3 so, there are 2 elements essentially, 3 nodes and total 9 degrees of freedom. Among these 9 degrees of freedom we have 4 of which kinetic degrees of freedom unknown which is 2, 7, 8 and 9. And then the corresponding the load vectors which is known to us is 1, 3, 4, 5, 6 so, with this we solved this problem and we found out the solution.

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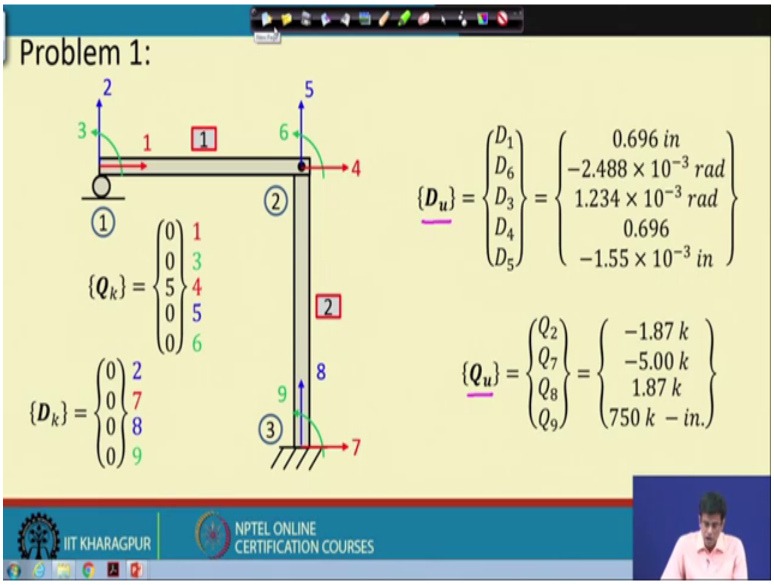


The solution is this we have solved in the last class. So, this is the D unknown; that means, the partitioning of after partitioning of the matrix you obtain this solution and then once we obtain D unknown. So, we obtain Q unknown by the second equation of the matrix equilibrium equation.

Now, here suppose if it is asked that you please find out the loads or the internal loading of the members. Then essentially you have to find out what the nodes acting at the node 1, node 2 and then node 3. So, in the node 3 for instance, the we know what is the to all loads are acting in the node 3 because, this is this will coincide with the internal loading so, this is Q 7, Q 8 and Q 9.

And then node 2 also we know what are the internal loadings. But, if suppose if I want to know what is the internal loading at node 2, we do not know the internal degrees of freedom corresponding to 4, 5, 6. So, in that case what should be the how should I find out?

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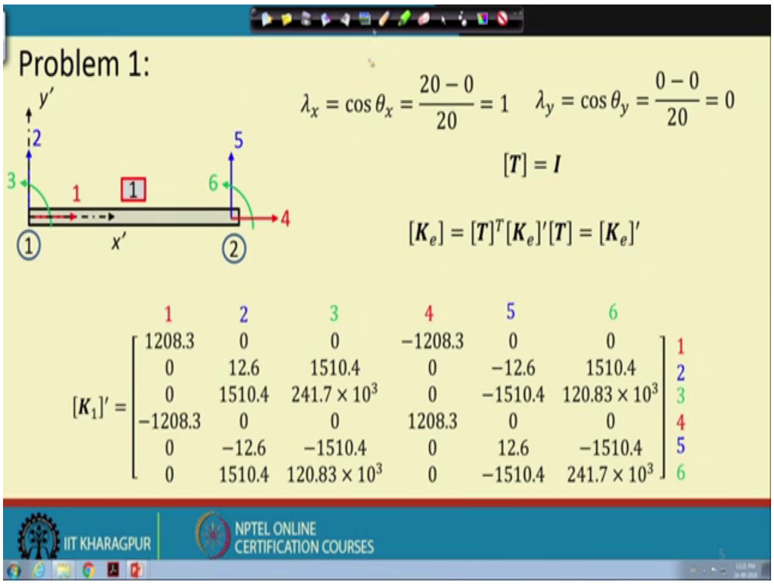


So, to do that let us take the element 1 simply so, this element if I take now we know from our previous calculation that this element is of length 20 feet. So, this is 20 feet already we have discussed or it is given this distance. And then what are the equilibrium for this element that we will explore to find out the internal loading in the node 2. Now, lambda x and lambda y also we know which finally, results the identity matrix T in that transformation matrix is identity matrix and finally, if you remember the formula for stiffness transformation so, this becomes K e is K e e hat K e prime.

So, this that means, the local and global stiffness matrix are same because, the element is entirely aligned with global x y axis or the structural x y axis. So, with this we know what is the stiffness matrix? And the stiffness matrix is of this form that we have we know it. Now, our objective is to find out what is the internal loading in this member. So, to do that before we start into that let us see what are the equations we need to follow?

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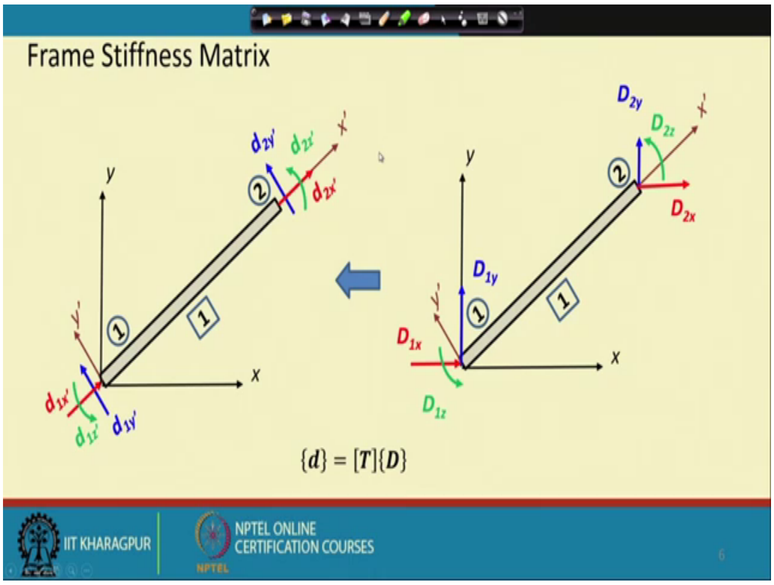


If you remember that there is a transformation which is if you know the global displacement vector capital D and then if you want to get the local displacement vector or deformation vector then this is the transformation matrix T. So, this we know from our previous discussion or even in the from the first class itself we know this.

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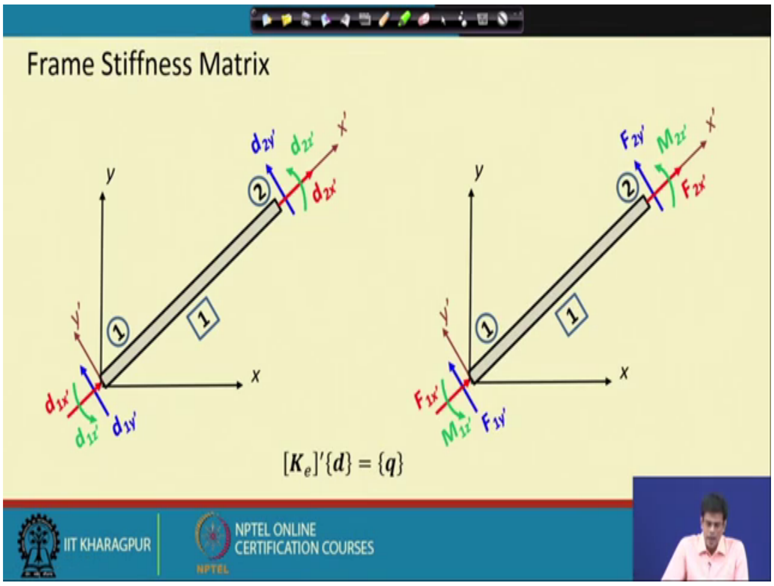


Now, another important formula that will use here is that the local equilibrium equation. For a member with small d is my deformation and q is my internal loading and K e prime is my local stiffness matrix then the equilibrium equation reads like this so, this form will use it.

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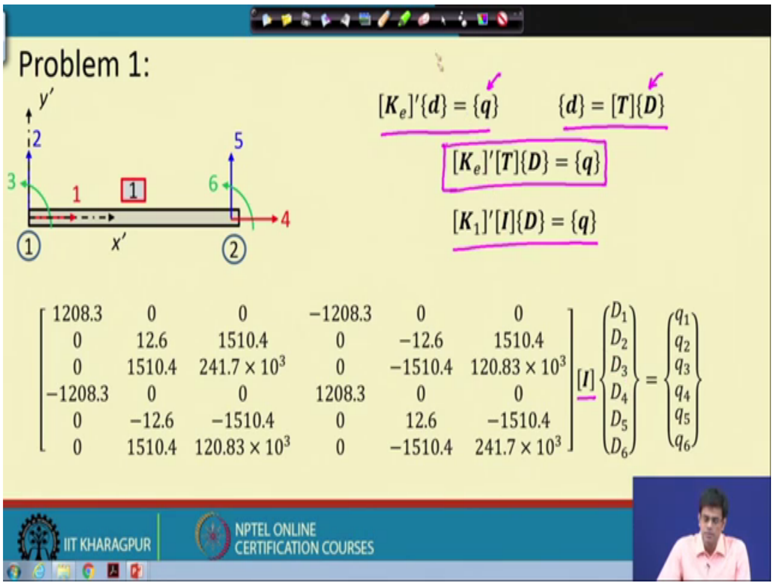


So, now for this case the K e prime d equals to q, where q is my internal loading. So, this I wanted to find out, now this d is again related to capital D in this way because, we do not know small d, we have found out only capital D. So, with this help of capital D and capital T we can add the transformation matrix we can find out small d.

Now, if we just substitute in this equation this and then this leads to this formula which is the q for the element small q or the internal loading of the element 1, which comes from the equilibrium of the element 1. So, as you know that the all element should be in all members of the structure should be in the equilibrium. So, by this logic this equation can be evaluated for finding out the internal loading of a particular member.

So, in this case the element 1 T is my identity matrix so, this becomes my equation for member 1. Now, naturally this will give me the K prime K 1 prime is this and then identity matrix and then this member contains 1, 2, 3, 4, 5, 6. So, D 1, D 2, D 3, D 4, D 5, D 6 and then corresponding q 1, q 2, q 3, q 4, q 5, q 6. So, this is my overall displacement and internal loading. Now, since we know D 1, D 2, D 3, D 4, D 5 we can find out simply q 1, q 2, q 3, q 4, q 5.

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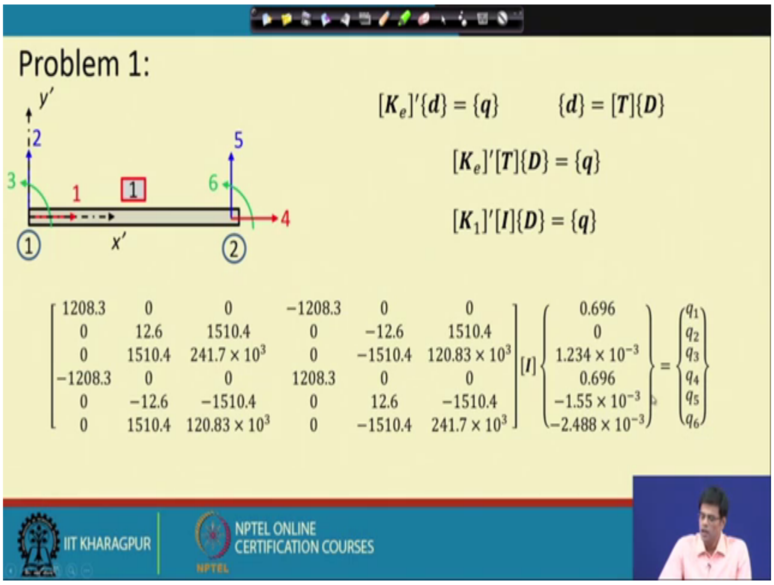


So, just by substituting this D 1 to D 6 that is that we have found out in the previous class if we substitute it here we get q 1, q 2, q 3 and qc, q 4 and q 6.

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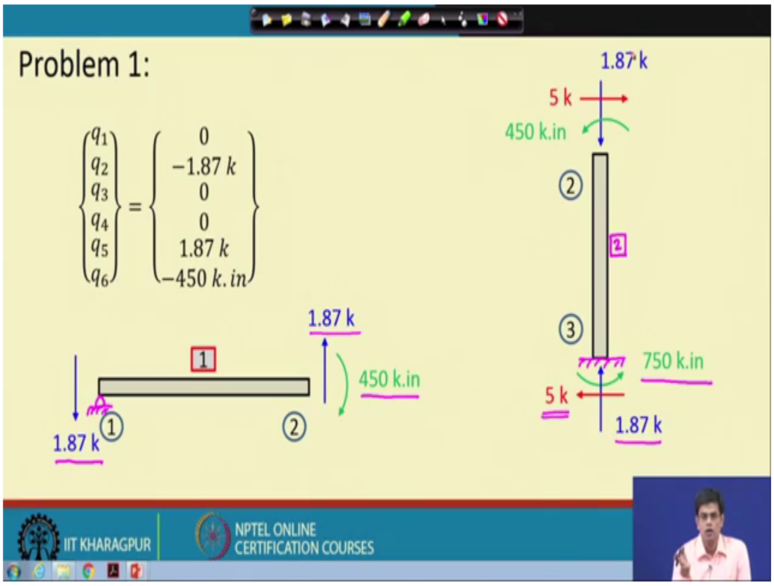


So, let us see how this q 1, q 2, q 3, q 4, q c, q 5 and q 6 so, it turns out this. So, now, if we draw the free body diagram so, if this member is having a shear force which is corresponding to q 1 sorry q 2 which is downwards minus 1.87 kilo pound. And similarly q 3 and q 4 that is the moment here is 0 and then the q 4 is the axial force in this side is 0 because there is no axial force in this member.

So, here if you look carefully that this support the node 1 was a roller support in a it was a roller support. Now, this roller support means; that means, moment has to be 0. So, this coincides with the; this coincides with the our finding. Now, again at node 2 there is no support condition was there so, but there is no axial forces or at all so, in that member to balance that so, there is no axial force. Now, there is a corresponding upward shear force 1.87 which is q 5 and then there is a clockwise moment that is negative moment is 450 kilo pound inch. So, this gives me the equilibrium of member 1.

Now, if you want to the bending moment diagram then you can just directly bending moment and shear force diagram you can there is not it directly draw this bending moment and shear force diagram from this equilibrium free body diagram. So, now similarly, we can find out for member 2 which if you find it out for q say q 4, q 5, q 6 and q 7, q 8, q 9 then this looks this. So, here it is interesting to see that this 1.87 kilo pound is acting this side.

So, downwards here, here it is upwards. So, will become 0 and then there is a anti-clockwise moment or the positive moment at this end for member 2 so, this is for the member 2. Now, these also balance so, there is no moment acting at this joint finally and then there is a external load fiber kilo pound is acting horizontally along x-axis which is again balanced by the fixed end. At this fixed end because if you remember this end was a fixed end so, this was balanced by 5 kilo pound. And similarly this 1.87 is again balanced here and then there is a counter clockwise moment of 750 kilo pound the inch.

Now, these 3 reactions are very much matched with our internal reactions that we have or the reactive forces which we have computed in the from the previous calculation. But, here at this joint 2 if you cut from the if you draw from the member 1 side and if you draw from the member 2 side this is the free body diagram or this is the internal forces that acting on this joint 2.

So, this also with this knowledge you can also draw the bending moment and shear force record so, this finally, completes the problem 1. So, this is a part of the problem which were left in the previous lecture. So, this for each problem you can once you solve the global degrees of freedom or kind of especially the kinetic degrees of freedom that is D unknown once you solve that.

So, you can go to the elemental level and find out the internal forces in the each member or the element. And then finally, once you found out the internal loading then you can draw the bending moment and shear force diagram. So, this is the final answer to the problem.

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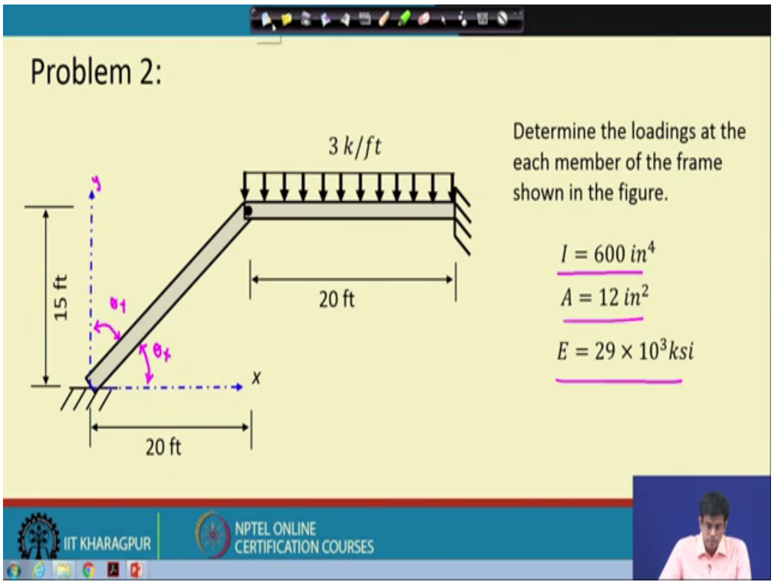


Now, let us do another problem which is an inclined member essentially this member is inclined along with an theta with x axis so, this theta can be found out. So, essentially what we need is theta x and theta y with the member that is inclined with the x axis. So, this is my global y axis so, this member both the ends of this member are fixed.

So, as we know since both the end are fixed. So, the all kinetic degrees of freedom at these 2 ends are known to us. And there is a UDL acting on the horizontal member that is 3 kilo pounds per feet and these are the problem that I, A and E. So, these with this we want to solve this problem. Now, to solve this problem the first step is to discretize again and find out what are the known degrees of freedom and unknown degrees of freedom, kinematic as well as the for reactive forces.

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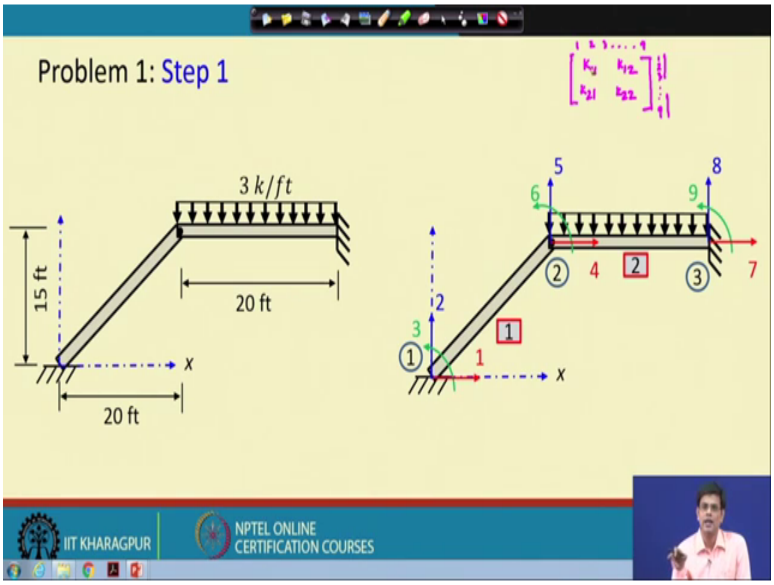


So, now here naturally the first impression is to discretize in this form because this is node 1, node 2, node 3 and there is a node 1 corresponding to 1, 2, 3, degrees of freedom node 2, 4, 5, 6 node 3, 7, 8, 9. So, among these 9 degrees of freedom 6 degree kinetic degrees of freedom are known to us because 1, 2, 3 we all know there is no rotations no horizontal and vertical displacement. So, 1, 2, 3 is known and similarly 7, 8, 9 is also know.

But, if you now see if we discretize in this way there is what we have seen it in the earlier example that we need to shift the rows from to get the partitioning of the matrix or to get the or to separate it out D known and D unknown. Because, you see your final global stiffness matrix K g will be of this form now in this K g there will be a K 1 1 K 1 2 K 2 1 and K 2 2.

Now, you see here this will come from 1, 2, 3 to 9 up to 9. And similarly 1, 2, 3 to 9, now to partition that matrix essentially you have 4, 5, 6 is my unknown so, 1, 2, 3 is known. So, this part will be known and there are some other part will also be known. So, middle part of the matrix has to be separated out from the rest of the part of the matrix. So, you need the row operations or the column and the column operations to separate it out.

Now, this can be avoided if you choose judiciously the degrees of freedom, but again this is just for hand calculations. So, in a coding where you leave this operations for the computer you really do not do that. So, I will explain one problem with this how you can judiciously choose the degrees of freedom.

So, that you do not need to do matrix partitioning again. But, again the cost will be you have to add the matrices properly while assembling so, instead of choosing this set up if I now choose a little in a different way suppose, I choose so, this is the thing if I choose in this form.

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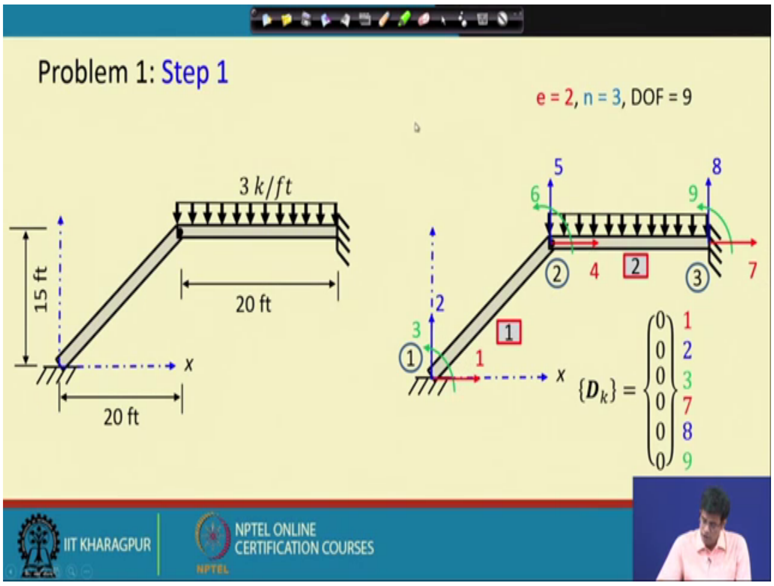


Now, instead of just shift these 2 degrees of freedom that is the if I just shift the these 2 degrees of freedom that is if 1, 2, 3 if I put here and 4, 5, 6 if I put here. T you see the node 1 is actually node 1 contains the 4, 5, 6 degrees of freedom and node 2 is essentially 1, 2, 3 degrees of freedom and then node 3 remains same with the previous case.

So, let us proceed with this thing and let us see how it will be done. So, if we choose this way then my 1, 2, 3 is my unknown. So, you can see one issue here that the first 3 degrees of freedom in a global stiffness matrix will be unknown. So, most likely we do not have to partition the matrix here. So, all the we have to partition the matrix but, the row operation and column operations probably will need not to do.

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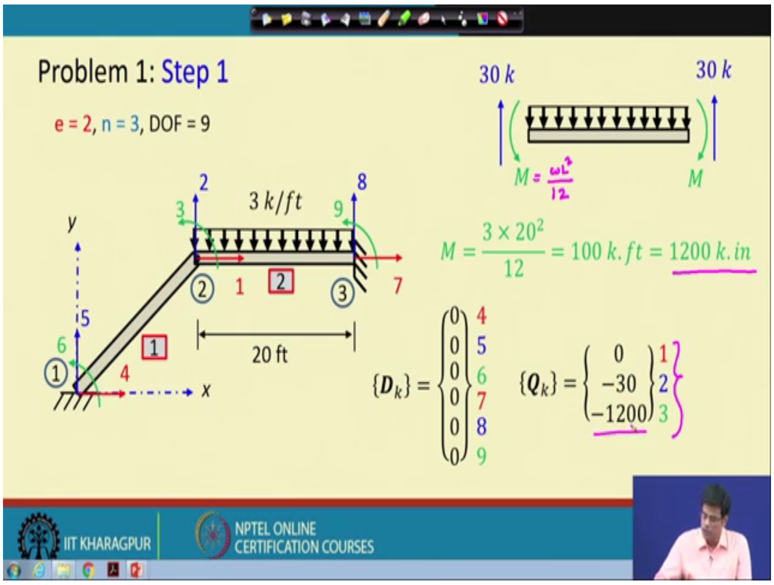


So, this we can explain once we assemble that we can see when once we assemble. Now, the cube the force vector so, since there is a udl so, there will be a fixed end moment. So, we know what is the fixed end moment formula for these things so, that is omega L square by 12 which is very well known so, and these are the moments.

So, this moment finally, comes 1200 ki kilo pound inch and there is a shear force corresponding to 30 kilo pound because, 30 and this is 3 into 20 so, total 60 so, 60 downwards so, divided at the 2 nodes 30 kilo pound and 30 kilo pound. So, here these at these degrees of at this node there is only applied force and these applied forces as 0 minus 30 and minus 1200. So, these are my 1, 2, 3 is my non zero means or the active force vector.

So, now here 4, 5, 6 and 7, 8, 9 are my known kinetic degrees of freedom. And so, my unknown kinetic degrees of freedom are known this displacement vectors are 1, 2, 3 and corresponding the force vector is 0 minus 30 and minus 1200. This probably it has been explained to you in the beam also. So, let us proceed now, to find out the stiffness matrix.

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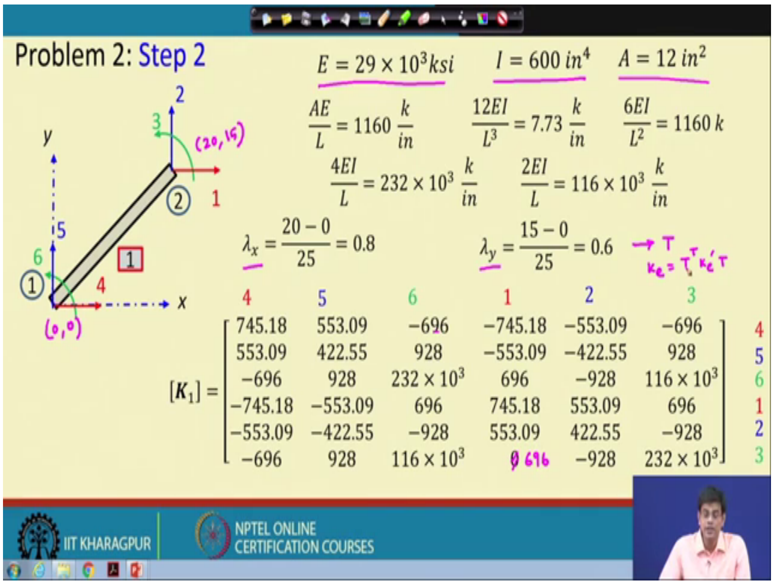


So, the main objective here to highlight the matrix assembling or the assembling of the global stiffness matrix. So, now, similar to the previous problem we just find out some quantities like AE by L, 12 EI by L cube, 6 EI by L square, 4 EI by L 2 EI by L and like that. So, these quantities are evaluated with the data given that is Young's modulus, I moment of inertia and area with this we can find out these quantities.

Now, this member is essentially if you see the coordinate of these members if you notice the coordinate of this member. So, this is the 20 15 is the coordinate for this node 2 and this is naturally 0 0. So, you can calculate lambda x, lambda y which is which relates to T matrix or the transformation matrix. So, the procedure is pretty similar with the previous case where essentially K e we write with T transpose K e prime T. So, with this we can get the final stiffness matrix.

Now, in this stiffness matrix the first 3 columns and first 3 rows are essentially corresponding to 4, 5, 6 degrees of freedom. And last 3 columns and last 3 rows are corresponding to 1, 2, 3 degrees of freedom. So, this is the only change and if you look carefully that after transformation the matrix is not essentially the usual frame stiffness matrix with these quantities. These things are 0 and all the things because transformation happened.

But, the property of the matrix remains same it becomes symmentric and this will be 696 I think. So, the property of the matrix remains same and but, the it is not the same form as we have seen for the frame stiffness matrix because of that nonzero transformation or non identity transformation.

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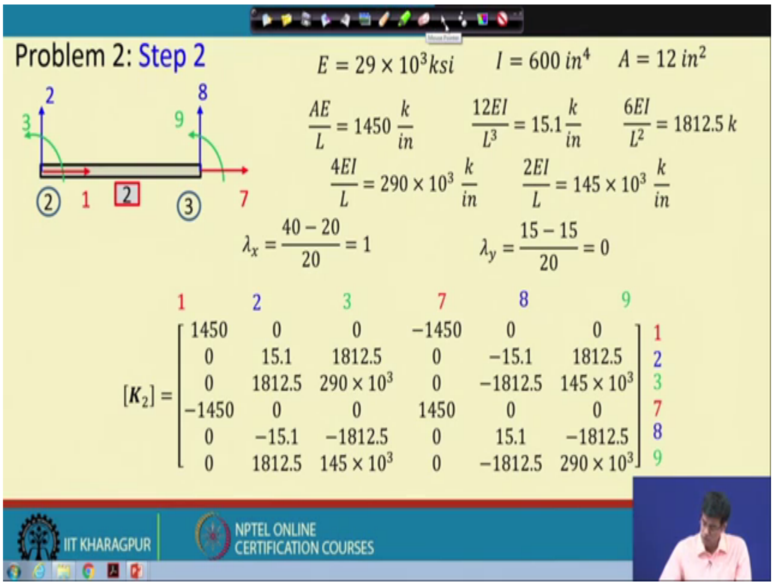


So, now similar to the this case we can find out the stiffness matrix for the element 2. Now, in the element 2 again the coordinates with the help of coordinates you can find out lambda x and lambda y. And since the length will be changed because that was an inclined length we considered there. So, here the these quantities will also will change even though E, I and A remain same with the member 1. Now, but length of member 1 and length of member 2 will be different so, that is why these quantities are differ.

Now, similar to the previous case we can find out the elemental stiffness matrix in the global degrees of global coordinate system. Now, this global coordinate system this T here is essentially a identity matrix so, it remains the same form with the local stiffness matrix or the K e prime. So, here the first 3 columns and first 3 rows corresponding to 1, 2, 3 degrees of freedom and last 3 columns and last 3 rows are corresponding to 7, 8, 9 degrees of freedom. So, this actually gives you the final stiffness matrix in the for 2 elements.

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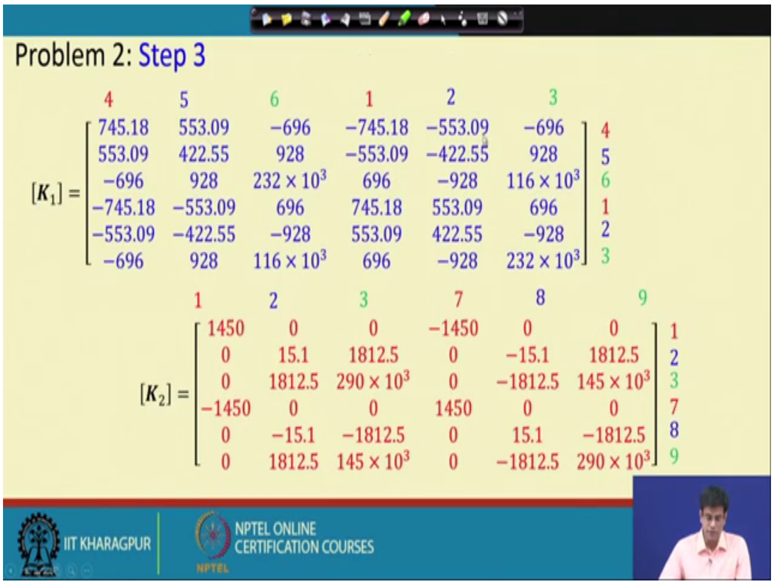


Now, the next objective is to assemble this stiffness matrix. Now, you see this stiffness matrix I have written in a same page so, that you understand this is my stiffness matrix. Now, the main objective is how to assemble this or what is the assembling procedure for this so, that we get the global stiffness matrix. That we will discuss in the next class so, I stop here today. So, in the next class essentially we will assemble and solve this problem.

Thank you.