Matrix Method of Structural Analysis
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Lecture – 27
Matrix Method of Analysis: Frame (2D) (Contd.)

Welcome so, we are discussing the frames 2D frames. So, in the last class we have derived the stiffness matrix for a frame and we are trying to derive now the inclined frame. What will be the stiffness matrix?

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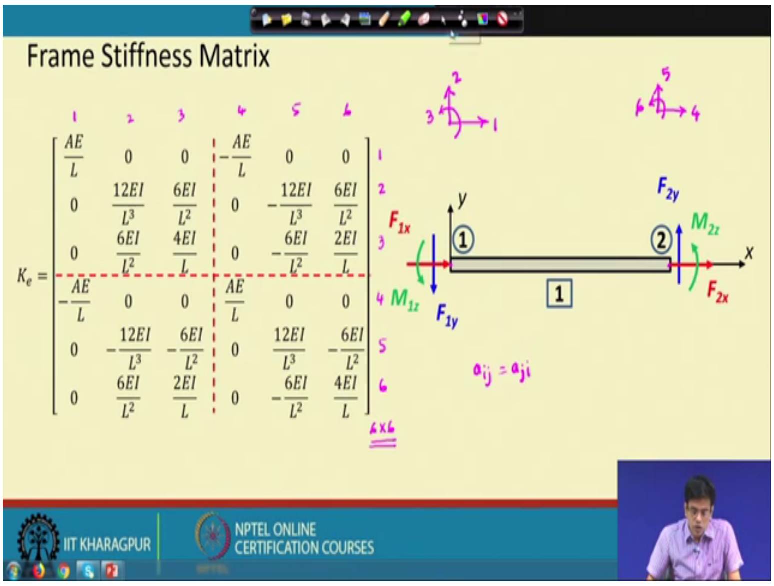

So, this is the frame stiffness matrix so, we know what are the frame stiffness matrix here. So, we know what are the degrees of freedom and essentially we find out 6 cross 6 stiffness matrix.

So, now in case of a in case of a 2 D frame what will be the stiffness matrix for a, what will be the size of the stiffness matrix, which is 6 cross 6. And we know the corresponding degrees of freedom. So, that is the first degrees of freedom, second degrees of freedom, third degrees of freedom, fourth degrees of freedom, fifth and 6 and these accordingly the columns are also 1, 2, 3, 4, 5 and 6 the degrees of freedoms are like this.

So, this is the first degrees of freedom, the actual displacements, then the transverse displacement and the third degrees of freedom is the rotation at node 1. And then similarly, for node 2 the first degrees of freedom is the 4 and then second degrees of freedom is the transverse displacement is 5 and then the third degrees of freedom is 6. So, according to these degrees of freedom what are the forces also we know; so, the 2 axial forces and moments. So, finally, the 6 cross T 6 stiffness matrix or the specially the elemental stiffness matrix.

So, now we know from our previous knowledge, this stiffness matrix if we take the determinant; then determinant will be 0. And another property we know about this stiffness matrix is that it is symmetric. So; that means, along this line. So, we know what is the symmetric matrix. So, in a matrix if the components are a ij. So, a ij equals to a ji. So, you know this is naturally the off diagonal elements.

So, here once we know this with this knowledge, then we progress for the inclined stiffness matrix.

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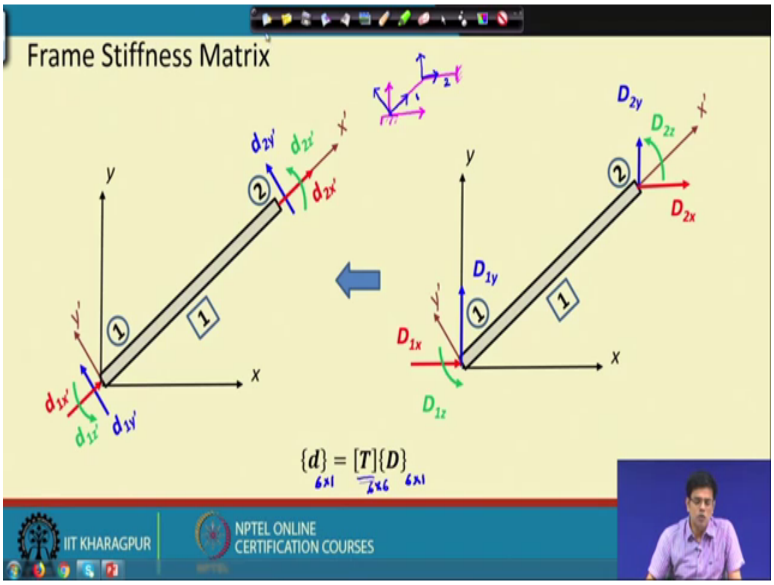

So, again we have also derived this, this when you have inclined degrees of freedom or the local degrees of freedom which is represented by small d. Then and the global degrees of freedom or the structural degrees, structural axis the degrees of freedom is that global structural axis I mean.

So, for instance this frame I discussed in the last class. So, it consists of 2 members suppose, this is a 2 member. So, this is my, the structural axis. So, the local axis for individual members I can do it like this.

So, for instance this is my individual members. So, in the individual members I can this is my first member, this is my second member. So, the local axis can it is this. So, this is my local axis for the frame. So, this is elemental 1, element 1 and this is element 2.

So, now for this case the local axis and the global axis for element 2 matches exactly because, the orientation is same, but for elemental 1, element 1 it is not. So, that is why we need to transfer this we need to have a mechanism by which we can transfer these global degrees of freedom and local degrees of freedom.

So, this transformation we have seen that is capital T. So, this capital T is the transformation matrix. We also know what is the size of this transformation matrix is 6 cross 1 because, capital D is 6 cross 1 and small d is 6 cross 1. So, this is naturally the transformation matrix. So, transformation matrix we know.

Similarly, we have also learned what is the force transformation. So, if there is a force transformation like this. So, e along the prime axis x prime, y prime the forces are F 1 x prime, F 2 x prime or if F 1 x prime, F 1 y prime and M 1 z prime for node 1 and similarly for the global axis.

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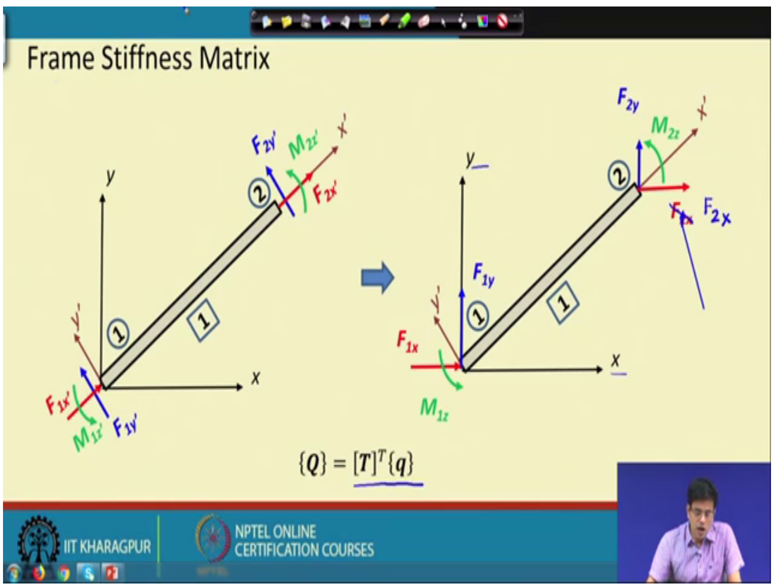

So, this is F 1 y, F 1 x and M 1 z. So, similarly, for node 2 F 1. So, this will be F 1 x, F 2 y sorry this will be F 2 x only F 2 x, F 2 y, F 2 z.

So, now we also know that if we know these quantities then we can go to this structural x, y coordinate system and this is T transpose Q. So, this with this knowledge we started to find out the stiffness matrix of an inclined member. So, let us now find out what will be the form.

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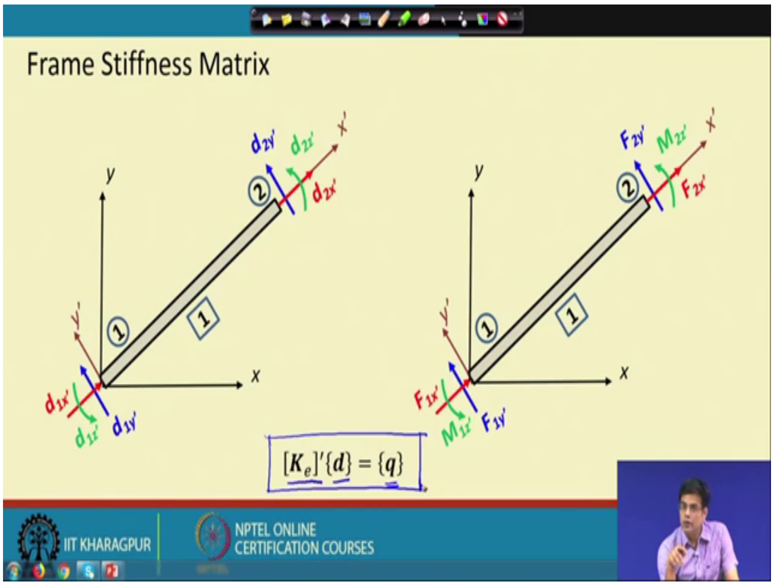

So, in a first let us understand in a local coordinate system or local the elemental coordinate system what is the equilibrium equation?

So, we know the stiffness multiplied by displacement equals to the force. So, this is the basic definition of stiffness. So, essentially when you have a unit displacement the force required for unit displacement is called the stiffness.

So, the equilibrium equation is for the local axis K e prime. So, I write it in terms of prime the prime represents the x prime, y prime coordinate system and this d; small d represents the kinematic degrees of freedom in the prime coordinate system. And small q represents the forces in the prime coordinate system. So, this is my equilibrium equation for the prime coordinate system.

So, we know this very well. So, now similarly, we can have a global equilibrium equation or global force displacement relation through that stiffness. So, that also we will see.

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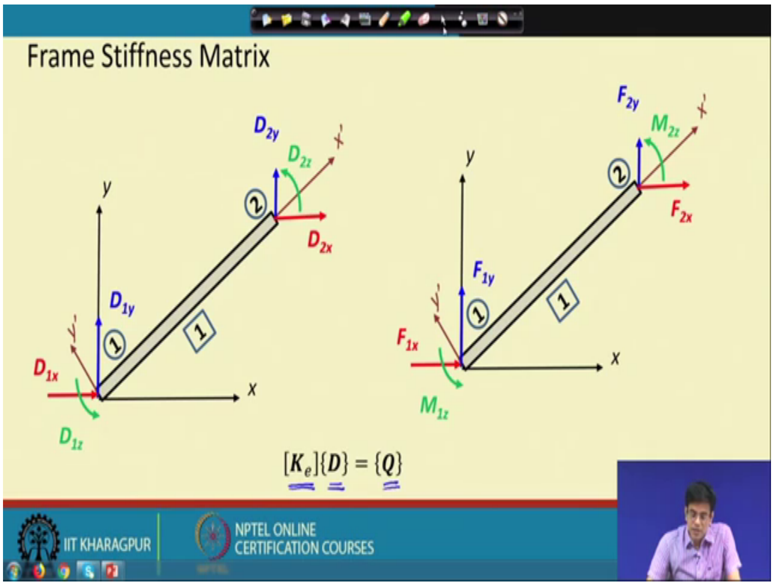

So, let us see how force displacement in the global coordinates system. So, this is my force displacement relation where I have not used any prime here and capital D is my global degrees of freedom. Or the degrees of freedom for that particular element in the global coordinate system. And capital Q is my, the forces in the structural or the global coordinate system. So, this is the equilibrium equation. So, this also will be 6 cross 6 matrix.

Now, this is my equilibrium (Refer Time: 08:14) equation. So, my job is to now find out what is the relation between K e and K e prime;so, the stiffness matrix in the prime and non prime coordinate system.

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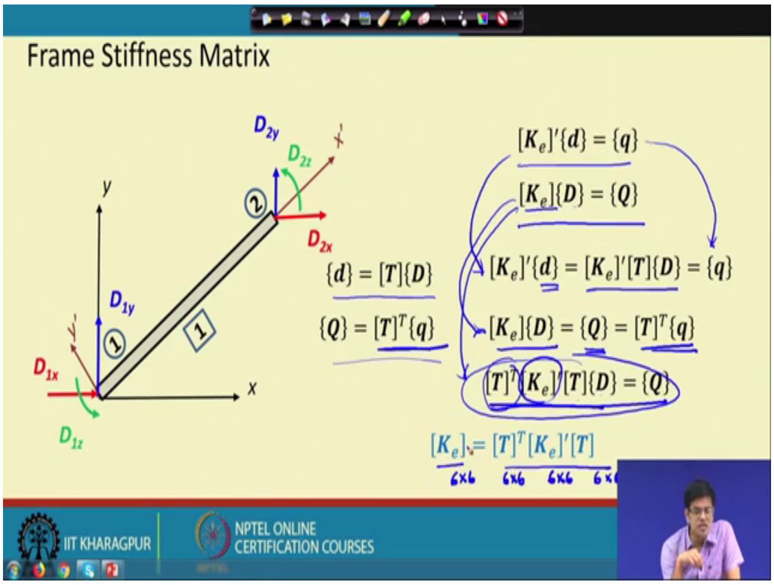

So, let us see how it looks. So, in a non prime and. So, this is a prime coordinate system the equilibrium equation and this is a prime know. So, this is a prime coordinate system and this is the prime coordinate system.

Now, we also know these transformations so, with this so, if I now write this K e prime D equals to I will just substitute this quantity T D. So, naturally small d in place of small d I substitute T and capital D. So, this is equals to the q, this comes from here.

Now, taking this thing again so, and this thing is K e capital D. So, this is non prime coordinate system. So, or the global coordinate system then this is equals to Q and this Q is essentially T transpose q. So, this I can write it very well.

So, now, if I equate Q from both the equation; so, that is again T transpose inverse. So, T transpose inverse is actually the inverse is transpose for this case. So, now here if I write if I compare these 2 equations. So, this is my the final equation that we obtain because if I take this thing this side and K e prime and if I write this T transpose T transpose Q in this side and then again inverse transpose this side and again this. So, it becomes T transpose.

So, now here this T transpose K e prime TD is Q. So, now, if I compare this equation with this equation; so, right hand side becomes Q, this becomes D only thing is that this is now in the prime coordinate system.

So, naturally the non prime stiffness matrix is this. So, in this you have seen in the truss case also; so, that means, the transpose of transformation matrix then the prime stiffness matrix and then again multiplied by the transportation matrix. So, finally, this matrix will also be 6 cross 6 because, this T transpose is 6 cross 6, K e prime is we know 6 cross 6 and then T is 6 cross 6. So, finally, this matrix K e will be 6 cross 6 matrix.

Now, let us see how it looks like or. So, if you know the T matrices, what will be the form?

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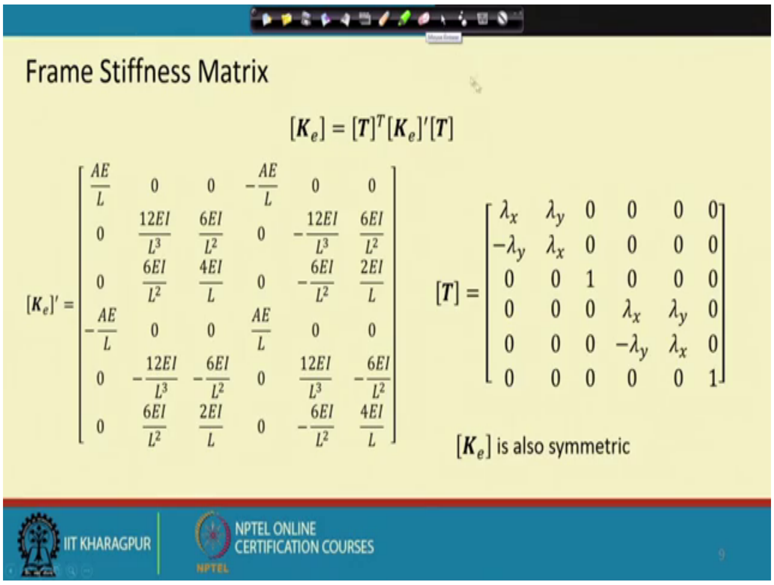

So, this is my final stiffness matrix and then this is my K e prime in the prime coordinate system we have derived this. So, these coefficients we know all. So, this l is the length of the element and then all those things if we know and then we also know the transformation matrix which relates the prime coordinates and the non-prime coordinate system. So, with this I can find out by multiplication we can find out the non prime stiffness matrix.

Now, it is interesting to know that this non prime stiffness matrix is also symmetric. This you can verify you just need to multiply this matrix with the prime stiffness matrix that is this and then again. So, transpose of this matrix with this prime stiffness matrix and then again multiply the transpose of that transformation matrix to get the. Finally, non prime coordinate system what is a stiffness matrix or global coordinate system of the or the structural coordinate system, what is the stiffness matrix?

Now, you can verify that this stiffness matrix is also symmetric and also you can verify that if you take determinant of this stiffness matrix. So, that is also comes to 0. So, this again leads to following conclusion that even though the transformation does not change the characteristics of the stiffness matrix or the properties of the stiffness matrix that is it is symmetric and it remains symmetric.

So, after transformation, second it is singular. So; that means, the there is dependent columns the columns are interdependent. So; that means, the determinant of this matrix is 0. So; that means, in other words it is wrangled efficient matrix.

So, so, these properties remain in the non prime coordinate system intact with the prime coordinate system. So, the transformation actually does not change any property in the matrix. So, this is important fact. So, we do not lose any property of this stiffness matrix.

So, now once we know the stiffness matrix in the rotated coordinate system from local to global. So, we can now proceed further.

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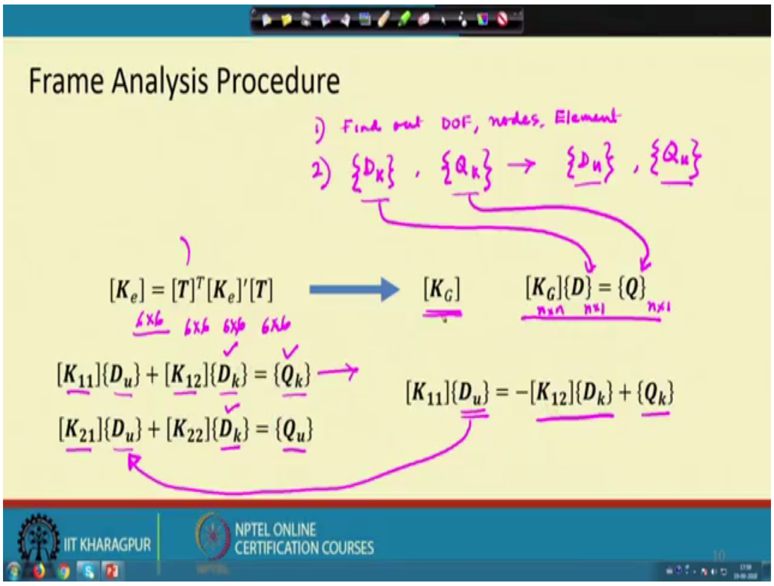

What is the frame analysis procedure? So, the frame analysis procedure in general is very similar to the stiffness analysis procedure where actually we have seen it for a truss, we have seen it for beam. For instance, if you have a structure if first you determine the degrees of freedom what are the degrees of freedom? Then actually you discretize or find out the elements and then nodes in the structure and then corresponding degrees of freedom.

So, the first step is this and then with after this you what you try to find out what are the degrees of freedom known to you, what are the forces degrees of freedom known to you. And what are the forces what are the displacement degrees of freedom known to you and what are the degrees of freedom you need to identify or you need to find out.

So, that after this you essentially form the stiffness matrix. So, in the stiffness matrix for truss you have found out 2 cross 2 stiffness matrix. So, if it is a rotated inclined. So, appro appropriate transformation you have to find out. So, then you essentially multiply with 4 cross 2 transformation matrix for truss.

And then you finally, get 4 cross 4 stiffness matrix for the truss. Similarly, here you get 6 cross 6 transformation matrix. So, the basic procedure remains very same.

So, the first step is essentially the discretization or find out the degrees of freedom, find out degrees of freedom, what are the degrees of freedom, nodes, elements. So, these choices also it has been discussed.

And then second thing is that what are the known degrees of freedom? So, suppose this is D. So, D known degrees of freedom you try to find out and then Q known degrees of freedom you find out. So, what are the forces you know and what are the forces you really do not know.

So, then the if there is a fixed end moment required for instance if there is a u D l that we will discuss again in this class probably it has been discussed in beam also. So, if there is a distributed load in the structure then fixed end moments you need to find out. So, these things you first calculate and know.

So, finally, once this, what are the degrees of freedom is unknown. So, D you need to find out and what are the force degrees of freedom you need to find out. So, with this essentially you compute the stiffness matrix local stiffness matrix and then you transform it to global stiffness matrix.

So, finally, you get the stiffness matrix in a global coordinate system from the local coordinate system. So, this is leads to the, this. So, in case of a truss in case of a truss it was a probably 4 cross 4 and then for a beam this is this was also probably 4 cross 4.

And then for a finally, for a frame it is 6 cross 6 matrix. So, these 6 cross 6 matrix once you know and through the transformation matrix this is also a 6 cross 6 matrix. So, and this also a 6 cross 6 matrix.

So, then the procedure is known as the assembling; that means, for each element you put the appropriate degrees of freedom and the sum. So, this a procedure is known as the union of the all elemental stiffness matrix in the global coordinate system and then find out the global stiffness matrix of the structure.

So, this global stiffness matrix K g that is also you know. So, this global stiffness matrix is essentially the singular matrix because, you have not put the boundary condition.

So, now this K g once you know this K g. So, what you do? Basically, this is the equilibrium equation in the global system or the global degrees of freedom. So, suppose the degrees of freedom of a structure is in cross in. So, this stiffness global stiffness matrix will be n cross 1 and n cross n and then D will be n cross 1, Q will be n cross 1.

So, among these some of the degrees of freedom you know, some of the force degrees of freedom also you know. So, you plug those information those informations here and you plug those informations here. So, finally, your unknown degrees of freedom will be this.

Now, once you know these quantities you basically find out in a something like this. This you partition basically the matrix partitioning also we have seen that is you partition this matrix K g is K 1 1 D u K 1 2 D K and Q K and then K 2 1 D u D u refers to unknown degrees of freedom and D K refers to the known degrees of freedom.

Similarly, Q K refers to known degrees of freedom for force and Q u refers to unknown degrees of freedom for the force. So, now, after this partitioning basically you know this quantity, you know this quantity, you know this quantity, but what you do not know is you do not know this quantity and you know this quantity.

So, you take the first equation. So, the first equation solution is very simple and this solution is essentially you have to find out the stiffness matrix part of the stiffness matrix K 1 2, you have to multiply with the known degrees of freedom.

And then it will add into the force that is Q K and then this is this becomes the force. In this side it goes to right hand side and then you find out the D u. Once you find out the D u you plug this information here. So, then you use the second equation to find out the unknown forces. So, this procedure becomes very simple and very well established to us because we have already used it.

Now, before getting into a particular problem I must tell you few things that even though in this course we are solving we are highlighting these procedures. So, some of the structure actually this method is not for the very simple 2, 3 element structure.

So, the objective of this course is to tell you that what are the process, what are the procedures involved for computing these deformations. And naturally we are going to use for a large degrees of freedom structure which by hand calculation you really cannot find out. So, you need to use the computer and you need to use the coding in appropriate coding languages and then you find out the deformation there are several softwares.

So, the objective here is to if you do not know the procedure very clearly you are actually not able to find out once the software does it is mistake. So, the final objective or the goal is this that you know the procedure how to solve this.

So, when software gives you for instance add pro or any other structures or CSI software's. So, any software that gives you some result you as an engineer basically you interpret those results.

So, once you try to interpret these results essentially you have to know whether the software says give incorrect result or not. So, unless you know what are the procedure software is (Refer Time: 22:37) software are following you cannot interpret or if there is a mistake in the result or if a mistake software does you cannot really find out what are the mistakes it does.

So, naturally the knowledge of this procedure is very important and these methods are not essentially for doing by hand. So, you can develop your own software too that is for instance you can write a Mat Lab coding or any language in Fortran or C Pascal or any any language even python and all those things. And then you write your own code and solve for any structure that is required.

So, then you will essentially understand where you can go wrong. And you can if the if there is a mistake or if there is a interpretation if you need to interpret a result for a large frame for instance there is a building frame 20 storied building frame or 30 story building frame and that too is a space frame. So, naturally that is not a 2 D frame. So, this is a space frame so, is a few thousand degrees of freedom for us very smaller problem. So, a few 1000 so, degrees of freedom.

So, if the procedure if you do not know how the software is doing calculations, you really cannot interpret this results. So, my main purpose of telling you these things that you need to know each integrals of this method to interpret the results that will be given to you. As an engineer you need to interpret those results. So, in the I stop here today.

So, in the next class we will try to solve some 2 element frame and then I highlight the methods what actually what are the details of this method. And finally, I will tell you what are the solution methodology.

So, thank you, I will stop here.