Matrix Method of Structural Analysis
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Lecture – 25
Matrix Method of Analysis: Beams (Contd.)

Hello everyone this is the last class of this week. What we have been doing is we have been trying we have been demonstrating the various steps involved in Matrix Method of structural Analysis of beam through 1 example.

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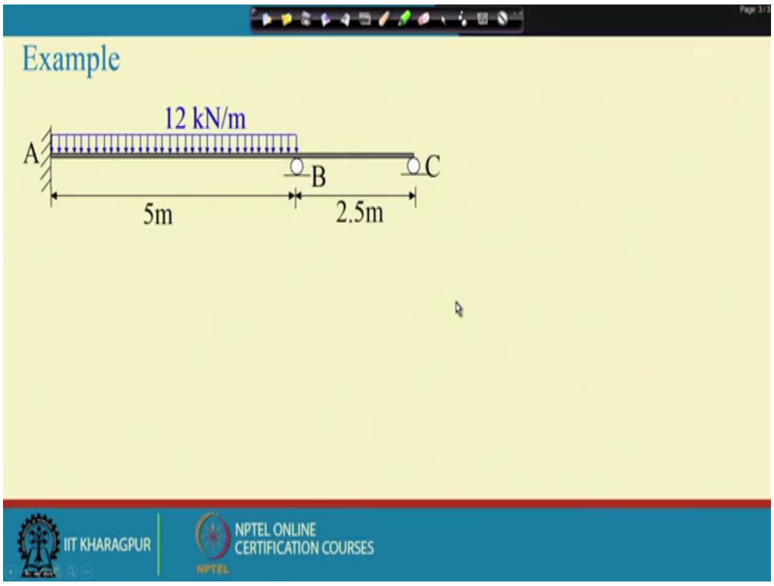

And the example that we have present in our hand is this one.

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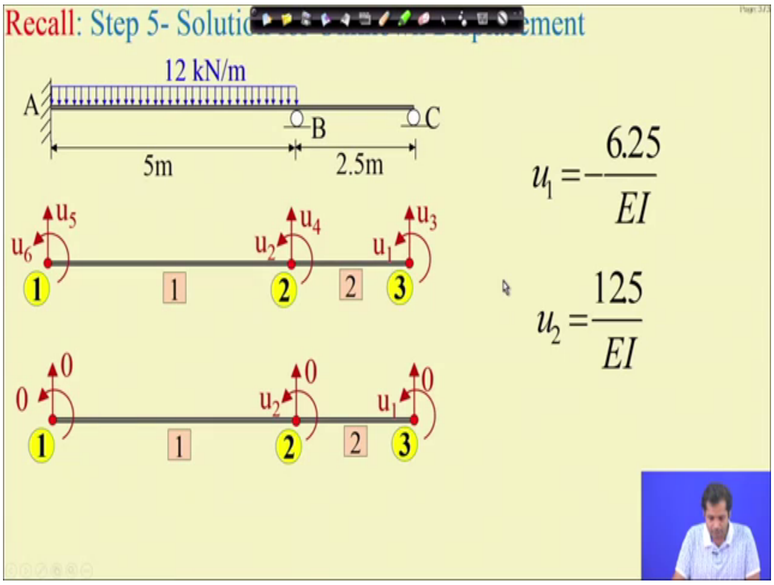

So, far what we have done is we calculated the we have done the step 5 which is solution for unknown displacements. We calculated the member stiffness matrix assemble them and then after assemble then we applied the boundary conditions and the calculated the nodal load vector.

And then solve for unknown displacement and for this the unknown displacements are 2 u 1 and u 2 all other displacements are 0 because of the constraint. And the values of u 1 and u 2 is this, we have already done it in the previous class. Now, today what we do is today will be the step 5, step 6 we will see how to calculate the support reactions and then member forces.

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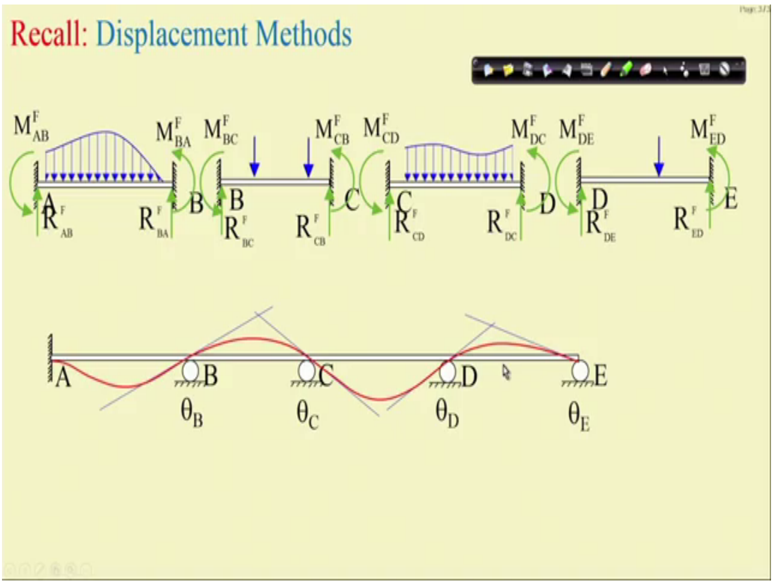

So, you see if you recall this is a very important slide. It says it has a core of the core concept of this entire exercise that we are doing.

Now, we have a beam. We identify the different segments first and then each segments are now divided into each segments are considered separately. Now each segments are now we have a 2 part the entire problem is now to divide into 2 part; 1 part is each segment is considered as fixed beam and therefore, we calculate the fixed end moments fixed end forces and friction movements which are called friction reactions.

Now, by making all the ends are fixed we essentially constraint all the degrees of freedom. So, there is no degrees of freedom in the member. So, only the member is whatever forces we have that forces due to the externally applied load. Now then in actual structure, in addition to the external action because of the externally applied load there will be deformation. In the first part there is no deformation in the second part we have just only deformation. And in actual structure what is happening both together right we have forces and the deformation.

So, therefore, whatever reaction the force you have that reaction force we have two contribution, one contribution is from the first part where is which is kinematically determinate structure contribution from that kinematically determinate structure only due to the external load. And then there will be another contribution because of the deformation of the structure.

So, total force total reaction at the support will be not only the total reaction at the support even the member forces, total member forces on the reactions will be the contribution from first part plus the contribution from the second part.

Now, first part is already we have with us the fixed end actions we will now concentrate on the second part and then add to the first part could to get the support reactions and the member forces.

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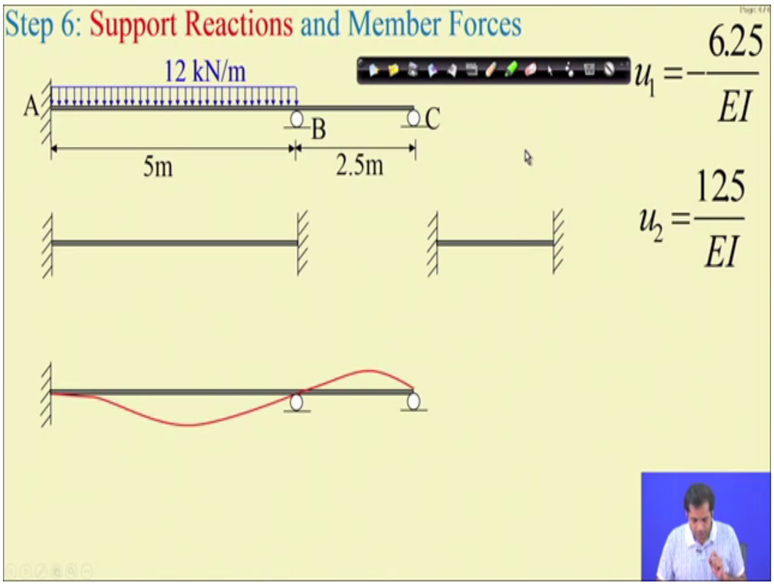

So, you see so, we have now this is the this is 2 segments and we. So, what we have is. So, for the fixed end moments if you recall these the fixed end this. So, what we have is, we have for this we have say use we have fixed end moments fixed end actions there will be fixed end actions remember anticlockwise moment and anticlockwise rotation is considered as positive.

And there will be a force here there will be force here and similarly we have this and then this and there will be force here and there will be force here. So, these are fixed end actions we already know what are the fixed end actions for this structure. And then we have can just now that just now we I mention this is for Kinematically determinate structure there is no degrees of freedom. All degrees of freedoms are constraint. Now, in addition now the structure when the deform because of the deformation there will be some actions at the at the joints action at the at the supports, and suppose these actions are this. Similarly these actions are this corresponding.

So, support reactions will be, total support reactions at every any joints will be this plus this. So, if I take this beam this beam so, at the at fixed end we have a force then moment and this force is say Y A vertical direction at a and this is M A moment at a. And then we have force support reactions at B suppose this is Y B and then support reactions are C this is Y C.

Then M A will be contribution from this plus contribution from this. Similarly Y A will be contribution from contribution from this and contribution from this. And Y B will be this contribution from contribution from this and then contribution from this. This will be Y B right and so, on. And similarly contribution from C will be it will be contribution from this contribution from this.

So, this is will be the total reactions. Similarly the member forces will also see the similar similarly we can obtain the member forces as well.

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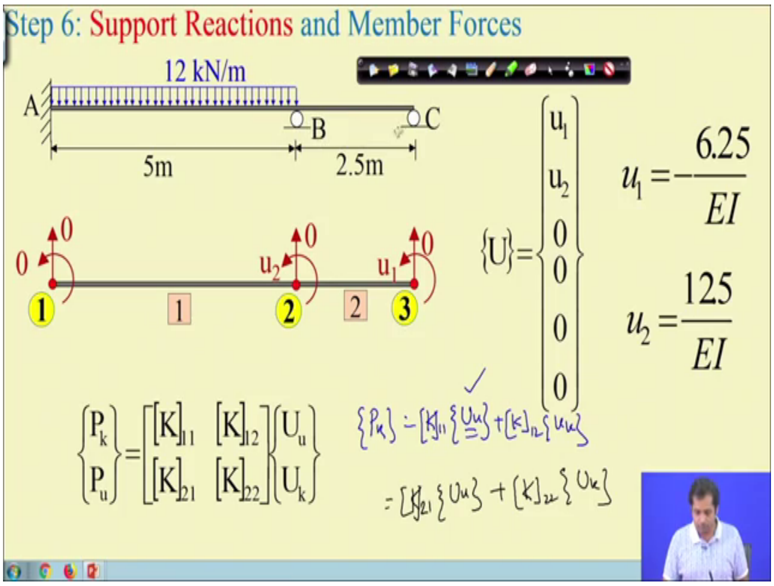

So, fixed end moments already we have, now we will how to how to get the contribution for the displacement part. So this is the displacement all displacements are we have just only 2 non zero displacement u 1 and u 2 all other displacements are 0 and values of u 1 u 2 are this we have already obtained it last class.

Now if you also remember in the last class we discussed if P is equal to K into U this is the relation force displacement relation. Then that relation can be partitioned depending on the unknown displacement, we can separate unknown displacement and unknown displacement. And similarly we can partition this stiffness matrix and the load vector. And if we do that these will be we get some think like this. And if you recall these P K are the nodal equivalent load vector, equivalent node vector at the joints were displacements are unknown and if we solve these equation.

If we solve P k is equal to K 1 1 into U u plus K 1 2 into U k. If you solve this equation you get U unknown U k is 0 and this one is known and this is nodal vector node if we have solve this equation you get U unknown.

Now, in order to get now we will use the second equation; second equation is you see what we want is if you recall in the second, if you just now we discussed we need the contribution this is these fixed end contribution already we know fixed end actions. We need what is the reactions generated due to the deformation and the deformation the entire deformations are now are represented by only 2 non zero degrees of freedom one is u 1 and u 2.

So, because of this u 1 and u 2 what are the reactions generated at this support because of the deformation. Now that, in order to get that we use the second part of this equation and the second part of this equation will be this. So, this will be K 2 1 into U unknown which is now know because we have already determined it plus K 2 2 K 2 2 and U known which is any way 0.

Now, these contribution these gives us the forces the reactions at the joints because of the deformation Let us find out that.

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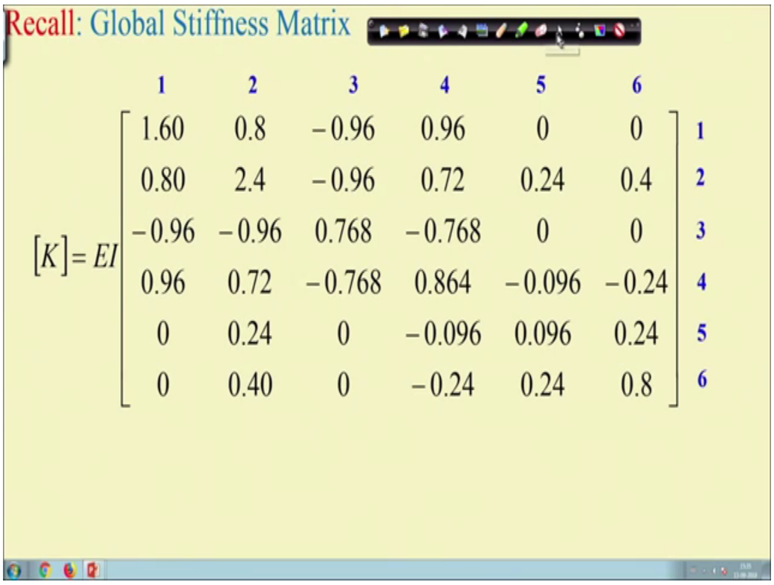

Now, if you recall our global stiffness matrix was like this and in these global stiffness matrix this is how we partition the matrix. This was K 1 1 and then a sub matrix and this was K 1 2 and this is K 2 1 and this is K 2 2.

So, now what we need is we need this part. So, we have to compute K 2 1 into U unknown now which is known means, we have to come we have to compute K 2 1 and then u 1 u 2. U 1 u 2 we have already determined the values. And K 2 1 will be this part, this is 4 by 2 matrix and u 1 this is 2 by 1 matrix which gives us the forces at the joint reaction at the joint only due to the deformation which is 4 cross 1. So, 4 unknown joint reactions we get. Let us do this exercise.

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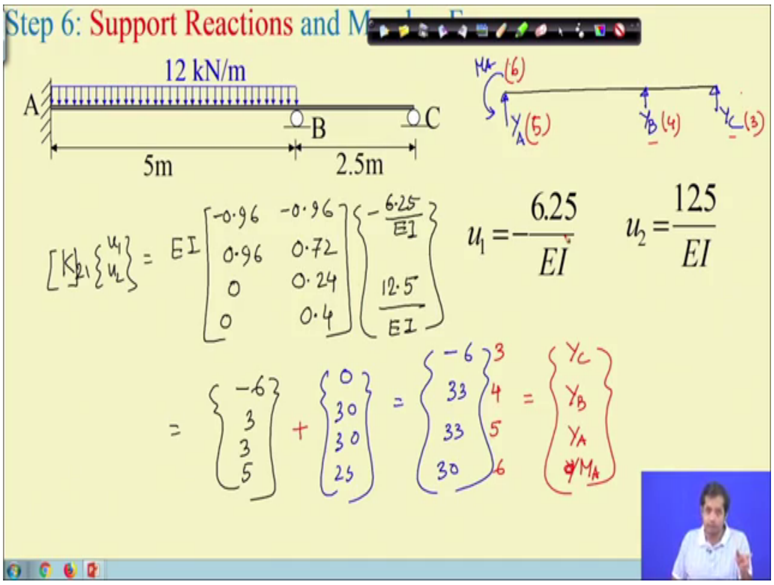

Now so, u 1 u 2 is this. So, let us find out what is this then K 2 1 into u 1 u 2 that will be K 2 1 is E I if you check, this will be minus 0.96 and then minus 0.96 and then 0.96 and this is 0.72. We have 0 here, then 0.24 and then 0 0.4.

Now, this is for this and then we have u 1; u 1 is minus 6.25 by E I and then 12.5 by E I. Now if we, this gives us. So, if we do this multiplication these gives us minus 6 this C i and this C i gets cancelled minus 6 3 then 3 and then 5.

So, therefore, total reactions will be now. Suppose these reactions are in this case. So, these reactions will be, suppose here if we draw this beam once again and the reactions are this is r this is Y B Y A and this is N A and this is Y B and this is Y C Y C we need to calculate now these reactions. But remember when we implement it we do not the degrees of freedoms are associated forces are not represented as M A Y A they are represented through some numbers right.

So, the numbers that we used if you recall the numbers we used is for this it is u 2 the this is u 3 this is this is third degree this is direction 3 and this is direction 4 and this is direction 5 and this is direction 6 right. So, what we get is we get this is.

So, now what we have is we can this total forces will be this plus in addition to that this plus we have the contribution from the fixed end reactions and that fixed end actions are

0 then 30 you can cross check from the previous class this is the and then corresponding values will be minus 6 and then 33 then 33 and 30 right.

So, what is this? This is associated to degrees of freedom 3 and then degrees of freedom 4 it is associated with degrees it is associated with direction 3 direction 4 direction 5 and direction 6. So, essentially this is direction 3 is Y C and direction 4 is Y B and direction 5 is Y A and then Y no sorry this is M A.

So, this is the react total reactions now let us check whether the equilibrium conditions are satisfied are not. Reactions wise total force here is 12 spanning over 5 meters. So, total downward 4 is 60 kilo Newton and then support reactions we have U Y C plus Y B plus Y A, Y C Y B and Y A 33 plus 33 66 minus 66 60.

So, equilibrium condition satisfy, this is important to check see equilibrium condition has to satisfy because that is the basis of entire all this formulation, but when you numerically implement it is good practice to cross check whether equilibrium, numerically the equilibrium being satisfied or not because that gives that gives you that gives a confidence over your on your numerical implementation.

So, this is how the reactions can be obtained. Now similar approach we can follow to find out the member forces. So, member forces will be we take one any member and then this member will have the member forces in that particular member will be having two contribution, one contribution from the fixed end action and another contribution from the because of the displacement. So, one contribution kinematically determinate structure no displacement and the second contribution will be only displacement no external load. So, let us do that exercise.

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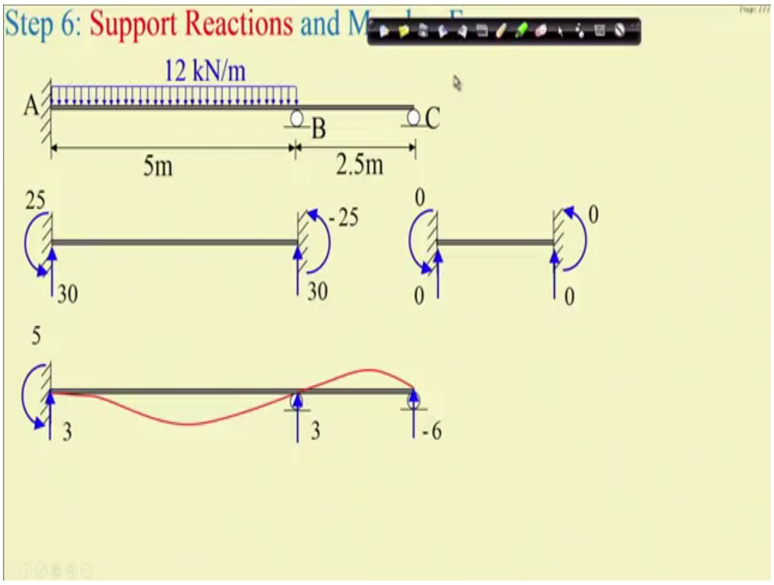

So, now we have just now discussed this.

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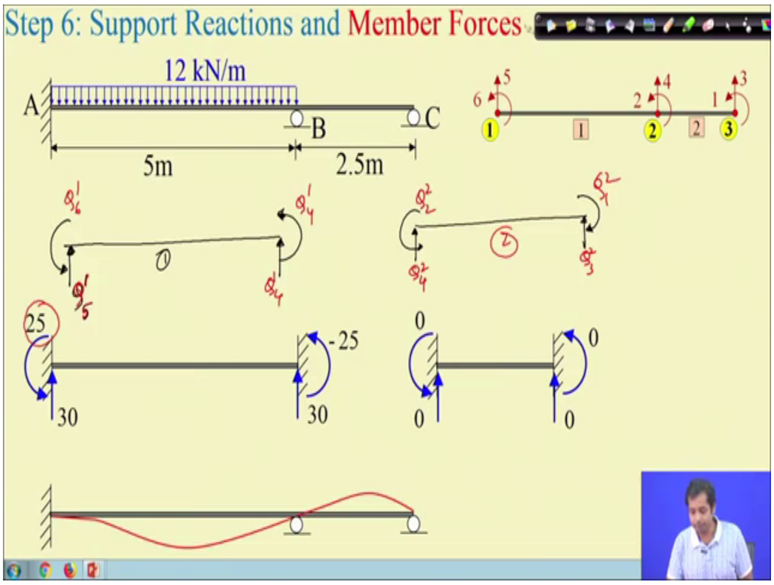

So, let us find out now member forces. Now, this is how the numbering of the members and joints are done. So, we have 2 contributions as I just now said one. So, member forces are what? Member forces is if we take any arbitrary member for say member number 1. This member number 1 this member number 1. So, in this member we have shear force and bending moment no axial force please recall, please remember in these formulation the stiffness matrix while constructing this stiffness matrix we assume there is no axial deformation taking. And therefore, there is no axial force only the response we have is the bending moment and shear force.

So, we have a bending moment here and then we also have a bending moment here and then we have a shear force here and we have a shear force here member forces. So, you see the bending moment and shear force the when I wrote, when I represent this direction there direction is represented at as we considered positive and negative in the in algebraic calculations. It is its bending moment can also be that the sense of bending moment in generally in beam if you recall sense of bending moment is either the bending moment is sagging moment or bending moment is hogging moment right.

Now, but when you numerically add them or do the algebraic manipulations between the moments then the sense that we have to consider is whether it is clockwise or anticlockwise, not sagging and hogging moment.

So, since our anticlockwise is considered as positive that is why this is shown like this. Now, so, suppose let us write it this is member number 1; this is member number 1. So, suppose this force is Q 5 for member number 1 and then this is Q 5 for member number 1 and similarly this is Q 2 this is Q 4 Q 4 for member number 2, for member number 1.

So, this is Q 4 member number 1 this is Q 5 this is Q 6 for member number 1. So, Q 6 is the bending moment Q 4 1 is the bending moment and Q 5 1 and Q 4 1 are the shear forces is. Now as similarly for member 2; member 2 will be we have force the bending moment and shear force can be shown like this and then force and let us call it as this is Q this is Q 4 this is Q 4, but for second member this is Q 2 for second member. Similarly this is Q 3 for second member this is Q 1 for second member.

Now, so, this set is bending moment shear force for member 1 this set is bending moment shear force for member 2. Now what would be Q dash 6? Q dash 6 will be contribution from this plus the contribution in the member in the member forces contribution from the due to the deformation.

Similarly, all these member forces will be contribution from the fixed end action plus the contribution due to the displacement. Contribution from the fixed end actions already we have, let us now find out what is the contribution for displacement.

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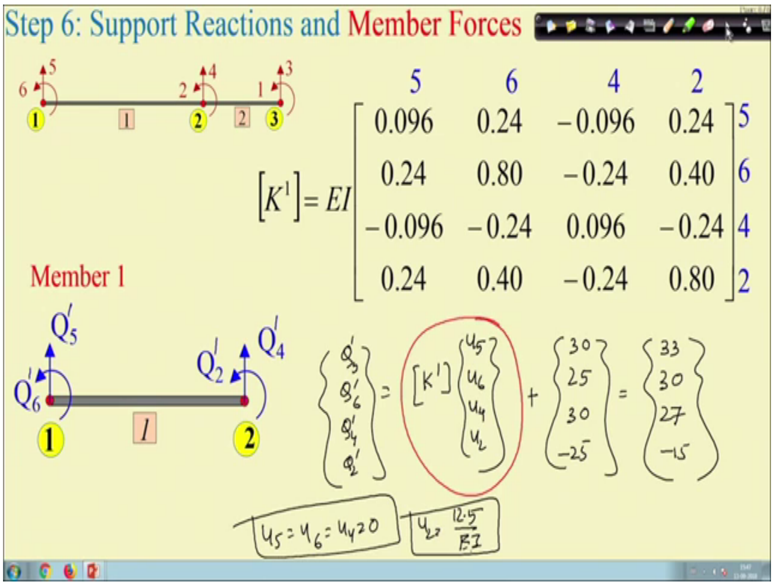

Now, in order to do that what we do is if you recall this is member number 1 and it is this stiffness matrix for member number 1. This is let us this is for 1 this is 1 member number 1. So, this is the corresponding stiffness matrix.

So, what we will have is we have say Q 5 member number 1 and then Q 6 Q 6 and then Q 4 Q 2; Q 4 and (Refer Time: 20:07) Q 2 this entire thing will be this stiffness matrix K 1 into displacement what are the displacement will be u 5 then u 6 u 4 and u 2. So, this is the contribution, this part is the contribution from, this part is the contribution due to the displacement and then in addition to that we have contribution from the fixed end moment right contribution from fixed end actions.

Now, plus contribution from fixed end actions now for this the fixed end actions are 30 25 you can check the fixed end moments and then forces minus 25. Now we already know that u 5 u 6 u 4 they are 0 u 2 is equal to 12.5 by E I. If you substitute that and then these expression the final will be it will be 33 then again 30 then 27 and then minus 15.

So, where u 2 is equal to u 2 is equal to 12.5 by E I and all other u 5 u 5 is equal to u 6 is equal to u 4 they are 0. So, K this matrix is this matrix, these are the degrees of freedom plus this is the contribution from the fixed end action and this will be the member force.

So, this would be the member force for member 1. Now, similarly exercise if you do it for member 2 let us do it member 2 will be.

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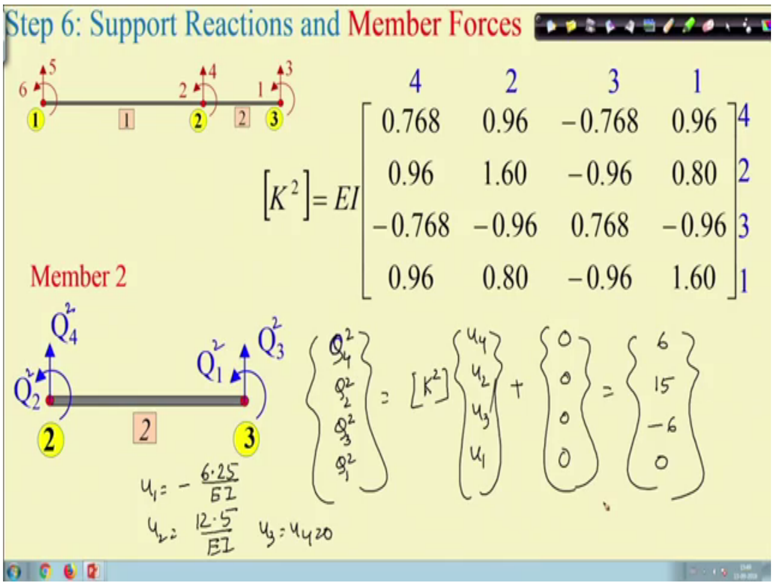

This is for member 2 and then again is you make it for you make it is for member 2, member 2, member 2 this is for member 2 right. And the similarly the member 2 will be it will be Q use Q 4 2 and then Q 2 2.

The order this is order because this is the how the degrees of freedoms are, this is how the degrees of freedoms are arranged and this is how depending on these arrangement your stiffness matrix is constructed and then u 3 Q 3 2 and then Q 1 2 2 stands for second member.

So, it will have contribution for displacement plus contribution from the fixed end action displacement contribution will be same this will be K 2 and then we have u 4 u 2 u 3 and then u 1 and plus the displace contribution from the fixed end moment and if you recall for this there is no there was no load on this pan.

So, fixed end actions where 0 all 0. In this expression we know that u 1 is equal to if you recall minus 6.25 by E I and u 2 is equal to 12.5 by E I right. And all other u 3 and u 4 u 3 is equal to u 4 is equal to 0. Now if we substitute this and K 2 is equal to this fixed end actions is this if we substitute that the values that we get is 6 then 15 minus 6 and 0.

So, this is the fixed end, this is the member forces for member 2. So, once we have the member forces from member 2 and if you recall how you can how you can constructed the bending moment diagram and shear force diagram. We can same way we can follow to contract the bending moment diagram and shear force diagram.

So, all the information we need to construct the bending moment diagram shear force diagram or bending moment to get the bending moment and shear force at a given point we have those information. Now, these information's are obtained through matrix method of structure analysis.

So, you know this is a very brief depending on the problems the dimensions will be different this scale will be different, but the essence of the steps involve in any problems in beam will be exactly the same. The similar exercise you can do it for different problems.

For larger problems there are issues implementation issues there are something to be we have to we have to consider we have to while the number giving the numbers and all. As I said earlier this will be discussed in detail in week seven when you talk about various implementations implementation aspects. So, before we before we close you see suppose your flexural rigidity is not constant with the same problem.

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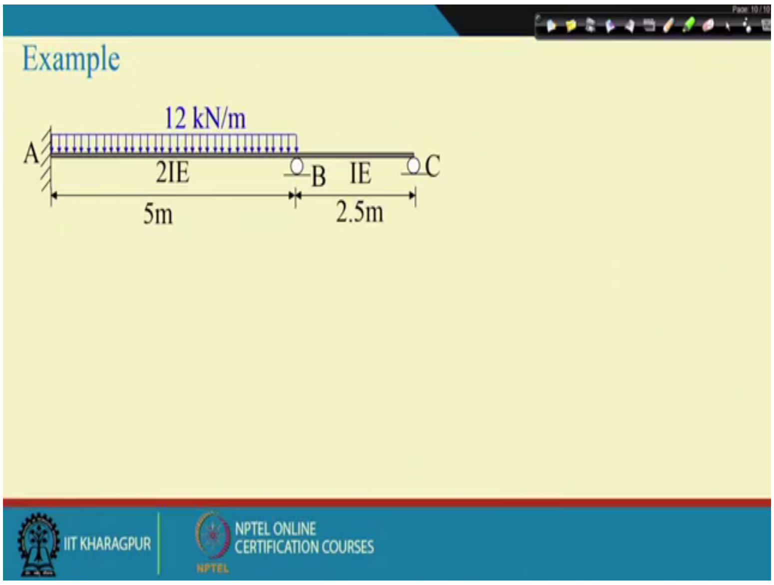

But if I in in the previous in the previous example the E I is constant for all the members right, but suppose E I is not constant in one member it is E I or you may have a you may have something like varying E I. So, you are either you are either you are geometry is continuously changing along the length you may different kinds of such geometry.

So, in this case suppose for the first it is E E I is it is 2 E I and it is E I 2 E I either they are young's modulus is the more or its shape is different. So, that the second moment of area is twice the second moment of area in the first the second one there may be different reason, but the total effect, net effect is suppose the E I is 2 E I here and it is E I here. So, the thing is exactly the same thing there is no difference at all.

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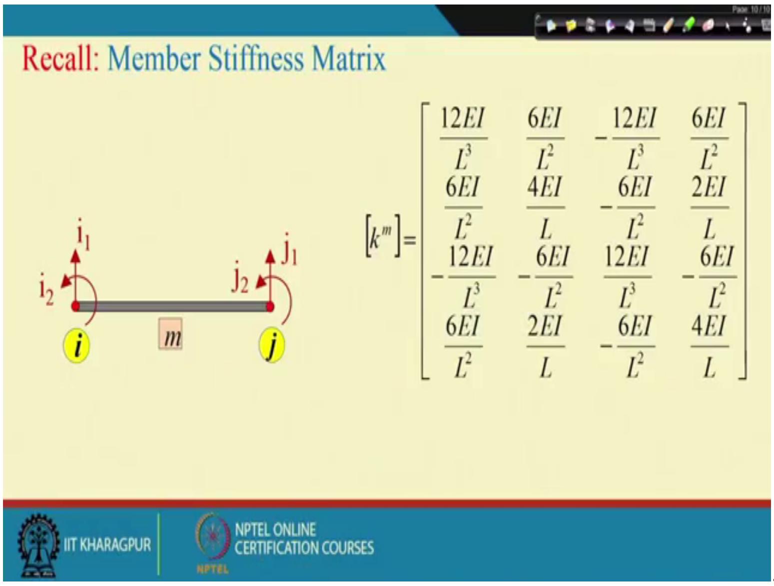

Only thing is you recall this is this is for any arbitrary member this is the stiffness matrix. And in this stiffness matrix these are the in this stiffness matrix these one where ever you have L this L is the length of this member length of this member and E I is the youngs modulus E I is the E I flexural rigidity of this member. For some member if you find it is 2 E I then instead of E I it will be 2 E I instead of all E I is it will be 2 E I. So, whatever is E I of that particular material you have to substitute that value here to get the members stiffness matrix for that particular segment.

So, then rest of the thing is exactly same once you have the stiffness matrix for all the members then assemble them to get the global stiffness matrix apply the boundary conditions, partition the matrices and then solve for unknown displacements. Once you have unknown displacements and then next you calculate the joint reactions joint yes joint reactions and member forces, but when you calculate joint reaction and member forces there will be two contribution, one contribution from the fixed end action another contribution due to the deformation of the deformation of the beam.

So, this is the method matrix method of structural analysis for beam and it is demonstrated through an example I would say very simple simpler example because in class room demonstration we had to take an example where the matrix size is small degrees of freedoms are less, unknowns are less.

So, there we can solve them here, but same thing you can, but the same exercise what we have done manually same exercise you can quote write a program and apply that for a large structure where we have many members and many joints, but the Essence of steps will remain same.

So, I stop at today, next week we will start the matrix method of structural analysis for frames for plane frame. You see you recall in truss problem the behavior of the trusses every member is subjected to only actual force either compression or tension. There is no bending, there is no shear in case of beam we have bending moment and shear force we assume there is no axial force.

Now, in case of frame we have to just combine this 2 we have a truss where only axial force we have a beam where bending moment and shear force in frame we have we have all we have axial forces we have axial force we have bending moment and we have shear forces.

So, three forces we have to consider therefore, for and then, but while constructing the degrees of constructing the stiffness matrix we really do not have to do all the exercise that we have done for truss and beam problem because we have to just slightly reorient these 2 stiffness matrices 1 for truss and 1 for beam we have to combine this 2 stiffness matrices to get this stiffness matrix for frame element. That exercise we will do in the next week I stop here today see you in the next week.

Thank you.