Matrix Method of Structural Analysis
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Lecture – 23
Matrix Method of Analysis: Beams (Contd.)

Hello everyone this is the third lecture of this week. We have been discussing the various steps involved in analysis of beams through matrix method of structural analysis. And if you recall the, these are the different methods, these are the different steps involved in matrix method of structural analysis and these steps we demonstrated in the context of truss problem, same thing we have been doing for beam.

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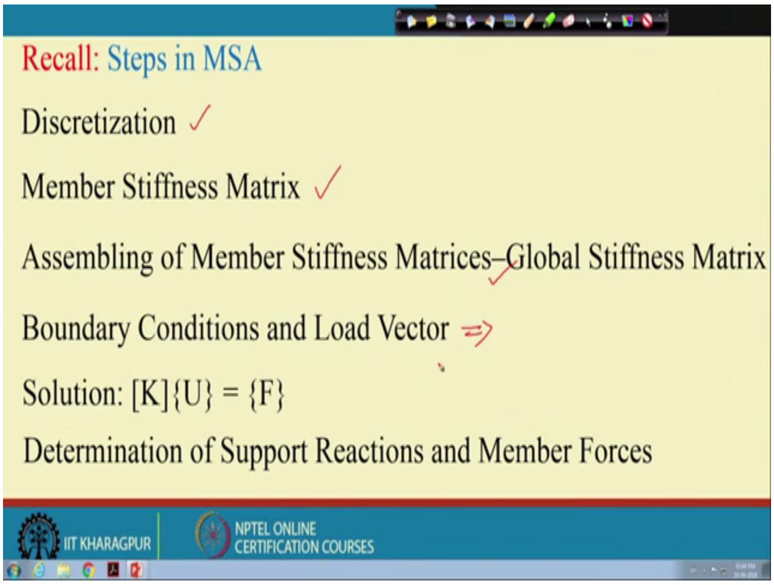

Now, we already discussed what is, what is how to discretize a beam and how to give the numbering. We also discussed what is the stiffness matrix for different members and then we discussed how to assemble these stiffness matrices to get the global stiffness matrix. Today, we will see how to apply the boundary conditions and then how to construct the load vector that is today's topic. And rest of the solution of these; solution of these the this system of linear equations and then determination of support reactions. We will not discuss these steps. We will demonstrate directly through an example in the subsequent classes.

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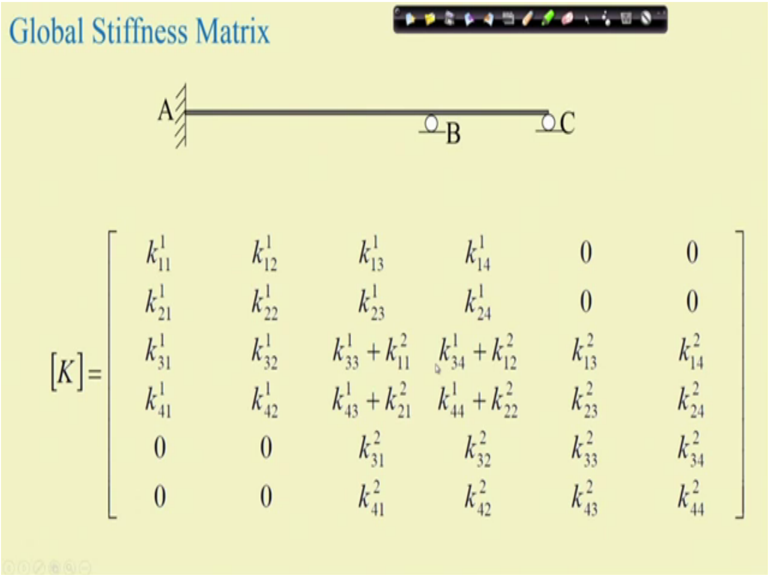

So, if you recall this is the problem we have chosen to demonstrate these steps and remember one thing is very important when we, our objective here to demonstrate the step that is why. Let us not right now bother about how to how to number different members, how to number different degrees of freedom, we will discuss that when we actually demonstrate these steps, when we actually find out the solution of that problem.

So, we have seen these global stiffness matrix of this beam is like this, it is a 6 by 6 matrix because we have total 6 degrees of freedom.

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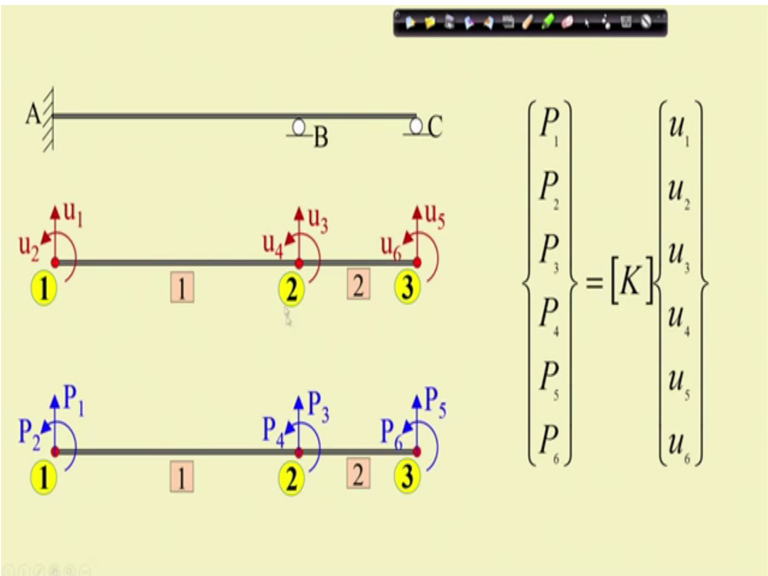

Now, so these are, this is how we have chose we have given the numbering of the degrees of freedom, but again let me repeat that once again. This is not always you have to choose the numbering like this, like we have given node number 1 degrees of freedom is u 1 u 2, node number 2 it is u 3 u 4 and node 3 it is u 5 u 6.

You can have u 1 u 2 for node number 2, you can have say u 3 u 6 at node number 1 and so on. So, how depending on this structure, depending on the unknown displacement unknown degrees of freedom how to give this numbering that we will discuss when we actually find out the solution, but here our objective is not to discuss the number input, here our objective is to understand various steps involved in this analysis method.

So, for instance if you have to the; if you have to the bound these are the displaced, these are the displacement and these are the load vectors and these displacement the load vectors are related to each other through this example; through this equation. Where K is the stiffness matrix, the global stiffness matrix and then this is the global stiffness matrix and P is the load vector, global load vector 6 components and U is the global displacement this.

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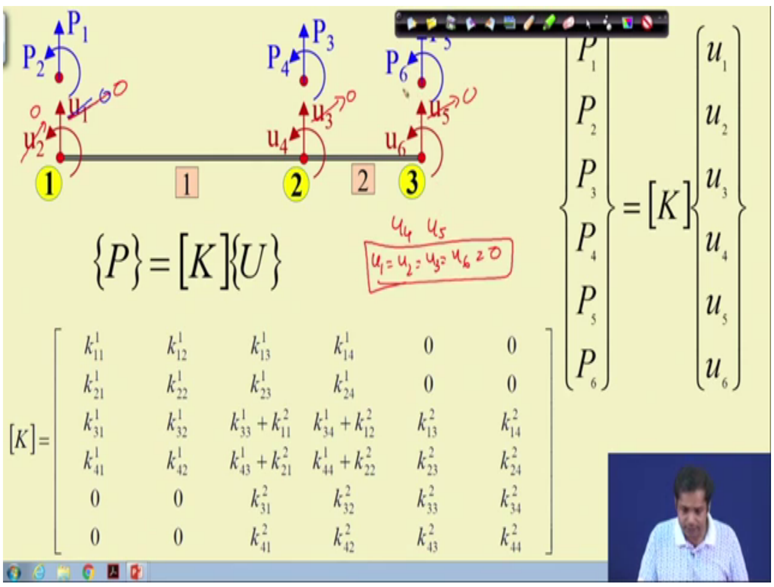

Now, next what we do is if we try to find out the; if you try to inverse this stiffness matrix you will see this is not invertible. This matrix is singular matrix and the physical manifestation of that singular matrix that is the structure is not stable. Structure is not stable, because we have not provided sufficient constraint to the structure, sufficient support to the structure and now we have to once we provide the sufficient support to the structure the structure becomes stable, but that information has to be translated in this equation and that information need to be translated in this equation through applying the boundary conditions.

Now, what are the boundary conditions we have here? For instance in this case if we take in this case we have all u 1 u 2 this is a fixed end. So, this is (Refer Time: 04:24) this is 0. So, this is take different color, so this is 0, then this is also 0 and then u 3 in this case this is 0 and then u 5 is also 0. So, only unknown displacements in this case u 4 and u 5. If we number it like this then u 4 and u 5 are the unknown displacement and u 1 u 2 u 3 and u 6 all are known displacement means they are 0 displacement.

You may not have always 0 displacement, you can suppose in the case of support settlement, you know the how much support is settled. So, if you know the displacement some known value of displacement you are providing to the structure then u 6 may not be 0 u that an u may not be 0 at that point u may be the specified displacement at that point, but in this case the that displacement is 0. So, that is the boundary condition.

Now, once we have the boundary conditions let us see how to construct the load vector. So, this is the boundary condition we already discussed. Let us see now nodal load vector, in order to calculate the nodal load vector we have to recall some of the things that we discussed, that we have already learnt in your structural analysis 1 analysis of indeterminate structure using displacement method specifically slope deflection method. And there u let us that information, that concept is required to construct the load vector.

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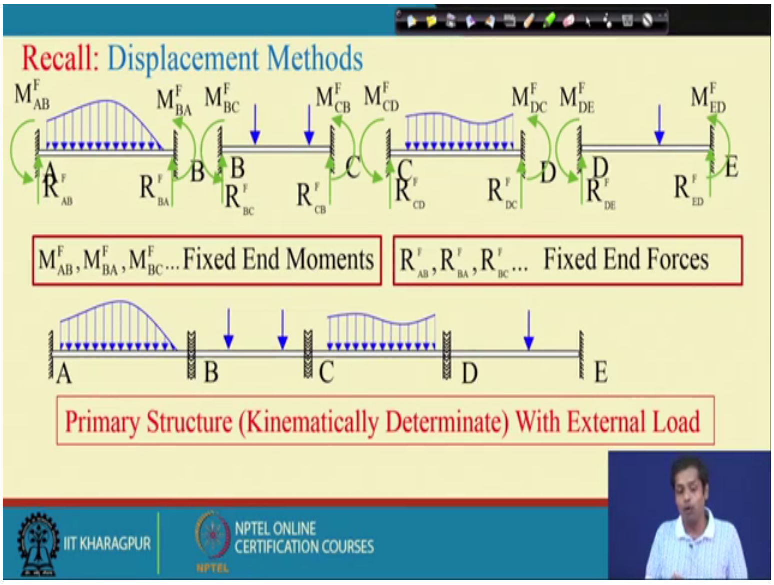

Let us recall that suppose you have a structure like this and then what we did in displacement method. First we assumed all these degrees of freedoms are constrained. There is no disk no degrees of freedom in this structure so, we make this structure Kinematically determinate.

So, this is the structure which is Kinematically determinate with external node, then what we did is, you we have we had taken the each segment separately. Suppose this segment separately, we took. Now all these beams separately, now they all are fixed beam, so they are fixed beam and subjected to the external load then we can then solve these using any method, we can solve this fixed beam separately and we can find out the moments at the support and those moments are called fixed end moments.

Now, so these moments are the fixed end moments. Similarly if you calculate the moments for B C, this will be the fixed end moments. As per our sign convention anti clockwise rotation and anti clockwise moment are considered as positive that is why in this fixed end moments the anti clockwise directions are shown as positive. Similarly you can have moments here, you can have these are all fixed end moments.

So, M AB is the fixed end moments in AB at end A. Similarly M F BA is the fixed end moment in segment BA at point B. Similarly M D is a fixed end moment in D at point D, F stands for fixed end moments. Similarly you can have fixed end reactions, reactions means in this case the vertical forces. These are the vertical forces.

So, these are called fixed end moments and these are called fixed end forces and together this M and R they are called fixed end actions. Now, once we have the fixed end actions what you did in the displacement method.

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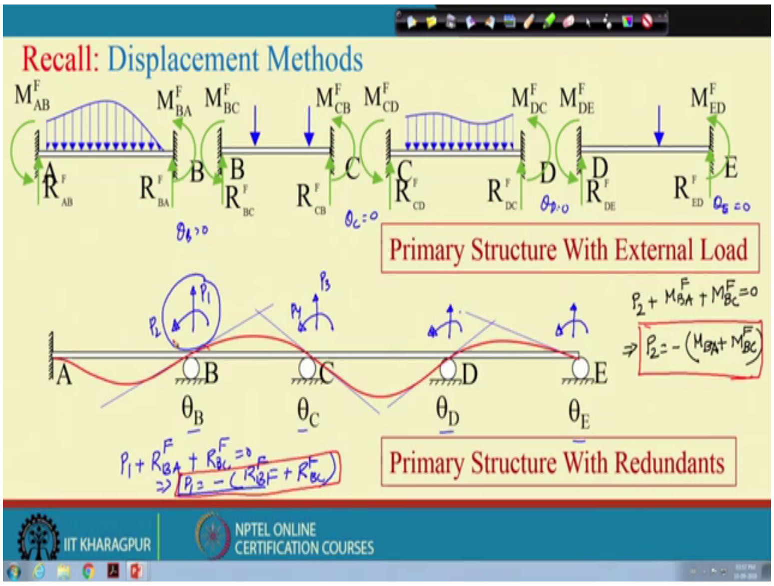

Now, in the next if you recall so this is a primary structure with externally external loading. Primary structures are statically Kinematically determinate structure, all the degrees of freedoms are constrained, but then in actual structure it deforms right and suppose this structure deforms like this.

So, in this cases; in this case in this in all these case you assume that theta B is equal to 0 then theta C is equal to 0 theta D is equal to 0 and so on, but then in actual structure we cannot have theta B, theta C, theta D theta E they are is equal to 0. They have some nonzero values. Now, if you assume that this red line is the deformed shape of this structure where you allow the deformation you allow these degrees of freedom then this is called, this is this structure is called primary structure with redundant.

Now, then what you did is, then now in order to get this deformation these red deformation (Refer Time: 09:08) you need certain forces right, but this structure there is no force the all the forces are with all the external load is only on the primary structure. Now in order to create this deformation what you have to do is you have to apply some force right, say for instance you have to apply some force and then some force like this; some forces like this.

Then the equivalent joint load see in the case of; in case of truss what happened is every we always apply load at the joints right. We do not apply load on the members because all that is how the truss needs to be idealized, but then in the case of beam we not only have force on the joint we can have also force on the member, but when you solve equation K u is equal to F, P is equal to K u that equation is define at the node. You get the nodal displacements hence therefore, you need the load that is specified at the node.

Therefore all the member load needs to be translated at the only we projected at the node and this projected load is called the equivalent joint load. Why it is called equivalent joint load because whatever effect you have that effect should be same. Effect should be same means the structure subjected to the external load the way the structure deform, the equivalent joint load gives you the same amount of deformation at the joint. That is why it is called equivalent joint load, but in this case the forces are applied at the joint and suppose these are called the; these are the equivalent joint load.

Now, the equivalent joint load since their effect will be, it will be same then it says that they that their equilibrium the these four sets and these fixed end reaction should equilibrate each other that is how you did in displacement method slope deflection method.

So, if they have to equilibrate each other then suppose this is P 1, this is P 1, this is P 2 and this is P 3 and this is P 4 then equilibrium says that P 1 P 1 plus R B A F plus R B C F that is equal to 0 and this gives you P 1 is equal to minus R B F plus R B C F fixed fix.

So, this is equivalent joint force at node number 1. Similarly if you moment equilibrium if you satisfy then the moment equilibrium will be P 2 plus then we have anti clockwise moment M B A F plus then we have M B C F that is equal to 0 and this gives you P 2 is equal to minus M B A plus M B C F.

So, this is the equivalent joint load at; equivalent joint moment at 2. So, P 1 and P 2 are the equivalent joint load at node number B. Similarly if you satisfy the equilibrium under joint we get the corresponding equivalent joint load, but remember these equivalent joint loads are not the support reactions. Support reactions are the equivalent joint load whatever you have, plus the because of this, because of this deformation there will be some contribution to the reactions and the total that reaction will be the effect of deformation plus this fixed end actions.

Now, next is so this is how we construct the equivalent joint load right. Now, probably if you still have some confusion when you actually demonstrate them through an example this confusion will be clear. So, we discussed that what is equivalent joint load.

Now, once we have the equivalent joint load for these problems say P 1 P these are the equivalent joint loads and they are the support displacement then we have and this is the this is how this load vector and displacements are related to each other.

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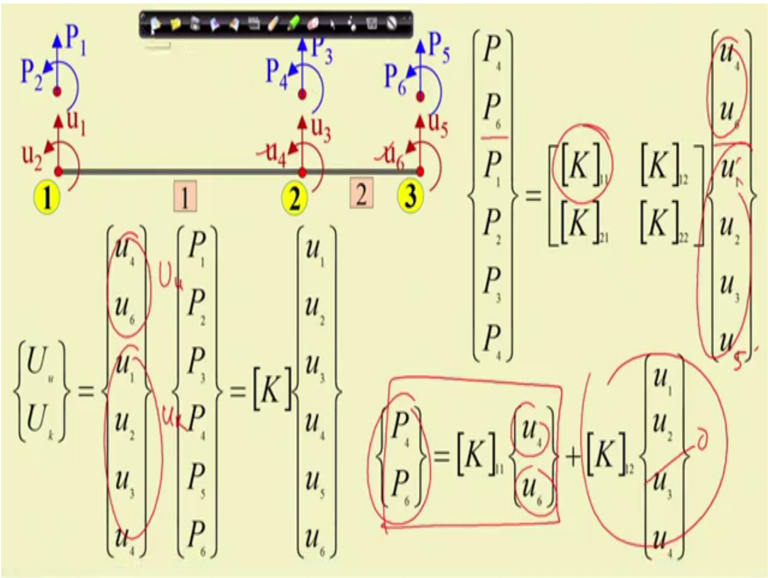

Now, then you see, this entire thing these entire load vector in this case can be rearranged in such a way that all the known displacements are kept one side and all the unknown displacements are kept one side.

For instance here in this case these are all unknown displacement and these are all known displacement. Suppose this is written as U unknown and this is written as U known displacement. Then what we can do is, so these are all known these are all unknown displacement and these are all known displacement.

Now, so yours force displacement relation was initially like this. Now if you rearrange them taking all the unknown displacement at one side and the known displacement at another side, I mean together then this rearrangement will be something like this. So, these are all known, this unknown. So, these are our unknown displacement, this side all unknown displacement and known displacement.

Now, when you rearrange this load vector and then displacement vector corresponding rearrangement needs to be done in the in this stiffness matrix and these exercise is called partitioning of this stiffness matrix. It is to be partition in such a way that known displacement and unknown displacements are taken separately.

Now, now once we partition the stiffness matrix, now look at this, look at this stiffness matrix look at this expression, from this expression we can write just from this the first equation, if we write then this equation become this.

Now, this is, in this case this is 0. So, entire part become 0 K 11 is the this part of this stiffness matrix P 4 P 6 are the corresponding nodal joint, equivalent joint load these and these we have already know how to; we already know how to determine them and then if we solve them then we get u 4 and u 6.

Now, once we solve it for u 4 and u 6 then u 1, u 2, u 3 if all other all other this is u 5, all other displacements are 0 so, we have information about the all displacements. Once we have the information about all the displacement the next step is to find out the support reactions and then other information about the beam behavior of the beam that we are interested in.

So, but that exercise how to find out support reactions and how to find out support reactions from this and then how to find out other information about the behavior of the beam we will not discuss through steps. Let us start an example in the next class and demonstrate all the steps that we discuss in first 3 classes and some of the confusion if you still have I believe that those confusion will be clear when we see some example.

So, we will have two example in this course, one example is the same example we will consider in the next class and then we try to, we will find out the solution of this example and then we will have one more example in the subsequent classes. I stop here today see you in the next class.

Thank you.