**Matrix Method of Structural Analysis**
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**Lecture – 22**
**Matrix Method of Analysis: Beams (Contd.)**

Hello everyone. This is the second class of this week. In the last class, we discussed how to construct element stiffness matrix for beam member. And then today, we will see if for a given problem several members we have, and then we have constructed the stiffness matrix for different members, then how to assemble them to get the global stiffness matrix that is the topic of today's lecture. So, today we will discuss the assembling of member stiffness matrices.

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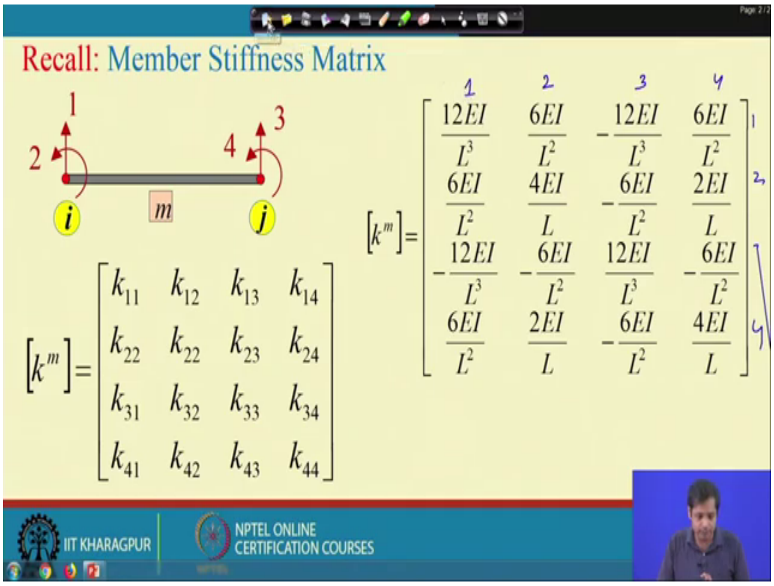


Now, just before that recall this we discussed in the last class. If we have a member, connecting connected between two point, two nodes i and j, and the degrees of freedom at i'th node is 1, 2, and degrees of freedom that jth node is 3, 4, then this is the stiffness matrix for this member general stiffness matrix for this member.

Now, what we have is ~~what we have is~~ this is if we recall, this is your node number 1 this is for node number 1, this is for 2, this is for 3, and this is for 4 right. And then and this similarly this is for 1, this is for 2, this is for 3, and this is for 4. So, and corresponding we can write this stiffness as stiffness as k 1 1 k 1 2 k 1 3 and k 1 4. So, um and we already discussed what is the physical significance of this k 1 1, k 1 2 ok.

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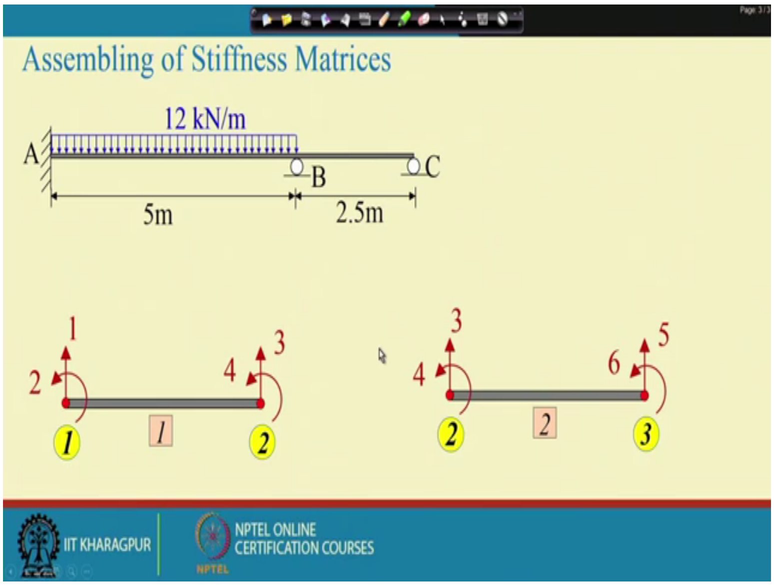


So, let us move on see how to do the assembling. Take an example first, suppose this is the example we take. Now, we are not going to solve this example today. Instead what we do is through this example for the given problem through this ~~example,~~example; we will try to understand how to do the assembling. We have done this for we have done this for truss problem. And we just discussed the same exercise for beam problem.

So, first but once we ~~once we~~ discuss a discuss everything towards the end of this week, we will take up this example, and then and then see how to solve them ok. So, the first thing is we have to discretize it once we have to discretize it like this, so we have two members now, member number 1, and member number 2. Then we have the degrees of freedom is member number 1 is connected between 1 2, and then 2 3 for member number 2. And this is how the degrees of freedom are defined for different nodes ok.

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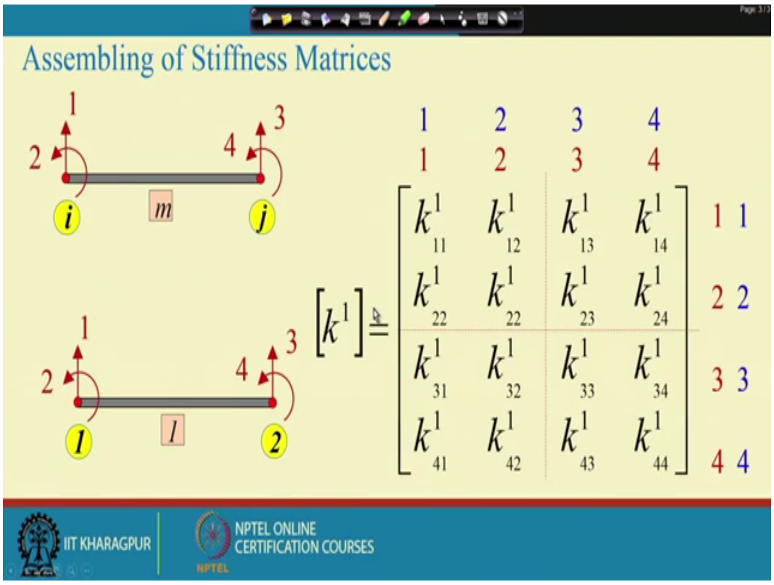


Now, so if we take member number 1, the corresponding member number 1 the stiffness matrix suppose the stiffness matrix is this, this super script 1 a stands for it implies that it is the stiffness matrix for 1 member number 1. Now, this is important this numbering is important. you see here two figures are there. This is figure number 1 and figure number this is figure number 1 figure number 1, and figure number 2. What is figure number 1 and 2? You see figure number 1 is the ~~is the~~ definition through which we have derived the expression for stiffness matrix ok.

Now, and in that definition we have we have given we identified the ith point degrees of freedom is 1 and 2, and jth point degrees of freedom is 3 and 4.~~;~~ ~~but~~ But you know for different, but when you when the member if we when we take a member from the actual structure, their actual degrees of freedom may not 1 2 and 3 4, it may be different ok.

So, but in this expression so the red numbering, here you can see on this stiffness matrix red numbering 1, 2, 3, 4 is it is the this how this stiffness matrix is constructed. And then the blue one is how this stiffness matrix with respect to the global numbering ok.

Now, for the member number 1 the global numbering and this idealize numbering, they are same. And therefore, this two are same. It will be and when we when we ~~when we~~ see the next member number 2, this will be more clear. So, this is the stiffness matrix, so member number 1. We are not writing the expression for this stiffness matrix like this x we are not writing this expression, the reason is we already know this expression. So, therefore once we have the final global stiffness matrix in terms of k 1 1, k 1 2, and so on. We can substitute this expression for a different members ok.

So, now let us to let us do it for 2nd problem. The 2nd problem this is the 2nd member this is the member number 1, and this is member number 2, now this is the difference. Red one it is the numbering of the nodes, while in the actual idealize system, but when but blue one is the global numbering. You see member number 2 was between node number 2 and 3 ok.

So, node number 2 is i in this case, so node number 2 is i, and node number 3 is j ok. Now, in the global degrees of (Refer Time: 05:23) as per the global definition, the 2nd node the degrees of freedom we defined as degrees of freedom 3 4, and the 3rd node degrees of freedom 5 6 ok.

Now, so but in the local ~~when we~~ when we write this stiffness matrix, in this form;~~,~~ our degrees of freedom is 1 2 here;~~,~~ 3 4 here. So, with the global degrees of freedom 3 4 is essentially, local degrees of freedom 1 2; global degrees of five freedom 5 6 essentially, local degrees of freedom 3 4 ok.

So, essentially just to make it simple,~~.~~ ~~If~~ if we the red one red numbering is with respect to figure number 1, and the in the global sense when we actually assemble them, the corresponding these will be corresponding to degrees of freedom 3 4. Similarly, these will be corresponding to degrees of freedom 5 6 ok. Now, once we have these stiffness matrices for both the members, next is to assemble them.

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Now, the reason why we as I said just now say the reason why we choose problem with two span, because we can do the assembling on the screen, but ~~for a~~ for a problem where we have many spans, and the many nodes, many members, and many nodes, and these assemblance we have to do using a computer code ok.

So, this is the degrees of this is the corresponding now this is the stiffness matrices written in terms of the it is identified corresponding rows and columns are x identified with respect to the global numbering ok.

Now, so as you as you recall the first step of this stiffness it is evident that we have total 6 degrees of freedom. So, essentially your this stiffness matrix size of the stiffness matrix will be six by size of stiffness matrix will be 6 by 6 ok.

Now, so first is so initialize we have to take a 6 by 6 matrix, all the elements of this matrix is 0. Now, next depending on the, their position the connectivity we have to populate this matrix, we have to we have to replace 0 with their corresponding values. Now, first let us check let us check with let us check this, now this is this let us check block wise first to make it clear (Refer Time: 07:50) remove all this zero's, so that yes so now all this are initial value is 0 now.

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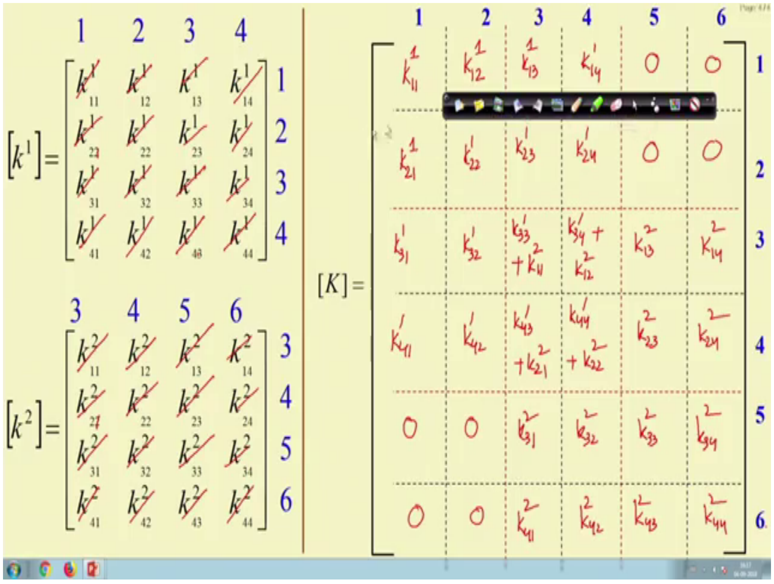


So, this is for a 1 1 right, this is for 1, this is for 1. Now, this is for 1 and this is for 1, there is no 1 1 here. So, straight away, it will be k 1 1 1 this value, so this value is already given. Then we have k 1 2 1 1 row and 2nd column 1, and then corresponding will be 2 there is no here. So, this is k 1 2 1.

So, we have taken this. Then k 3 4 k 1 3 4, 1 and then 3 is this there is no 1 3, so it will be k 1 3 1. And similarly, this will be k 1 4 1, so this is this is we have already substituted this ~~value,~~value; we have included this value in the global one stiffness matrix. Then k then 1 5 6, there is no 1 5 6 here there is 1 5 6 here, so this will be 0 and 0. This was for the 1st column, 1st row.

Let us see the 2nd row 2nd row, the 1st column is 2 1, we have 2 1, this is 2 1 though this will be k 2 1 k 2 1 1 this gone, it will be 1 ok. Then k 2 2 k 2 2 will be this k 2 2 1, this is gone. Then we have 2 k then 2 3, 2 3 there is no other 2 3, so this will be this will be k k 2 3 1, and similarly k 2 4 1, so this is gone, this is gone. So, this will be 0, and this will be 0 right. So, this is for 2nd row.

Now, let us do it for 3rd row 3rd 3 1. We have 3 and then 1, this will be 3 1 this is 3 1 this will be k 3 1 1. Then 3 2 3 2, there is no 3 2 here, 3 2 will be this value k 3 2 1. Then k 3 3 here we have k 3 3 is this, and then we have k 3 3 here is this. So, k 3 3 will be this plus this.

So, this will be k 3 3 1, this will be k 3 3 1 plus this will be k 1 1 k 3 3 k k 1 1 2 ok, so this two. Then k then 3 4 we have 3 4 here, and we also have 3 4 here. So, 3 4 here is this, and then 3 4 here is this. So, this will be k 3 4 1 plus k 1 2 2. k 3 4 1 plus k 1 2 k 1 2 2.

So, this is for this. Then 3 5, we do not have any 3 5 here, we have 3 5 is this one, and 3 6 is this one. So, this become k 1 3 2 this becomes k 1 3 2, and this becomes k 1 4 k 1 4 2. So, this is for the 3rd row.

Do it for this 4th row. 4th one is this 4 1, there is no yes there is 4 1 is here. 4 then we have 1, this is k 4 1 we do not have 4 1 here. So, it will be k 4 1 1, and then k 4 this gone. Then k 4 2 1 is this k 4 2 1 is this. k 4 3 k 4 3 what we have k 4, and then k 3 here also we have k 4 and k 3. k 4 3 is this one here, and k 4 3 is this one here.

So, this will be k 4 3 1 k 4 3 1 plus 4 3 k 2 2 2, then it will be k 4 3 1 k 2 1 k 2 1 2, this will be 1 ok.

Now, let us do it for then k 4 4 k 4 4 is this one is k 4 4, and then k 4 4 is this one. So, k 4 4 1 k 4 4 1 small k 4 4 1, and this case it will be k 2 2 2 k 2 2 2. This is for k 4 4. k 4 5 there is no k 4 5 here, k 4 5 we have this is k 4 5 k 2 3 2 k 2 3 2, and k 4 6 will be k 2 4 2 k 2 4 2. This is for k this is for the 4th column, 4th row.

Now, similarly let us do it for let us do it for I can put it here yes that is better let us do it for 5th 5th column 5th row. So, 5 1 there is no 5 1 here there is no 5 here, 5 1 straight away will become 0. 5 2 there is no 5 2 5 2 straight away, we can have 0. when we have 5 3 5 and then 5 3 is k 3 1 2 k 3 1 2. Then 5 4 5 4 will be k 3 2 2 k 3 2 2. And then 5 5 will be 5 5 will be k 3 3 2 k 3 3 2, and finally 5 6 will be k 3 4 2 k 3 4 2.

And then last last row 6 6 1 there is no 6 1 here 0, there is no 6 2 here 0. And then 6 3 6 3 is this k 4 1 2 k 4 1 2. 6 3 6 4 will be k 4 2 2 k 4 2 2. 6 5 is k 4 3 2 k 4 3 2. And 6 6 final is k 4 4 2. So, we have taken all these elements and substituted here. So, this is now our global stiffness matrix ok.

Now, look at global stiffness matrix is symmetry or not, it is symmetric k 1 2 this k 1 2, 1 and k 2 1 k 1 2 1 k 2 1 these are ~~stiffnesss~~stiffness. These all the~~-~~, these stiffness matrices member stiffness matrices are symmetric, and then global stiffness matrix is also symmetric. And but, but you know when we when you write the computer code or when you do this assembling manually, you you start with writing all the though you know that this is a symmetric matrix.

Do not take it for granted for the time being when I say do not take it for granted, what I what I want to mean is when you write this stiffness matrix write each component separately, and then check whether this stiffness matrix is symmetric or not that would be a check whether you are assembling is done properly or not ok. Now, so this is the global stiffness matrix.

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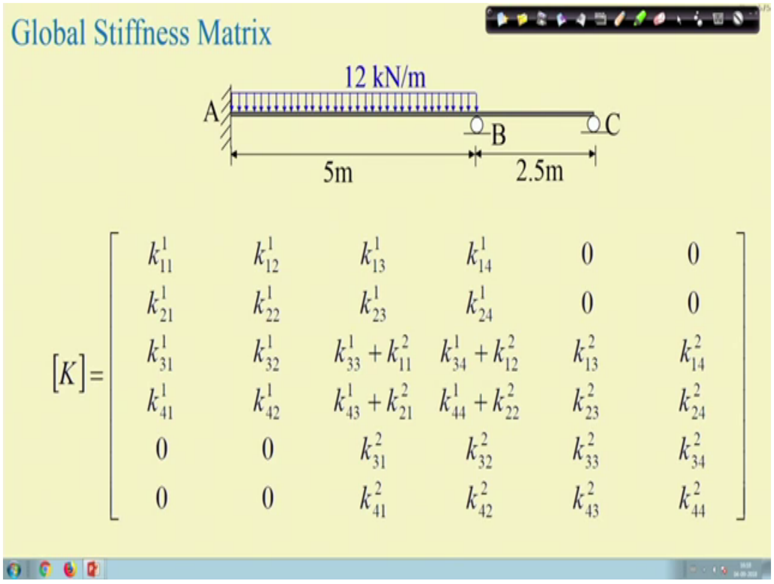


Now, let us let us let us write it let us see the final form of the global stiffness matrix, so for this member the global stiffness matrix is this. Remember, this is the stiffness matrix written in terms of k 1 2, k 2 3, and so on. But, we know what is the expression of different components in terms of length, in terms of flexural rigidity, we will do that.

We will substitute those values here, and get the global stiffness matrix. When we actually solve this problem, but as I said here our ~~our our our~~ purpose our objective has not been to solve this example, our ~~our~~ objective has been to see how the assembling is to be done. Once we have understood that, how the assembling is to be done. The next when we actually solve them, we will write all these values in this assemble stiffness matrix, and then solve it.

Now, before for the solution we have done the first step discretization, second step is member stiffness matrix, third step assembling this stiffness matrix, which give you global stiffness matrix, then the fourth step required is the boundary conditions and the load vector. This stiffness matrix will be this stiffness matrix will be the determinant of this stiffness matrix will be 0, you substitute all this value, and find out the determinant you will see this is 0.

The next is next step is to apply the construct the load vector, and apply the boundary conditions that will be topic of the next week next class lectures. So, next class we will see how to enforce boundary condition, and how to construct load vector. So, I stop here today; see you in the next class.

Thank you.