Matrix Method of Structural Analysis
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Lecture – 20
Matrix Method of Analysis of Trusses (Contd.)

Hello everyone. This is the last lecture of this week.

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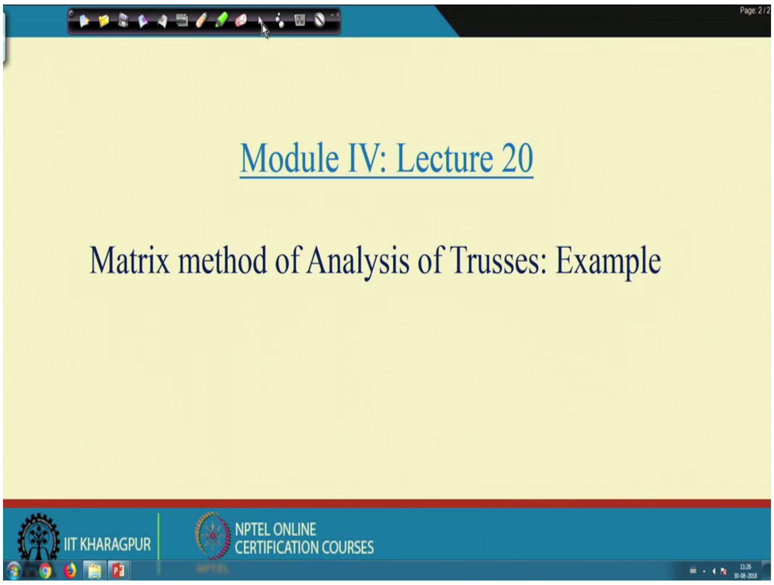

What we have done so far is we discussed what is the underlined philosophy of Matrix Method of Structure Analysis and in this week we demonstrated that philosophy through an example of truss. Now here what we have seen is suppose a truss is given to us, then we have to first divide the truss into its elements into its members. And then write the force displacement relation for each member; when you write force displacement relation we need the stiffness of each member.

So, the first is to construct these stiffness matrices, for each member which you called element stiffness matrices. Now once we have the element stiffness matrices for all the members, then you have to assemble them, all the matrices to get the global stiffness matrix of the entire structure. Now once we have the global stiffness matrix, then apply the boundary conditions. Whatever information we have about the problem in terms of boundary condition, in terms of nodal loads we have to substitute that information in that global force displacement relation and then calculate the unknown displacements and subsequently the member forces.

So, we have demonstrated this, in the last 4 classes we have demonstrated this steps through an example. Today, we will just do the similar exercise summarize the entire steps through one more example, the last example that we took is were statically determinate truss, here we take an example for statically indeterminate truss. And summarize all these steps through solution of that indeterminate truss. So, today's topic is again the matrix method of analysis of truss with one more example.

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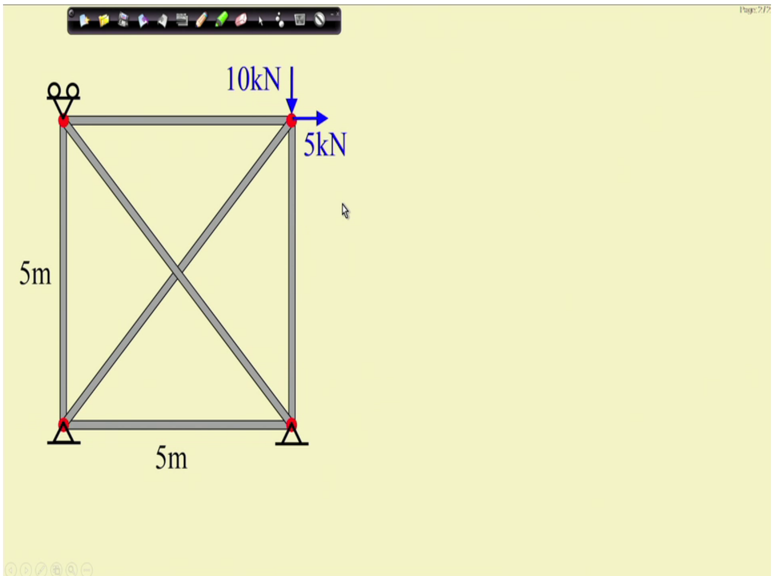

So, let us take these example. So, you can say that your number of number of unknowns we have in this case is 2 hinge joints and then one roller joints, 2 hind joints gives 4 reactions and one roller joint gives another one. So, total 5 reactions we have. And then we have 6 members in the beam. So, 5 plus 6; 11 unknown we have and the number of joints we have 4. So, 4 joints give us 2 into 4; 8 equations; so, 8 equation, 11 unknown. So, this is a indeterminate truss.

Now, please note that this is these 2 members, this member and these 2 diagonal members they are not connected with each other that the middle this is not the connection. So, it has only these 4 joints. Now let us try to analyze this structure through matrix method of structure analysis. If you recall the first thing is we have to in order to identify this notes and the elements, we have to give them some numbers. So, first remove all the support and the forces on the member.

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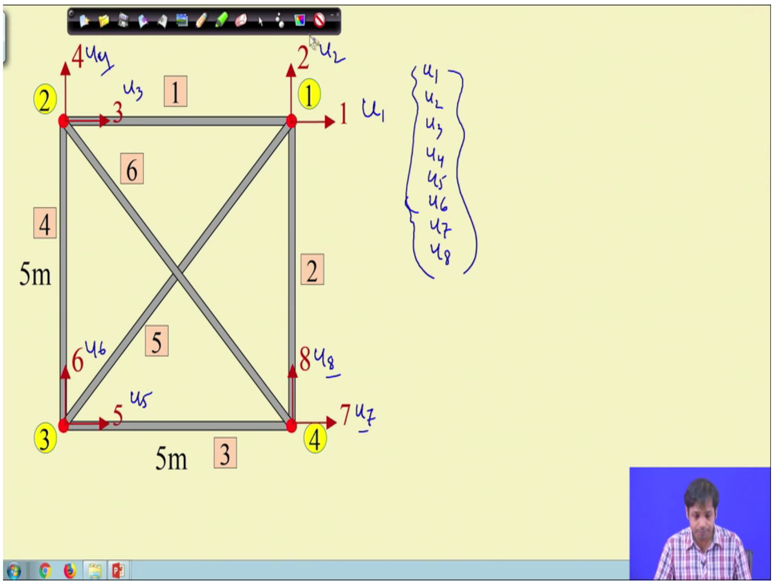

And then, these are the numbers that we give for the members. So, a 6 members, in this case numbering is given as 1, 2, 3, 4, 5 and 6; 5, 6 are the diagonal members.

Now, similar to the members we have t o give numbering for the nodes. So, these are the node numbers; 1, 2, 3, 4. Now at every node we have 2 degrees of freedom. So, degrees of freedoms are also need to be numbered. For instance, suppose node 1, we have 2 degrees of freedom; one is horizontal displacement and the vertical displacement. See horizontal displacement is displacement u 1 and this is u 2.

Similarly, node number 2, horizontal displacement is u 3 and u 4. And node number 3, horizontal displacement is u 5 and then u 6. And then last node number 4, horizontal displacement is u 7 and u 8 vertical displacement. So, total 8 displacements we have and those displacements are can be written in a vector form as u 1, u 2, u 3 u 4, u 5, u 6, u 7, u 8. These are 4 8 displacement.

Then out of 8 displacement we can say that u 5, u 6, u 7, u 8 and u 4 they are 0, we will come to this point when we enforce the boundary conditions. It is just the numbering of the degrees of freedoms, nodes and the members.

Now, once we have numbered all member nodes and associated degrees of freedom, now next what we have to do is, we have to know break all these we have to take every member separately, all members separately and write down their stiffness, the expression for the stiffness matrix.

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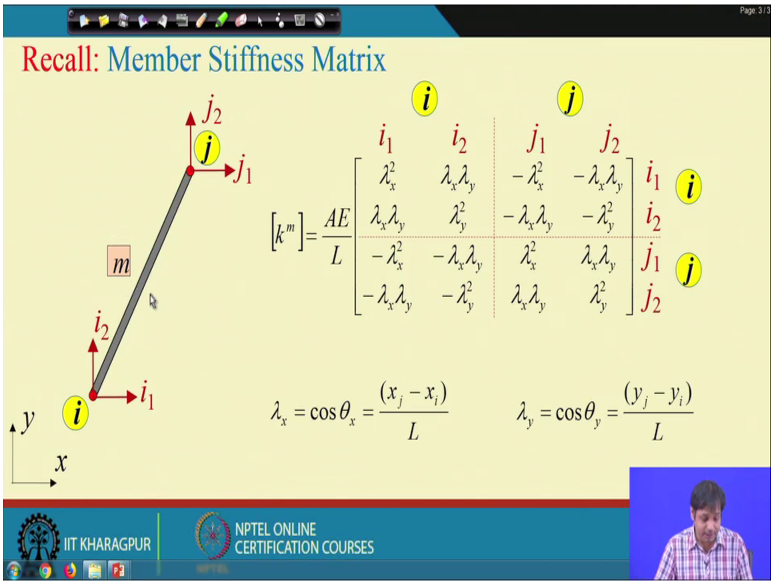

Now, if you recall, that if we take any member, suppose which is connecting between i and jth node and the member is say represented as m. And which has degrees of freedom i 1, i 2 and j 1, j 2 at the jth node and i 1, i 2 at the ith node. Then these member stiffness matrix can be expressed as this; where the lambda x and lambda y gives you the information how these what is the orientation of this member; what is the how it is oriented with respect to horizontal x axis ad how it is oriented with respect to y axis? So, lambda x and lambda y are defined as this, that we have discussed in first lecture of this week.

Now so, what we do next is, this is for any arbitrary member m, now we apply this same thing for every member. So, in order to do that we have to find out what are the lambda x, lambda y for different members. Now, suppose for all the members AE is constant. The cross sectional area in Young’s modulus is same for all members.

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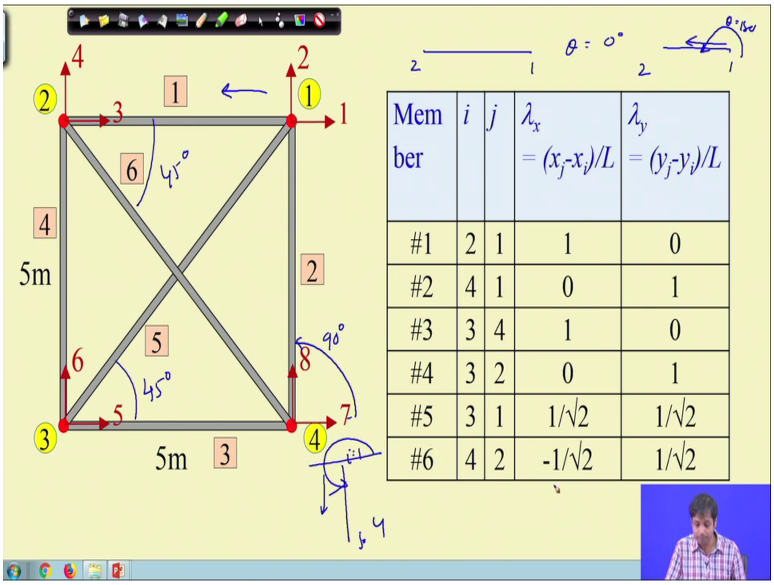

But the length is not same for different members this is 5 meter, 5 meter, 5 meter, 5 meter and the diagonals become 5 root 2. Now, you see these angles are 45 degree. These are angle this is 45degree angle, right; this is 45-degree angle. So now, for each member if we write an ij at this each this is very important, how you are connecting how you are the member number is fine, but another important thing how this member the member is always between 2 nodes right. How it is what is the order we are representing that node by the connecting nodes? For instance, if you take member number 1, then member number 1 is connecting between 2 and 1.

So, for us in this case we take i is equal to 2 and j is equal to 1. You can you could take i is equal to 1 and j is equal to 2; means, you can go in this direction as well, but whatever way you go whatever way you number to start with we will discuss what is the most efficient way of numbering these members and the node numbers at the 7th week when we do when we discuss about implementation issues.

Let us not bother about that issues right now. So, you can represent you can use the connection of member 1, either 2 1 or 1 2 that is up to you, but based on that connection you have to define the angle. For instance, if we take member number 1 and this is connecting like this, we are going in this direction 2 1; that the angle is the theta becomes 0 degree. But again if you make 1 2, means you go in this direction. So, this angle becomes theta. The theta becomes 180 degree. So, any orientation you go, but whatever way you define that orientation you have to consider theta accordingly.

So, in this case suppose member 1 is oriented as 2 1, i is 1 j is i is 2 and j is 1 member 2. Member 2 is this one, this one we take in this direction like this. So, we can take this angle as theta which is 90 degree. But if you take in this direction, if you go from this direction, means if we take that i is equal to 1 and j is equal to 4, means you are going in this direction. In that case we have to take this as theta.

So, in this, but we have taken since 4 to 1 in this direction our theta becomes 90 degree. So, similarly member number 3, member number 4, member number 5 and 6 we can define these this is the way. But as I just now say it is not compulsory that you have to choose i and j like this only, it is just for demonstration you can take i and j just reverse order as well or the mixed order as well that is up to you. But whatever way we define we have to define we have to take the angle accordingly.

Now, once we have taken this i and j makes this calculation of lambda x and lambda y which is straightforward. So, these are the corresponding lambda x and lambda j's lambda ys for different members. Now once we have the lambda x and lambda y for different members. And then these we substitute this lambda x and lambda y in these expression for every elements, for every members and calculate their respective element stiffness a member stiffness matrices.

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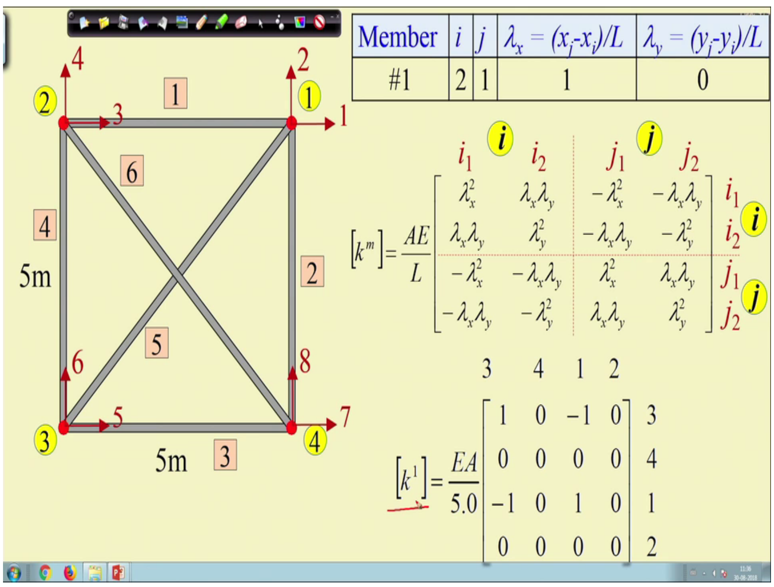

Let us do that. So, first take member number 1, member number 1 is between 2 to 1. And then this is the general expression for stiffness matrix. And if you substitute these lambda x and lambda y, here expression of k 1, please note that this k 1. This k 1 which is for stiffness matrix for 1, first element is this.

So, similarly we can calculate stiffness matrix for other elements, say I take a member number 2.

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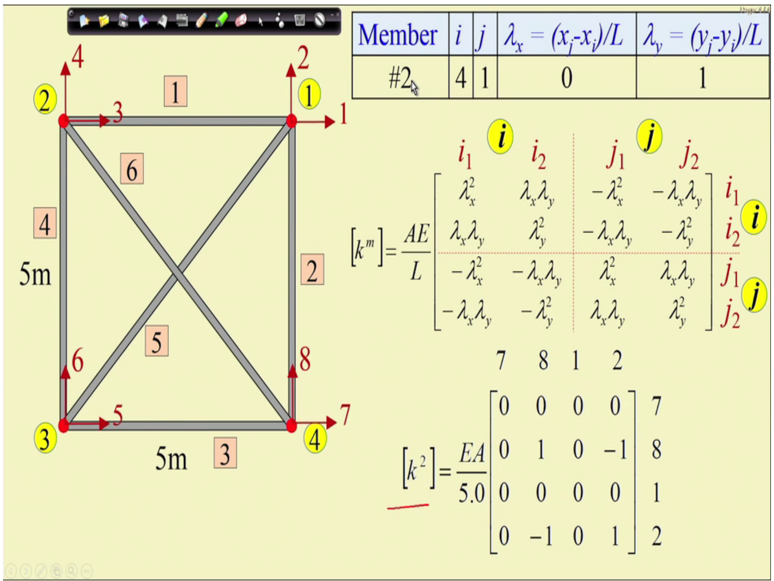

If we take member number 2 come in between 4 to 1 and corresponding lambda x lambda y these, substitute those lambda x and lambda y here and we get k 2 as this. Length is 5 mm. EA we are keeping it as EA, because EA is constant for all these members. Do this exercise for member 3.

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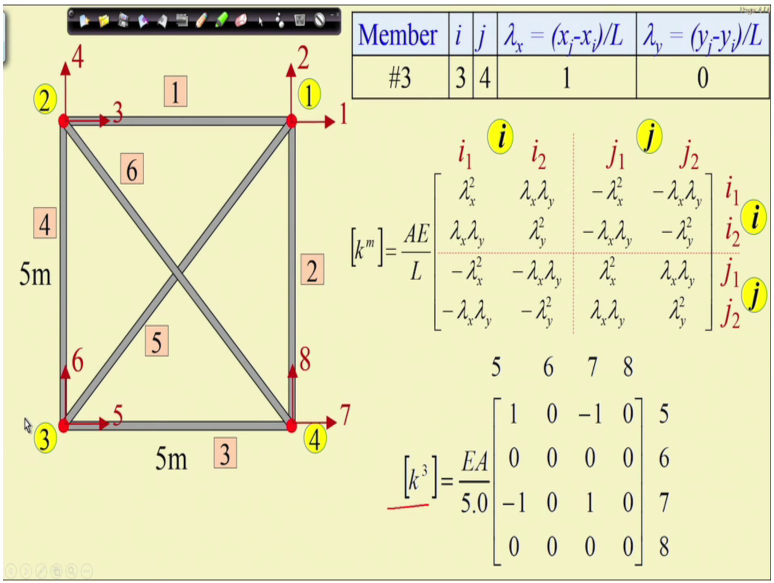

If we do this that for a member 3, member 3 is connecting between 3 and 4 and 3 and 4 corresponding lambda x lambda y substitute this, we get this is k 3.

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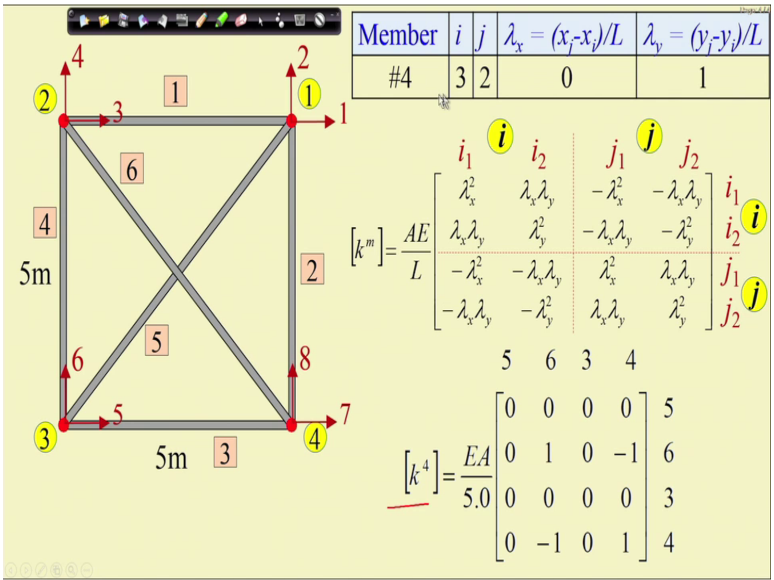

Similarly, for member 4, member 4 again we can get k 4 as this.

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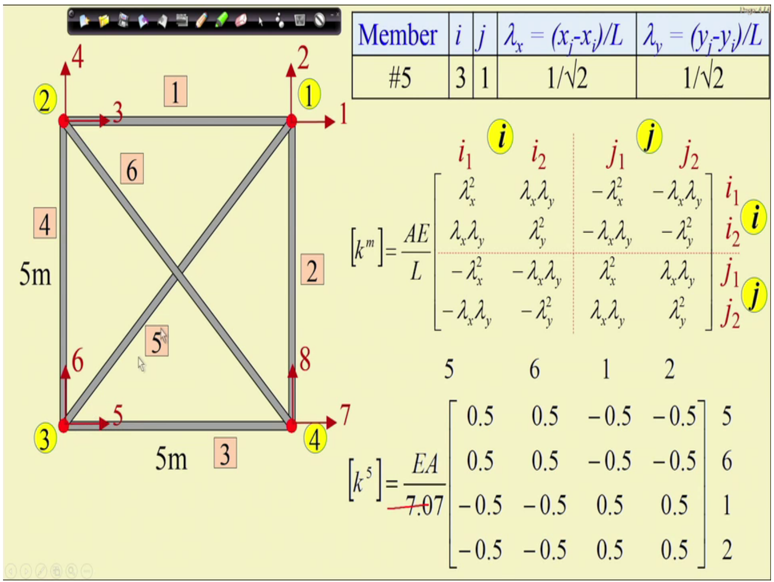

Member 5 is this one, the connection between 3 to 1 and corresponding angle is 45 degree and associated lambda x and lambda y this substitute this, and we get k 5 as this.

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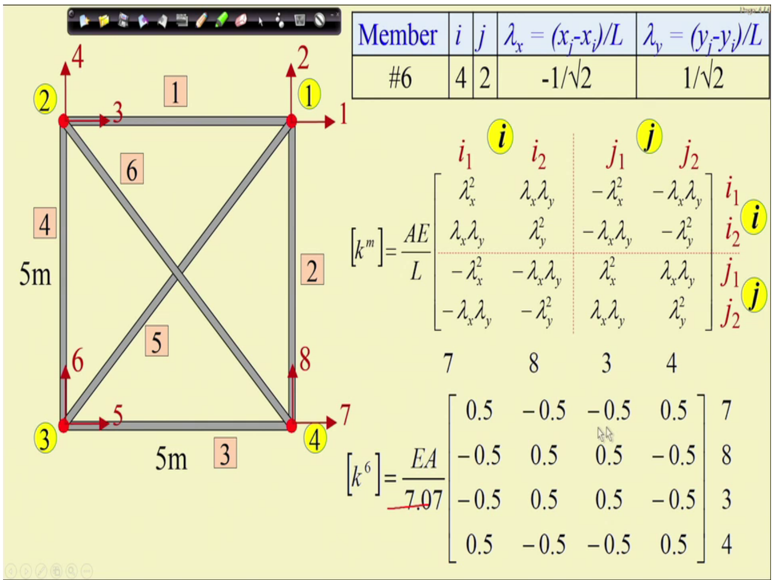

And then similarly for k 6, same exercise and k 6 will have this. So now, we have all these stiffness matrices, all stiffness matrices for all the members. And let us see all of these stiffness matrices together. And these stiffness matrices are this.

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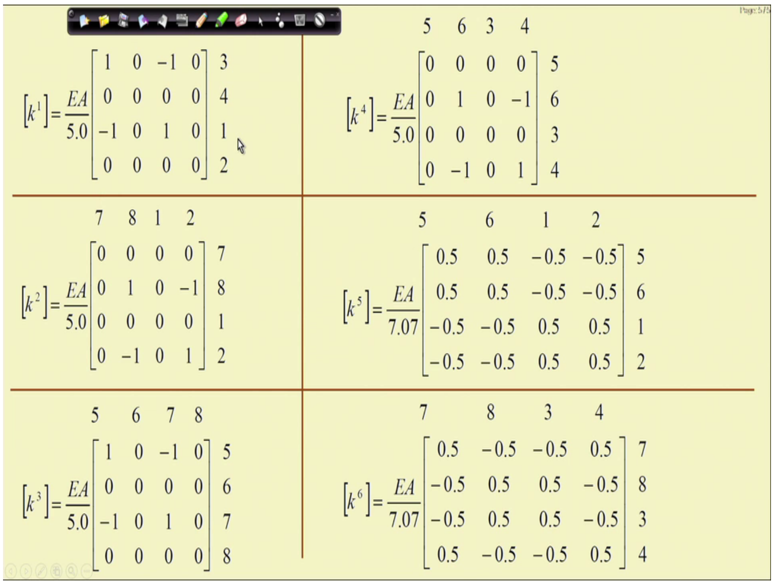

This is 6 members, 6 stiffness matrices. Now, once we have those stiffness matrices for all members, then next is this we have to assemble them. And this is a very important part of this assembling, as far as calculation of member stiffness matrix is pretty straightforward because you have the expression, you have to substitute lambda x lambda y for that particular member.

But the most important part here is assembling. And there the how to how to numbering them and all these things those issues will come. But for the given numbering giving way of numbering, let us see for this problem, how these assembly needs to be done. Will not assemble the entire stiffness matrix, what we do is, we take just one note and discuss the assembling. And then take the rest and then finally, show the I will show you final results. This assembling we have already discussed detail in the previous in the in the previous examples that we discuss in the previous classes.

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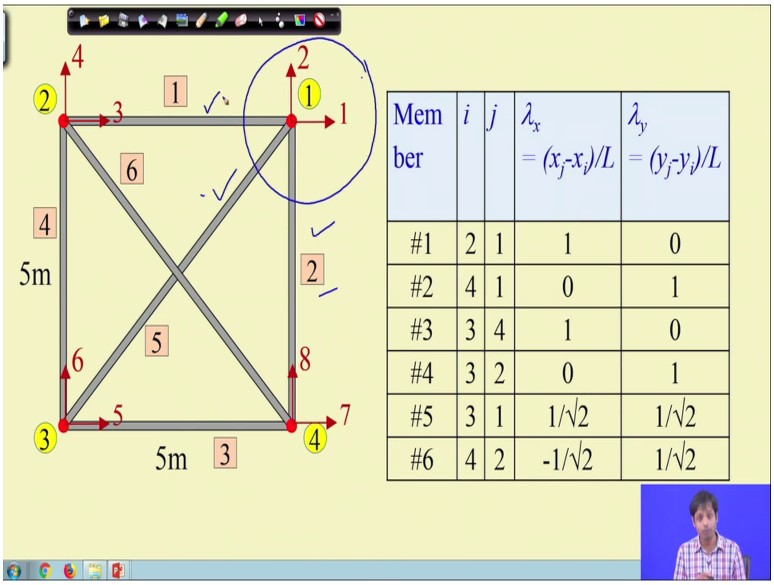

So, let us consider this member number 1. Let us consider joint number 1, and see why how this assembling at this point we have. See at joint number 1, we have 3 members, member number 1, member number 2 and member number 5.

So, so, in the stiffness matrix the term associated with member number 1, in that term there will be contribution from second member number 2, member number 5 and member number 1. Let us find out that contribution. Let us and take the values in the stiffness matrix for member number 1, for joint number 1.

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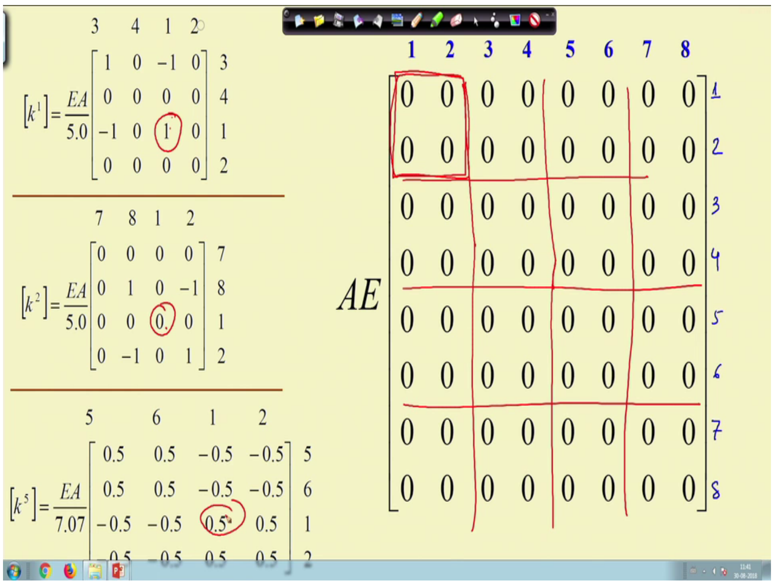

So, these are the 3 stiffness matrices member number 1, 2, 5 because only these members will contribute to joint one. Now see we have 8 degrees of freedom in this structure. So, global stiffness matrix will be 8 by 8 matrix. The first step is you take a global stiffness matrix and initialize them as 0. AE we can keep outside because it is same for all the problem.

Now this is important so, these each row and each column so, this is associated with ith the first degrees of freedom. This is second this is third similarly this is for first second third. So, let us write this is 1, 2, 3, 4, 5, 6, 7, 8.

So, this what we are interested now is to find out. So, these are if find out this contribution. So, just if I draw this just for the now. So, this block will give you this block gives you what is the contribution from I from first node to the first node. This gives you what is the contribution from first contribution at first node first node from the from this from the from the degrees of freedom 3 4. And these gives you what is the contribution of second third node from first node, from third node this gives the contribution at first node from the 4th node like this.

Now, we will we will see what is the this block would will concentrate on this block see the values that these block and rest of the blocks will just give you the final expression. Let us let us do that you see now let us see term by term, now let us take first term this one.

Now, this one is 1 1 right. Now 1 1 means we have 1 1 here, you can keep it so that you can see this yes. We can have we have 1 1 here, this is 1 1 term. This is 1 1 term and then another 1 1 term we have we have 1, we have another 1 1 term here. And then we have another 1 1 term here.

So, what we have to do is, so 1 1 term will be this plus this plus this, right. And if you if we do that, then this term will be first term will be this.

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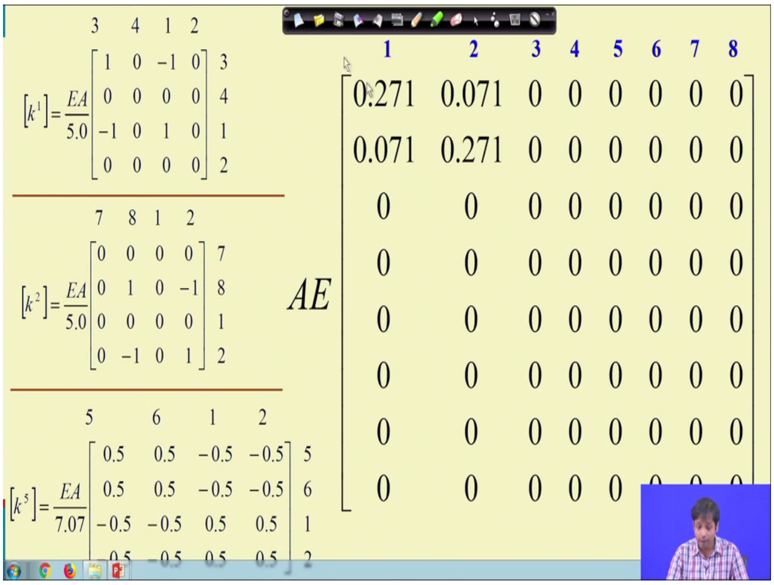

Let us do this, then 1 1 let us do then 1 2 term this is 1 to term 1, 2, 3, 4 and so on. 1 2 term, 1 2 term is you see here you have 1 2 term is this one. And then we have 1 2 term is this one. And then we have 1 2 term is this one. So, this 1 2 term will be this plus this plus this and if you substitute this here, these values will be this.

Now, similarly let us see what is 2 2 1 term. 2 1 term is this is this is 2 1 term. This is 2 1 term. And then this is 2 1 term. And correspondingly this is 2 1 term. And if we substitute add them, and then substitute that then this corresponding value will be this. And without actually doing these exercises you can straight away write 0.071 because the stiffness matrix is symmetric matrix. But it is good to write calculated and write it will be a check whether you are getting this whether your calculations assembling is correct or not.

Now, let us find out the first last one, which is 2 2, 2 2 we have here we have 2 2. This is 2 2, and then this is 2 2, and similarly they are also we have 2 2. Now if we add them, then final expression for 2 2 will be this. This will be 2 2.

So, similarly we can do this exercise for all these all the degrees of all the joints and all the degrees of freedom. And if we do that your final stiffness of stiffness matrix will be like this,.

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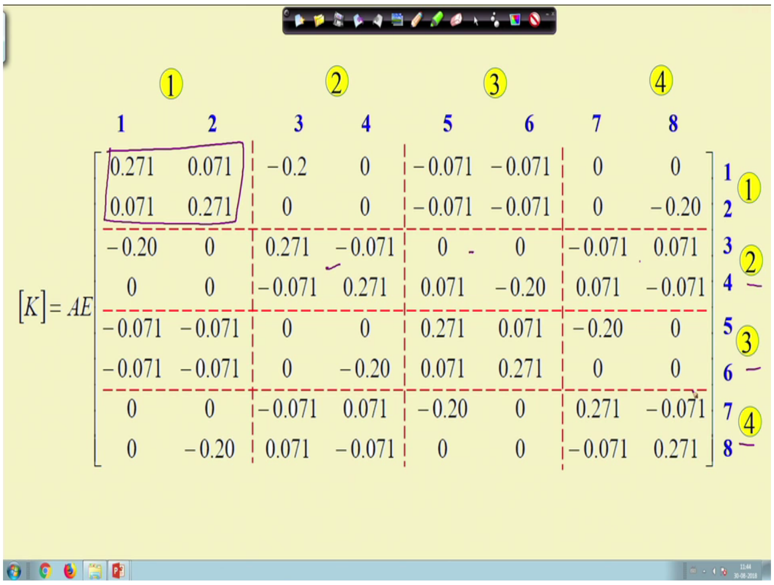

So, this is the first block, we discussed only the first block here. Only this block we discuss how to get this block. Rest of the things is if you apply the same thing.

Now, this is again this block is contribution at node number 1 from node number 1. This is contribution at node number 1 from node number 2 contribution node number 1, from node number 3 contribution from node number 1 from node number 4.

Similarly, these block is continued contribution at node number 2 from node number 1 contribution at node number 2, from number 2 contribution at node number 2, from node number 3 contribution at node number 2, from node number 4 and so on.

Now, so, this is our global stiffness matrix and you can say that these global stiffness matrix is symmetric. Now you try to invert these global stiffness matrix. As we said, this global stiffness matrix is singular matrix the determinant of this matrix is 0. And it is physical consequences, because you have defined the structure, but we have not yet given the boundary condition. The boundary condition makes our structure stable. The mathematical consequence of this is your singularity in the stiffness matrix. And it is physical manifestation is your structure is not stable. The structure is we have to structure does not have sufficient constraint.

Now, then we have to provide the constraint. Then what are the constraint we have in the in the problem?

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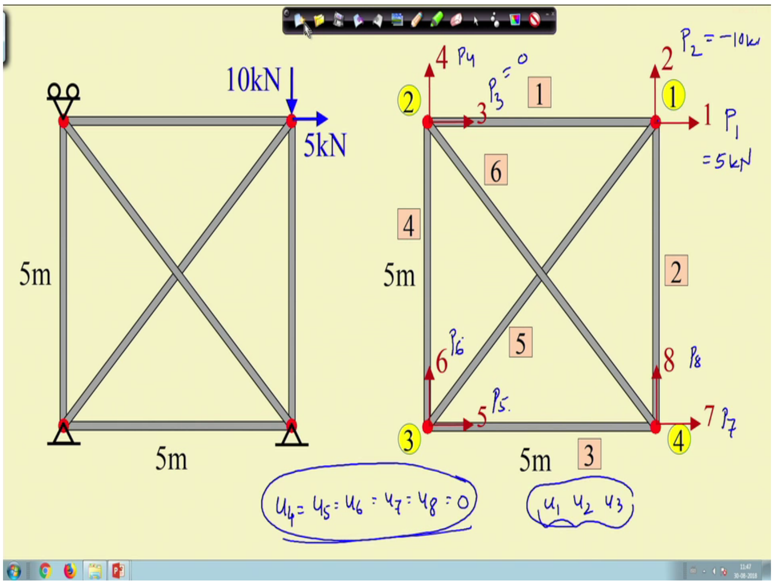

Now the constraints are you look at the examples these are the look at the figure if we see then let us write down the constants. We have say u 4 is equal to 0, u 4 then u 5 u 6 all are 0, u 5 is equal to u 6 0 is equal to u 7, is equal to u 8 all us are 0, these are 0. So, only non0 us will be u 1, u 2 and u 3. So, essentially you have to find out. Solution for u 1, u 2, u 3 rest other degrees of freedom u 4 to u 8 this is 0. So, these are constrained.

So, this is for u similarly what information we have for load one? Suppose if this is P 1 this is P 2 load one means member load. This is P 3, this is P 4 and then P 7, P 8, then P 5, and then P 6, P 6. See we do not know what is the, what is the what is the what is P 5 what is P 6, because they are the they gives you the reaction and joints.

But looking at this point one can certainly say that result net resultant of the force here; will be P 1 will be 5 kilo Newton, and then P 2 will be minus 10 kilo Newton. And then P 3 is equal to there is no P 3 in this direction. All other all other P 7 P 8 we cannot say, because right now we cannot we cannot tell their value, for that we need to find out the support reaction. So, they are essentially you have to find out at the end of the day.

So, this is the information about the degrees of freedom we have and this is the information about the loads on the structure we have and that. We have to use this information to get the solution. Now so, based on these information let us partition the stiffness matrix, this is where we partition the stiffness matrix.

We take all these known displacements and we separate it know as known displacement and unknown displacement. And in this case unknown displacements are u 1, u 2, u 3 and u 1, u 2, u 3 and the known displacement all these displacements are 0.

Now, we partition it like this. Now, so, once we have partition it we can write the force displacement relation as they, this is the global force displacement relation.

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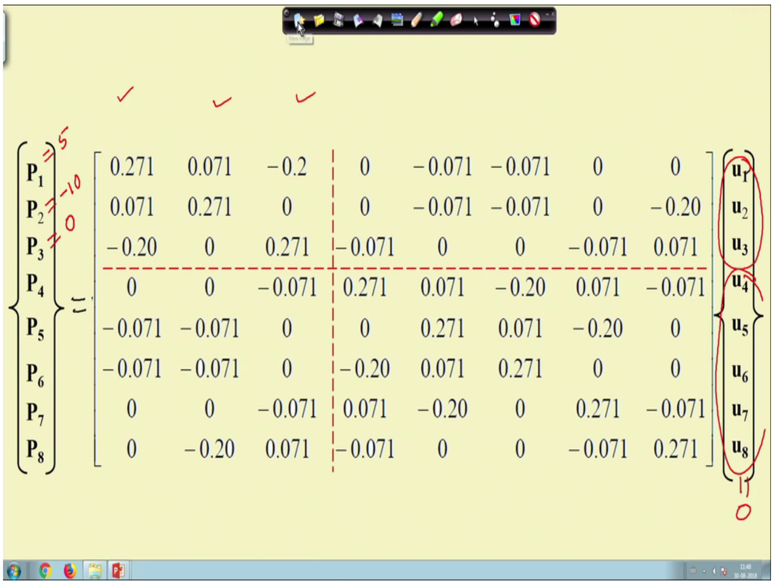

This is a force is equal to there will be an equality here, these force equal to the stiffness into displacement.

Now, on the among all these displacement, these displacements are not known. We need to find out these 3 displacement, but other displacement these displacements are 0. Among all these we can say that P 1 just now we saw that P 1 is equal to 5 kilo Newton. P 2 is equal to minus 10, and P 3 is equal to 0. So, what we do is, we substitute those values in this load vector and the displacement vector. And if you do that this will be our load displacement relation.

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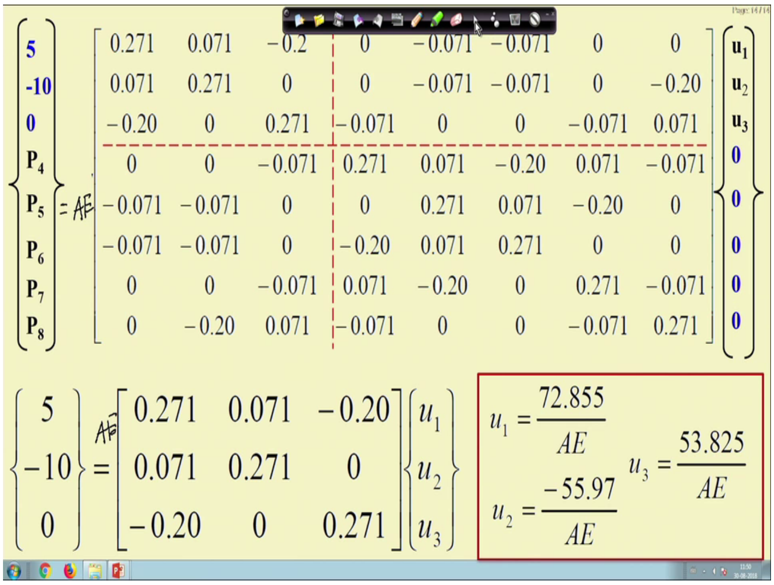

Now, so, what we do now is, so, if we can take this part. So, this part will be from this from this we can writes this part, your this part will be this will be is equal to this part multiplied by this part. This part will not be there because this part multiplied by this 0 is 0. So, this will be this multiplied by u 1, u 2, u 3. And if we do that if we write it; so, we have so, this is this part this part only written.

And rest of the part becomes 0 because these are all 0. Now this gives us now you invert this is called reduced stiffness matrix reduced stiffness matrix after you apply the boundary condition constraint. As I just now said if you take the determinant of global stiffness matrix it comes 0, but now if you take the determinant of these reduced stiffness matrix this is not 0.

So, means your again your physical manifestation is this if the provide a support to the structures of the structure is now stable. And if you solve then you get a unique displacement field or unique solution.

Now, from this we can solve for u 1, u 2, u 3 and if we solve it the solution will be this you can check with the solution. There will be an e AE term here which is missed, please correct it. There will be this will be this and there will be an AE term here. And this is equal to there will be an AE term here. So, the u 1, u 2, u 3 displacement will be this.

Now, a equal axial rigidity if you make AE more your displacement will be less it means that AE makes your structure more rigid. That is why it is called rigidity.

Now, once we have the displacement field now you see all the displacements are known u 1, u 2, u 3 and rest of the displacements are 0. Now next we have to find out what are the forces other unknown forces right.

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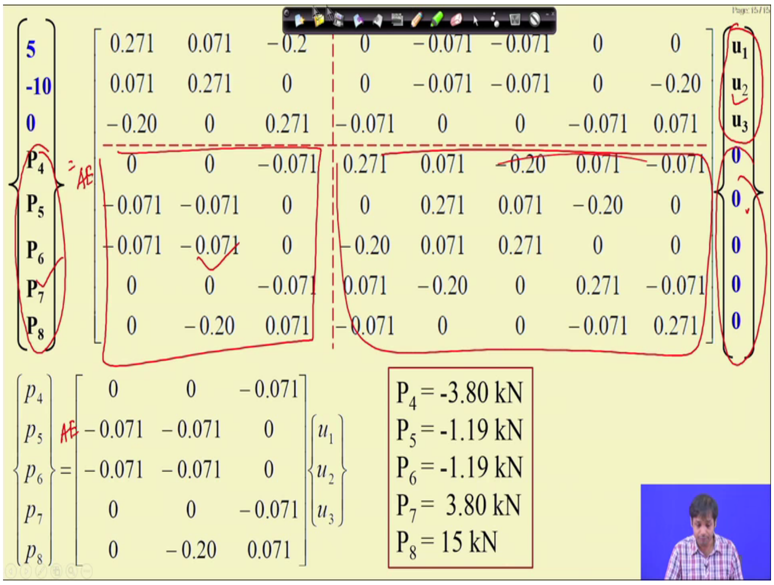

Now, use the second part of this, second part see we could say, but by just looking at the structure you could say that P 1, P 2, P 3 are 5 minus 10 and 0 respectively. But we did not make any comment on P 4, P 5, P 6 and so on. Now we have to evaluate them. We can evaluate them by this block. You see, this will be again, this expression this will be, this will be, this part multiplied by this part, plus this part this part multiplied by this part, but since it is 0, this part contribution will be 0, essentially. So, essentially you have this will be this multiplied by this and this is written here,.

Now, u 1, u 2, u 3 already we have the expression here will be an AE term here. U 1, u 2, u 3 already we have the expression and if we substitute them, we can solve for P P 4, P 5, P 6, P 7 and P 8 and if you do that of solution will be this will be our solution you can verify the solution. So, this is all these P's ok. Joint forces and from that you can easily determine what are the reactions.

Now, once we have determined what are the reactions, these P 4 P 5 these forces will give you the reactions and then we have also determined what are the displacements. Next one next thing is left it is your what are the member forces. Let us find out what is the member forces.

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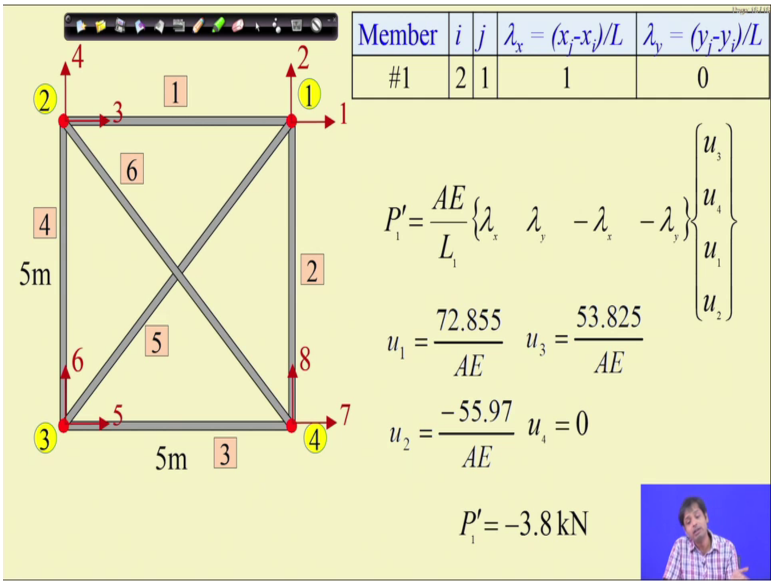

Now, if you recall our expression for member forces will just give demonstrate that with one a one member and rest of the thing is same thing you can again apply. Take member number 1. If you recall the expression for member forces is this if P 1 dash is the force in member one. Then this is the expression ok. And u 3, u 4, u 1, u 2 are the associated degrees of freedom.

Out of that we have already know that what are the expression u 4 is equal to 0, we already know. And rest u 1, u 2, u 3 we have already determined. Lambda x lambda y for member number 1 is given here. If you substitute all these values here and solve for P 1 dash, and P 1 dash or P 1 u 2 is given here. U 4 is equal to 0 and if we get it P 1 dash will be minus 3 point 8 kilo Newton and check that.

So, you can apply the same thing for all the members we know lambda x lambda y we just have to substitute. Lambda x lambda y values here and then associated degrees of freedom.

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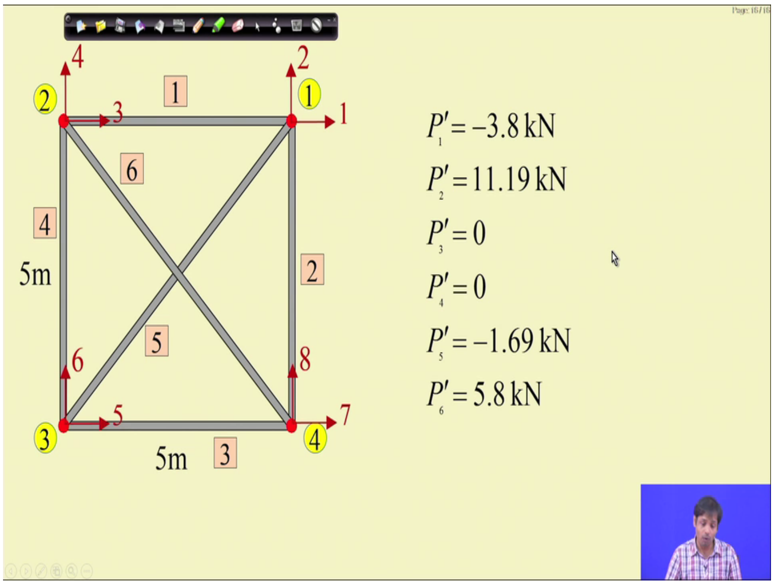

And if we do that for all the members, then the finally, the member forces for all the members will be this.

Now, you can check a solution check whether your equilibriums are satisfied or not whether the forces are correct they are compatible your support reactions are forces they satisfy the equilibrium and those cross checks you can do.

Now, this is a summary of the entire methods, but that is true and examples. Now same thing you can apply for any method. Since we have to we have to we have to show all these stiffness matrices in a single screen, calculate all the forces.

Therefore, the problem that we chose in both the problem, the first one, and the this one the your number of number of the unknowns or degrees of freedoms are less. So, that we can solve it here without using any computer but you know in actual structure you do not have just 4 joints, you have say 100s of joints 100s of members. Your degree your stiffness matrices that you come across that is a in terms of several thousand’s by several thousands.

But so, in order to solve them we have to use any computational tool, but the essence of that computational tool the algorithm that the algorithm which constitute the solver is this the exactly the same procedure, but in a larger scale. In a where your number of members and modes if the dimensions and mode, but the steps are concerned, they are exactly the same that we have discussed in this week.

We have to number the first we have to number the members number the joints number the degrees of freedom, and then write the stiffness matrices for each member. And based on their numbering assemble them; and then once you have the once you assemble them glade the global stiffness matrix. Apply the boundary conditions, reduce stiffness matrix, solve for unknown displacements. Once you know the unknown displacements, and then you calculate the unknown forces, unknown joint forces. And once you have both, then we can apply we can find out the member forces right.

So, this is the matrix method of structural analysis for truss. There are some issues the implementation issues just as I said how to give these numberings, and what is the most efficient way of numbering for a large structure, those things we will discuss. Those issues implementation issues will discuss in the 7th week.

Before we demonstrate this method for other structural idealization and other structural idealization that we consider in these courses beams and frames. So, truss we have completed in this week. Next class next week we will discuss the matrix method of structural analysis for beams. So, I stop here today. See you in the next class.

Thank you.