Matrix Method of Structural Analysis
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Lecture - 02
Review of Structural Analysis - I

Hello everyone. Today is the second day of our ongoing course on Matrix Method of Structural Analysis. In the first lecture, we tried to understand the reason a good reason for learning matrix method of structural analysis and what is the limitations of the main analysis procedure that we started in our first course on structural analysis, and then how this course how that concept is can be taken forward to solve to analyze the structures that we come across in real life ok.

And now, this week and at the next week we will be revisiting some of the basic concepts of structural analysis I. So, today we will start with degrees of freedom constraints and static equilibrium.

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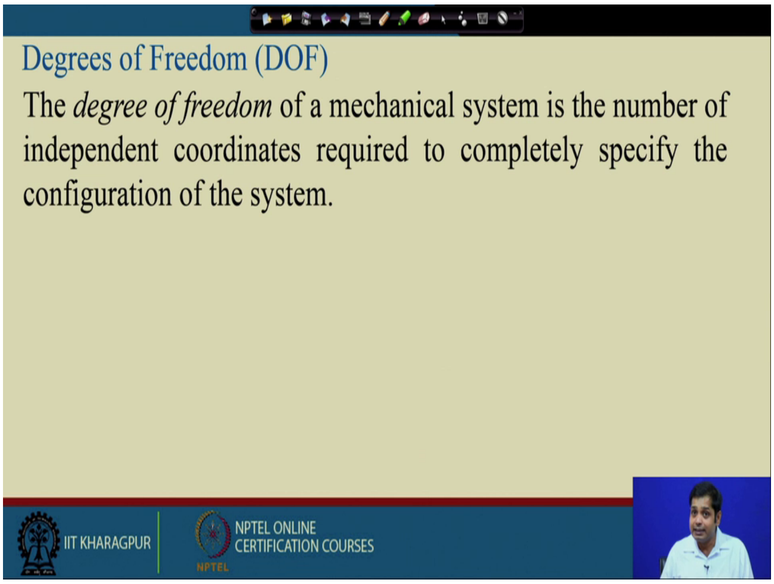

You see, what is degrees of freedom as name suggest: it is the freedom of an object a how many see if we take an object if you take an object for instance, if we take a pen, this pen any object in three dimensional space this skin this pen can move in this direction, this pen can move in this direction, this can move in this direction and this can this can pen rotates like this, this can pen rotate like this and this can pen this can pen can rotate like so.

So, this pen has freedom to move in this these three directions and similarly this pen has freedom to rotate about this about three different axis. So, this pen has six degrees of freedom. In fact, as in the three dimensional space, that is the maximum degrees of freedom can any object can have ok.

So, degrees of freedom; so it is a freedom of an object to move. Now the degrees of freedom if I have to define, the definition goes like this the degrees of freedom of a mechanical system, if the number of independent coordinate require to completely specify the configuration of the system. Just now the definition that I gave and the definition written here, and you see there is there is there is a common link there is these two definitions actually tells you the same story.

I leave it to you to realize that, and as we go further we will move, we will understand this why this is what does it mean independent coordinate independent coordinate to require to completely specify the configuration of the system right. So, these are the degrees of freedom.

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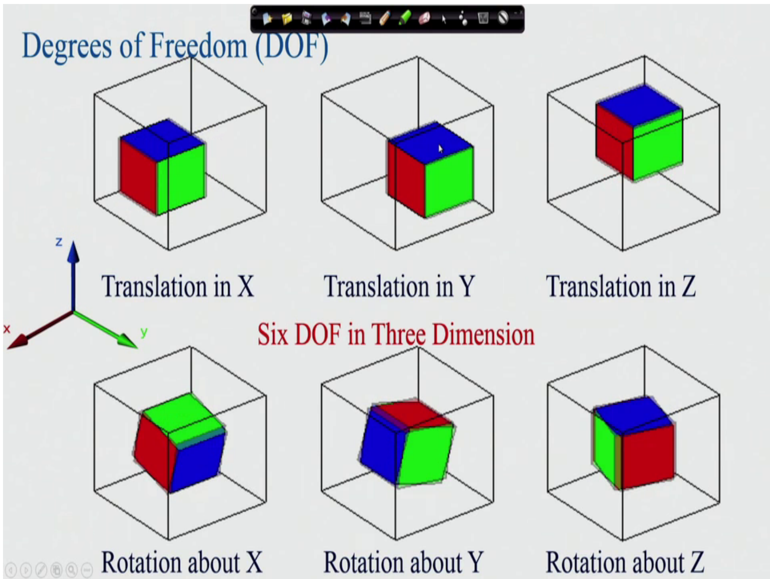

Now, you see what when you these are different degrees of freedom in three dimension. Just now I showed you there are three translations, these three translations and in rotations about three axis.

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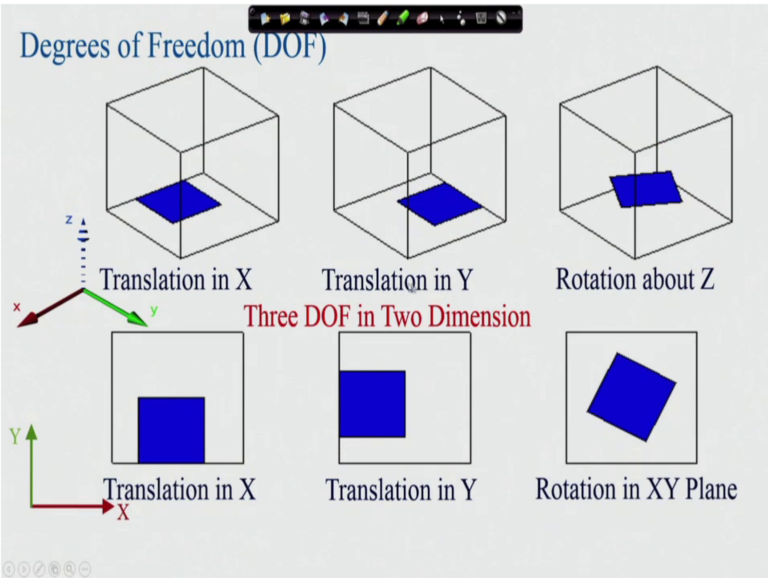

Now, if we take a problem in 2 D, then in two dimensions we have three degrees of freedom, this axis is where the object lies in plane. So, you have you can have two rotate, two translations and one rotation two translation and one rotation. So, these are the three degrees of freedom we have of an object in two dimensions ok.

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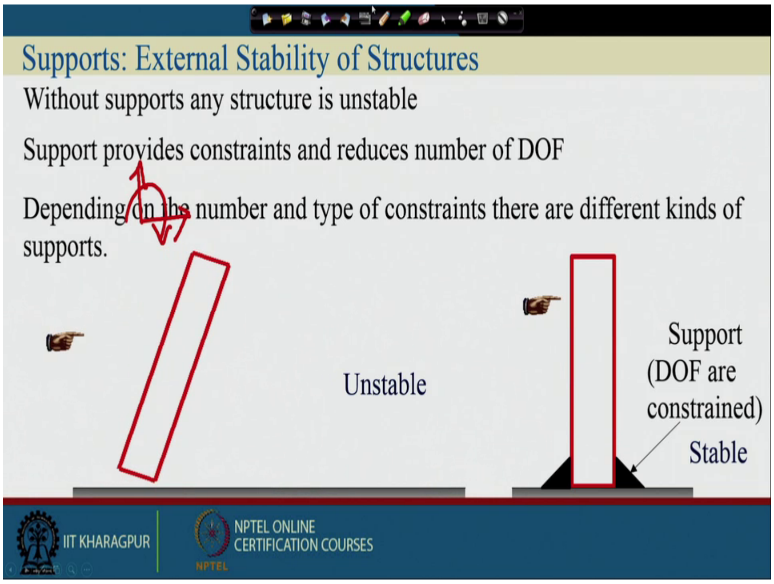

Now, now let us see without support; now what is support? You see a structure is as I is as I say a structure has freedom or any object has freedom to move in three dimensional space. But we do not want our structure to move freely in three dimensional space. So, need to constrain some of its freedom and that is done by providing support. So, what support does is s s without support without providing any support a structure is unstable. For instance, if we take a structure like this, there is no constraint. So, these it is a two dimensional problem two dimensional object. So, these object has three degrees of freedom, it can r r move in this direction, it can move in this direction and it can rotate about this axis right.

So, it can move in this direction, it can move in this direction and it can rotate about this axis. So, these are the degrees of freedom it has, but now if we apply if we apply a load, then the structure becomes unstable this. But you know we want our structure to be stable we do not we do not want structure to move freely in space and for that, we need to provide we need to constraint some of its degrees of freedom and that is done through providing supports.

So, what support exactly does is, if we same object if we provide support then if we apply a load like this unlike the unlike the previous unlike the previous one, if we apply load here this structure is not freely cannot freely move. So, it is to support provide some of the constraints, support constraint some of the degrees of freedom right.

So, now, depending on the number of type of constraint, there are different times or support. Now the thing is in order to make a structure stable, we need to decide based on some again based on some conditions, now how many in order to make a structure stable how many degrees of freedom to be constrained. And depending on that how many degrees of freedom to be constrained, we can have different kinds of supports right.

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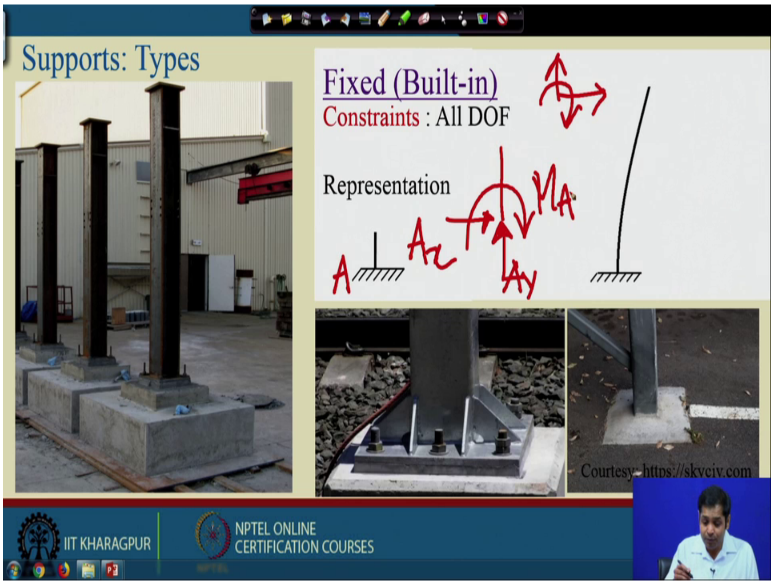

For instance we have fixed support. Fixed support means this is the this is the way we represent fixed support you are so, you all are familiar with all this these are some examples of fixed support. So, what are the diff in fixed support the degrees are constraint are all degrees of freedoms are constrained.

So, if this is a fixed support, it this point cannot move at this at any in two dimensional space, this can any point can move in this direction this direction and can rotate here, but since the support is fixed, it cannot move in any direction right. Now how constraint, how this freedom, how the how these degrees of freedom are constrained support provides support reactions right.

So, what is the if we have to draw the free body diagram of this support, if you recall the free body diagram of the fixed support is, if this is the fixed support a free body diagram is then you have you have this force, this force and a moment. So, this is the free body diagram of this support. So, this is say it this is support is A. So, in this is A y this is this is A x and this is MA right these are the support A x A y A M A are the support reactions. If it is fixed support we have three support reactions ok.

Now, go next if we have.

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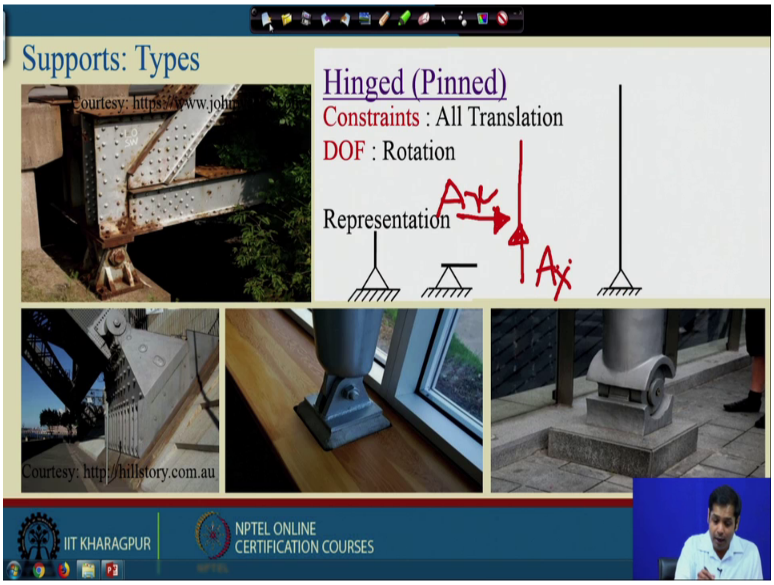

If we have another support; another support is the hinged support hinged support is represented like these are some examples of hinged support. So, as you can see here these points cannot move in this direction and this direction, but these point has freedom to rotate. So, in this case the support has only one degrees of freedom unlike previous case in the fixed support; there was no degrees of freedom degrees of freedom was 0. In this case it has one degrees of freedom and that degrees of freedom is the rotation.

So, if I have to draw the free body diagram of this support free body diagram. So, this is the support and then this is the support reactions. We can have we have only two support reactions, this is A y in and this is A y and this is A x this is A x. There will be no moment here because rotation is not constrained. So, this is also we have same.

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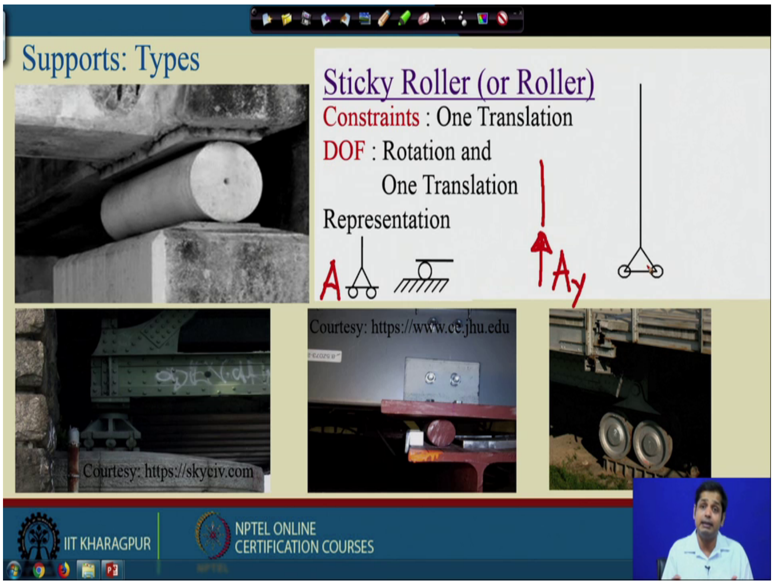

Now, next another support is another support is roller support. As you can see here the motion in this direction is constrained, but the displacement if the moment in this direction is a moment in this direction is allowed. So, it has rotation degrees of freedoms are rotation and one translation. So, this can rotate about this and this can translate about this.

So, what are if I have to draw the free body diagram of a roller support, a free body diagram will be only this. There will be if this is A, then this will be this is A Y. There will be no horizontal reactions here, because the horizontal moment is allowed, there will be no rotation here; there will be no moment here because rotation at this point is also allowed.

So, these are different kinds of supports we can have. Similarly in three dimension, these the supports will be similar to this it could be roller, it could be hinged support, it could be fixed support, but again if it is roller then in which direction it is roller, in which direction the degrees of freedoms are constrained if it is hinged, then which direction rotation is allowed depending on that you can have degrees of different degrees of freedom at different supports ok. You can have a support in combinations of if you take any structure, then there are there will be many supports in the structure and those support system will be can be the combination of different kinds of supports ok.

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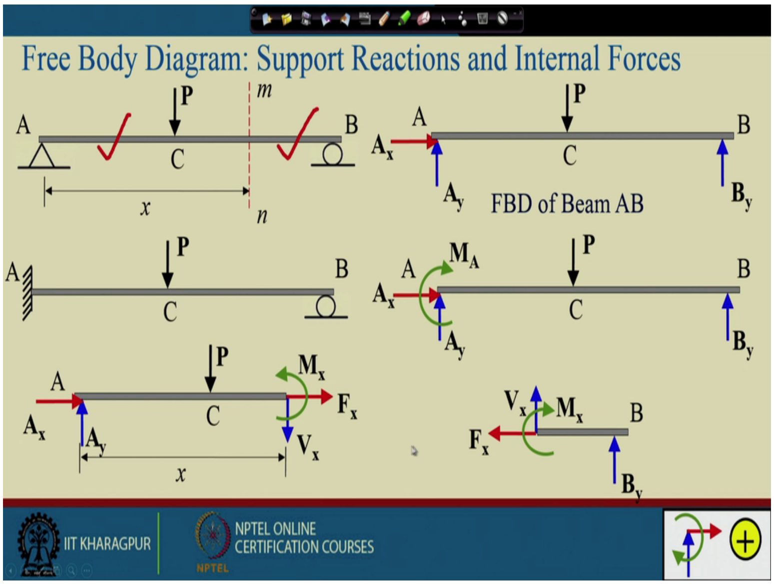

Now next is. So, what is free body diagram? Again as I say this is just we are reviewing the reviewing the basic concept, if we take a if we take a simply supported beam, which is subjected to concentrated load. Suppose I want to draw the free body diagram of the entire system then the free body diagram of the entire system will be if it is hinged support, if it is roller support at B, roller support has one constraint. So, there will be one reaction, suppose and then hinged support has two constraint two reaction there will be no moment here, and suppose this is B y and A y and A x are the support reactions at A. So, this is the free body diagram of the entire structure.

And if I change the support condition instead of hinged if you make it fixed support, then the free body diagram will be again for B it remains same, it is just a only one reactions here and since it is fixed support, there will be three reactions to forces and one moment and these are two forces and one moment. And these are the forces and moments and this is the sign convention when I write moment as this. And remember when you talk about sign convention, sign convention is very much subjective. I am following this sign convention in this problem, but you may follow a different sign convention for your problem.

But whatever sign convention you use, you be consistent with the sign convention ok. Now similarly if I take a section here at section mn, and if I have to draw the free body if we make this section and cut the entire structure in to two part and then one part is this one, one is one is this part another one is this part and if we have to draw the free body diagram of this two, the free body diagram will be this part free body diagram will be these are the support reactions and then these are internal forces.

Since it is a beam problem, so, we have three forces one is axial force F x, shear force V x, at the moment M x this VFM are the internal forces. And similarly for this part of the structure B y is the support reactions and similarly these are the internal forces. And remember these internal forces for the equilibrium at this joint, this forces and this force the either side both side of whatever forces you have on the either side of the section, they should balance each other. So, these F x F x they will cancel each other, M x and M x will cancel each other that is the re that is the reason why these are shown in opposite direction and similarly V x if it is downward here, it should be upward here, because they will they will cancel each other and then only the equilibrium at section n will be maintained right.

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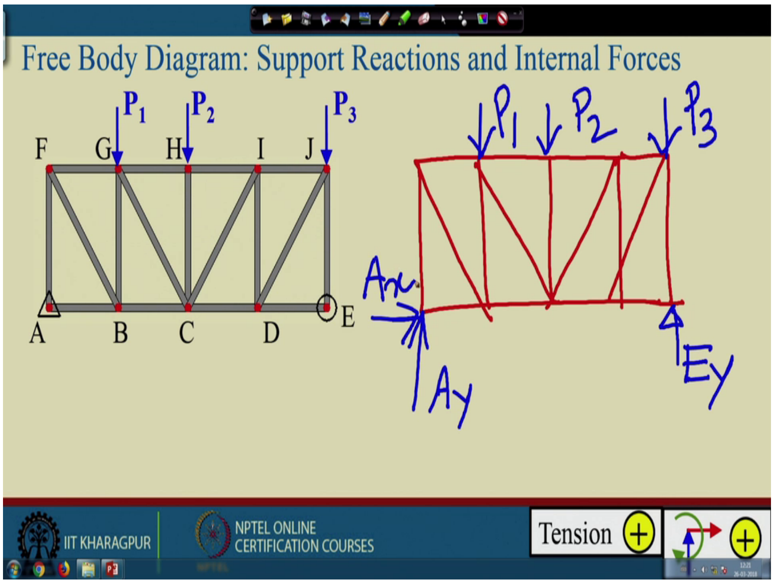

Now, we can have say it is a truss problem, similarly if I have to draw the free body diagram of the entire structure, the free body diagram of the entire structure will be entire structure will be this these are different members. And these are different and these are different these are support, these are different members and then we have these forces we have, this is force this is force. And then we have another force P 3 there if P 3. So, this is P 3, P 2 and then P 1 and then we have support reaction at E it is roller support. So, there will be one reaction only which is E y E y and then at A there will be two reactions, one is this direction another one is this direction, this is A y and this is A x A x. So, this is the free body diagram of the entire system right.

Now, if we take a section here, suppose one section here and then we break the entire structure into pieces then what will happen? And take different pieces, then we have to show the internal forces right.

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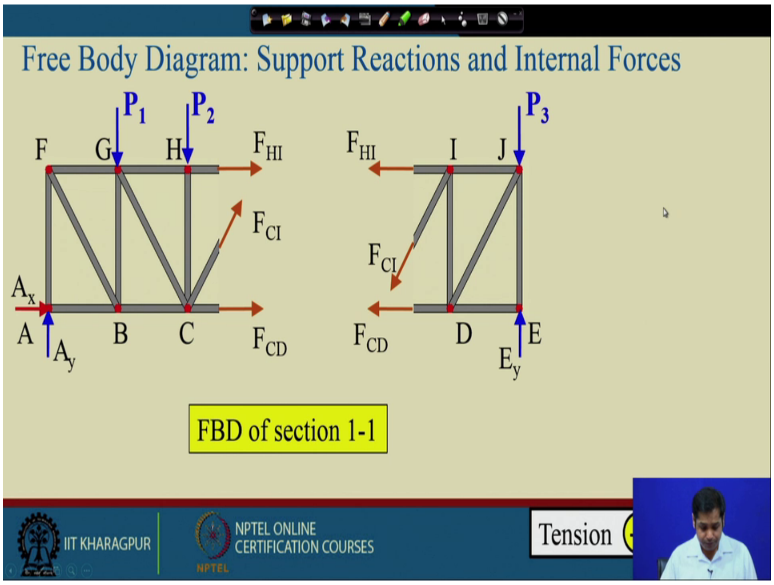

These are the since it is a truss problem, internal forces means unlike beam where we had axial force, shear forces and bending moment all since it is the truss problem every members are all members are two force members. And therefore, the forces only we have in the member is the axial force, along the line of action along the a along the axis of the truss.

So, these are these are ss these are the axial; these are some these will be the internal forces in the member right and that also we have seen how to represent these are the internal forces in the member. So, if we draw the free body diagram of the entire system, the free body diagram will be something like this. These are the support reactions and then F HI F HI this is the force in member HI. So, again this when you are taking this section if this force is this direction, for this section you have to show force in opposite direction then only the equilibrium will be maintained and that is true for all forces. So, this is the free body diagram showing support reaction in internal forces and similar free body diagram you can do for any structure right.

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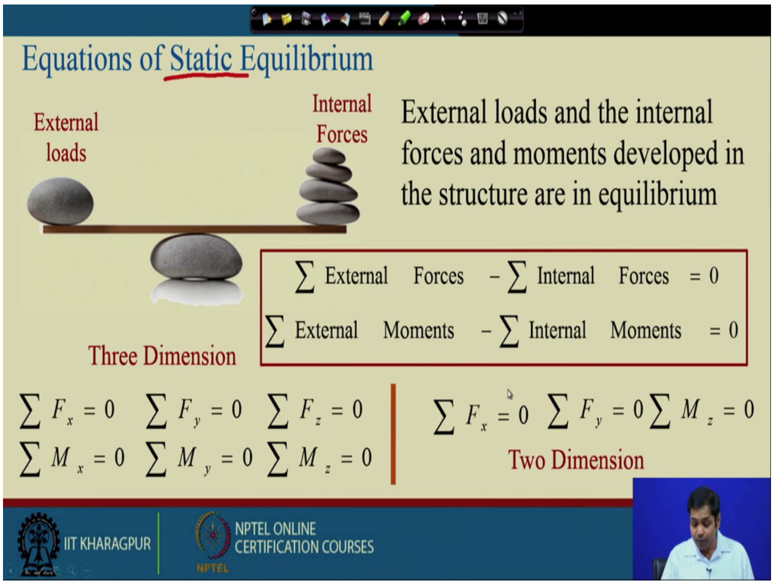

Now, so, what is equilibrium? And the equilibrium is. So, in the free body diagram just now what we have? We have support reactions, we have external loads and then we have internal forces right. Now equilibrium is when all these forces and the moments they cancel each other they balance each other they make force or any make moments in the structure is 0. So, then only say that this is in equilibrium or to be specific, this then we say that this object is in static equilibrium. This term is very important this term is very important static equilibrium.

Whenever we talk about equilibrium in this course, it is understood if it is not exquisitely mentioned, it is understood it is static equilibrium ok. Now the equilibrium is when external loads and the internal forces and moments developed in the structure are in equilibrium, and the equation for equilibriums are this is the summation of external forces summation of internal forces that is equal to 0 and the summation of external forces, the support reactions are also included here. And the summation of external moments and internal moments about any point, any axis they should also be 0.

Now, if this philosophy has to be written in terms of mathematical equation and these equations are this, these are called equations for static equilibrium. If you remember in the previous class we show one is ideal one once have a physical assistant, then idealization and then writing the or representing the idealized system through some mathematical equations these are the equations. These are one of those one of those equations equilibrium equation static equilibrium equations for three dimensions and for two dimensions. Two dimensions because there are two dimension three equations because there are three degrees of freedom and three dimension six equation, because there are six way six different ways six independent way, an object can move in three dimensional space ok.

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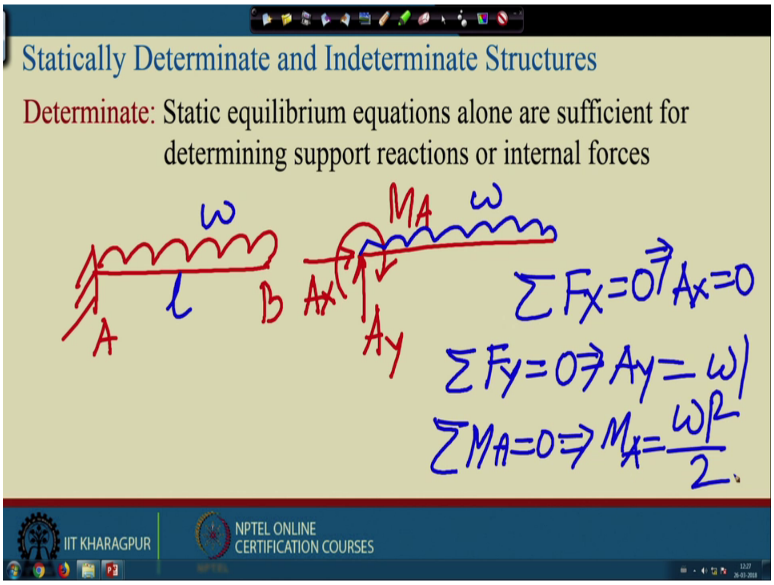

So, let us what is determinate at indeterminate structure. Now the determinate structure has you have already we already know the determinate structure is static again. Again I will go back to that, that steps in engineering analysis, once we write the once we write the mathematical equation and then next part is to solve that mathematical expression right. Now the thing is another thing is if equilibrium equations are one part of those mathematical of that mathematical representation. Now when we try to solve those equations; if the equilibrium equations alone are sufficient to have the solution of the idealized system, then we say the structure is determinate structure ok.

And for instance for instance if we have a simply if we have a cantilever beam, which is subjected to say uniformly distributed load and then this is A this is B, and which we write the free body diagram of the entire beam, this is A this is A y and this is A x horizontal reaction and then you have moment M A, and then there is and then on top of that, we have external load we have external load like this say this is w this is w.

Now, once we have this, now write summation of F x is equal to. So, we have three unknowns here A x A y A y A x summation of F x is equal to 0 if you write is F x is equal to 0 if you write then this gives us A x is equal to 0 A x is equal to 0 right directly this gives you A x is equal to 0. If you write summation of F y is equal to 0, F y is equal to 0 then A y we get A y we get if this length is l, A y A y you will get w into l. And if you get summation of say M A is equal to 0 its moment about A moment about A is equal to 0 and this gives us M A is equal to wl square by 2 right.

So, we have we. So, three equations three unknown and all these equations are equilibrium equations and for this structure these are called determinate structure these are called determinate structure for any structure. If it can be done these structures are determinate structure. But you know many structure or rather I would say most of the structure that we come across, those structures are not determinate structure.

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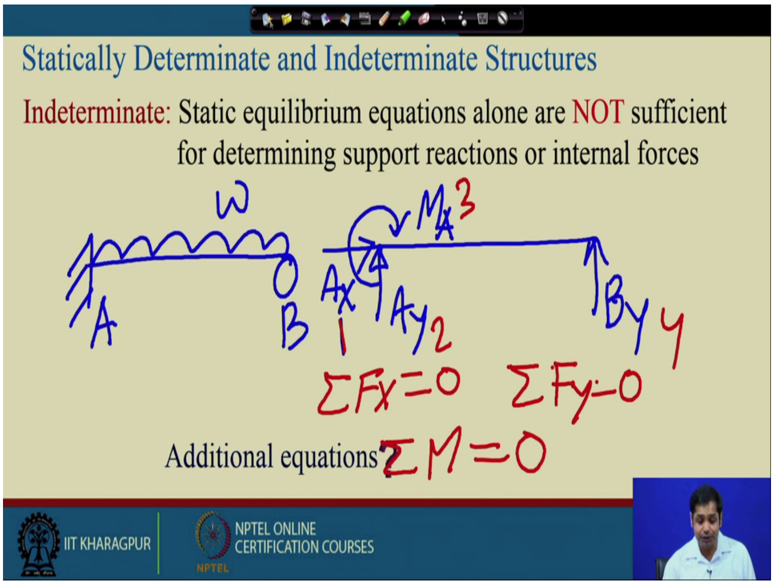

Because the equilibrium equations alone cannot give you the solution of those structure and those structures are called indeterminate, though this is the example that I showed those structures are called indeterminate structure.

Indeterminate structure is only static equilibrium static equation alone are not sufficient to determine the support reactions and internal forces. And for instance if we have the same example same example, but cantilever beam, but make it another support here. So, it is subjected to again uniformly distributed load now what are the unknown if it was a free body diagram? This is A, this is B. So, we have A x A y and then just A x and then moment M A and then here also we have B B B y right

So, number of unknowns we have number of unknowns four one is one, this is one and then 2, 3 and then 4 number of unknowns are 4 what are they how many equations we have? We have only three equation summation of F x is equal to 0, summation of a summation of F y is equal to 0 and moment is equal to summation of say moment of about any axis say about any axis will be 0 only three equations we have.

So, here we cannot solve the entire system by only these three equilibrium equations. So, in that case what we need is, we need an additional an additional equation we need an additional equation we need an additional equations. And if you remember the when you solve indeterminate structure you studied compatibility equation. Again we will go back to again that flow there if you remember and go back to the first class, we wrote equilibrium equations compatibility equations and boundary conditions. Equilibrium equations only for determinate structure, equilibrium equations class compatibility equations or some other equations we will see what those equations are and along with the boundary conditions right.

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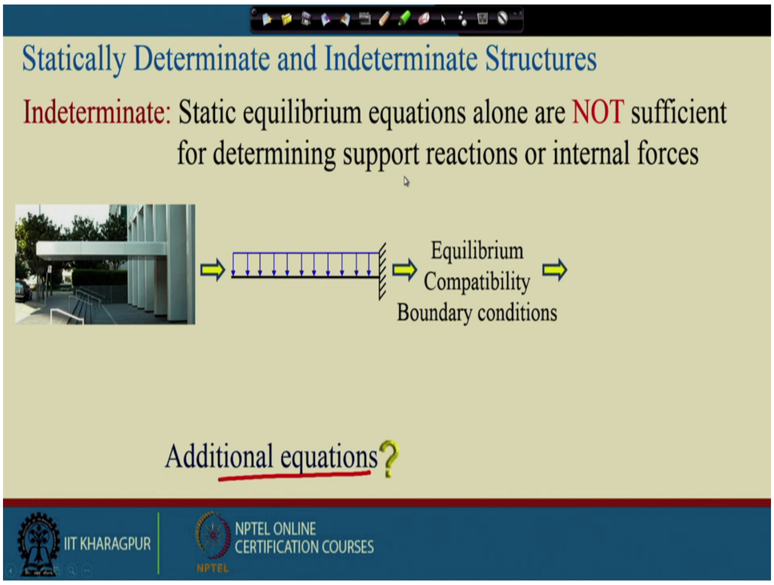

So for those, structures are called indeterminate structure.

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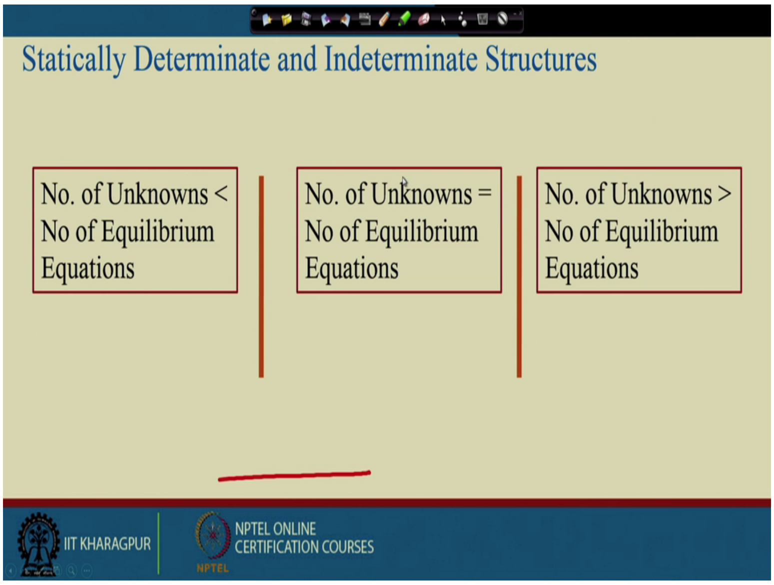

So, we have based on the based on the how many equations are available and it how many unknowns are unknowns present in our analysis, we can have three different three different three different cases. One case is when your number of unknowns less than the number of equilibrium equations and another case is when your number of unknowns are exactly equal to the number of equilibrium equations, and number of unknowns are greater than the number of equilibrium equations.

In the first case is the second one is determinate structure, because your number of unknowns are same as equilibrium equations equilibrium equation can be used alone to solve the number of total unknown. And then second if the number of unknowns are greater than number of equilibrium equation. So, we need additional equation without that additional equation the structure cannot be analyzed. And then the first one the physical interpretation of that is the if the number of unknowns are less than equilibrium equation, the physical interpretation is the structure becomes unstable.

You see here your unknowns are only two unknowns vertical reactions here vertical reactions here, but the equations are available is three equations are available. Summation of forces in vertical direction, summation of forces in horizontal direction and summation of moments are 0. So, this is unknown this is this is an unstable case right. So, what we do next is, we will see once we have understood what is or rather once we have revisited, what is indeterminate structure and what is determinate structure then next question comes is how to quantify the indeterminacy of a structure.

If it is determinate then it is very straight forward you have an equilibrium equation exactly same as the number of even unknown and solving. But if in case of indeterminate structure, where your number of equations are number of unknowns are more than the number of number of equations, then the question comes is how many additional equations are well how many additional equations are required. So, we need to quantify the indeterminacy of the structure.

So, next class what we do we will see, how to determine indeterminacy of a structure a two kinds of indeterminacy, one is static indeterminacy and kinematic indeterminacy. Then, we stop here see you in the next class.