Matrix Method of Structural Analysis

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Lecture – 19
Matrix Method of Analysis Trusses (Contd.)

Hello everyone, welcome you all. Now, in the previous class we discussed how to enforce boundary conditions once you have already assembled the stiffness matrix. And then we discussed how to partition this stiffness matrix based on the known information about degrees of freedom and then you can solve for unknown displacement.

Now, what we do today is once you have obtained the unknown displacement how to determine the next information that we need about the; an about the structure is what are the boundary, what are the support reactions and the member forces.

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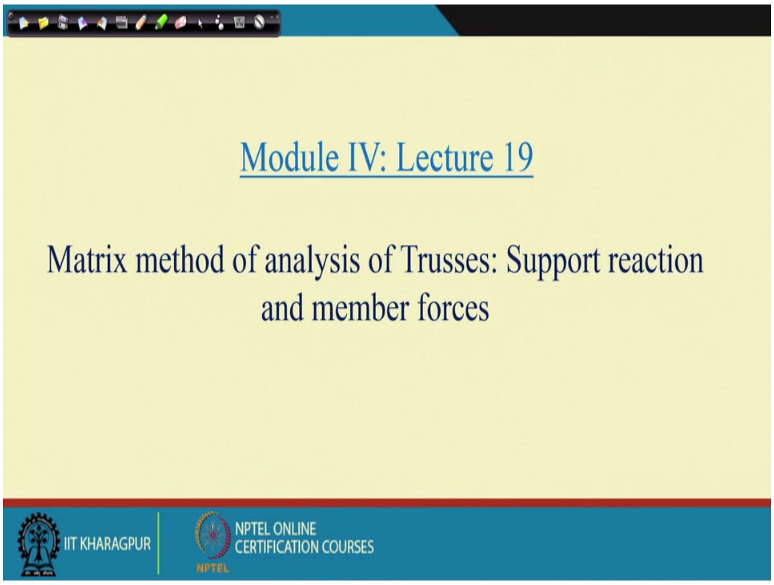

So, today that is what we are going to see. So, today is analysis of truss, support reaction and member forces.

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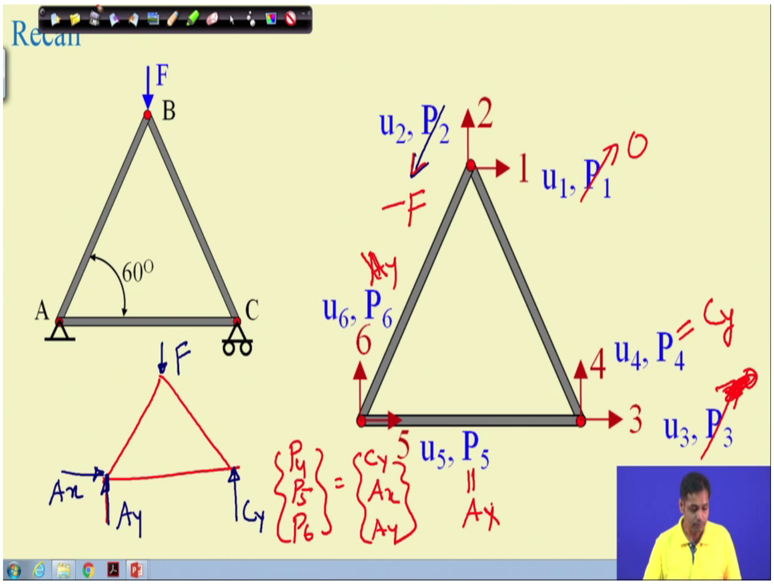

Now, recall this is the problem that we are discussing, and then these are the corresponding degrees of freedom and the forces and then we have the global stiffness matrix like this.

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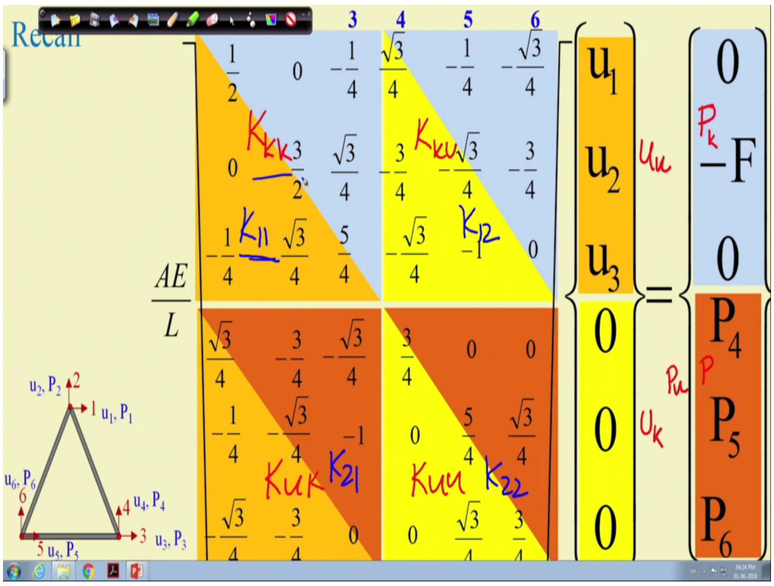

And this global stiffness matrix relates the global load vectors and the global displacement field. And then we know some of the information about the displacement and that information is given here and we know some of some information about the force field and those information's are also those information's also given in the load vector, ok.

Then if you recall what we do is we partition the matrix partition the all the load vector displacement vector and consequently the stiffness matrix as well. And the partitioning is done based on what information you know and what information you do not know. So, in this case we this was our u, we do not know this information so this is u u, we know this information so this is u known. And the fault force vector this is force this is P known and this is P unknown this is P unknown, ok, this is how we partition the load vector.

And corresponding we once we have partitioned the load vector and the corresponding the stiffness matrix can also be partition like this, ok. So, this portion, this portion that the this portion gives you, ok. So, this we that is we name this is K known a K kk and this is we used K ku and this is we used K uK, though uk and ku they are same because the matrix the entire matrix is meta matrix is symmetric matrix and this was is K uu. This is how we use the partitioning, but there is no hard, and fast rule that we use the these you use kk K ku something like this in there are many books where instead of K kk it is written as say it is written as say K 11, K 11, K 12 and then K 21 and K 22.

So, essentially how you define, how you represent it is not that important. Important is you understand the partitioning either you can represent this matrix by this or these or any other that you were comfortable with ok.

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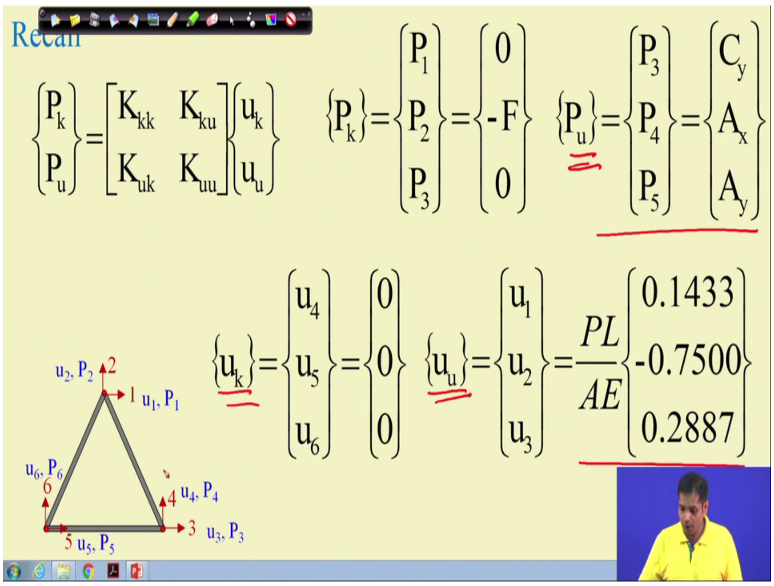

Now, once we have that then if you recall that we can write the entire load displacement relation like this after partitioning we can, this is the entire load displacement relation and where P K are the P 1 P 2 P 3 which are 0 which are which are 0 minus F and 0 and then P u is equal to this, ok.

Here you see if I if I go back, if I say here you see if I draw the if I draw the stiffness if I draw the free body diagram of this of this structure, free body diagram will be, there will be a support reaction there will be a support reaction this is say Ay and this is say Ax and this is Cy, this is Cy, and the force is this is the force F we have, ok.

Now, from this free body diagram if we compare this free body diagram with this with this diagram, then we know that P 2 is equal to minus F anyway we discuss it already P 2 is equal to minus F and P 1 is equal to 0 there is no force and similarly P 3 is equal to 0. But then other 3 unknown forces they are related to support reaction as this. So, sorry not this one yeah ah other 3 forces are related to like this say P 4 is equal to Cy and then P 5 is equal to Ay and P 6 is equal to this is a Ax ax and P 6 is equal to a y.

So, if we write P 4, P 4, P 5 and P 6, P 6 they are essentially Cy cy Ax and ay. So, if we can determine these unknown force unknown unknown P's so that is equivalent to determining the reactions, ok. Now, so this is the partitioning. Now, so that that is what he written here, so P unknown is equal to which was P 3 P 4 P 6 is equal to support reactions at Cy, Ax and Ay. So, this is unknown that we have to determine.

Now, u unknown that we already determine in the last class that was the solution from the last class and then u known already 0 because that is what the boundary conditions, ok. Now, what we will do today is we will determine what is P u which is support reactions and then once we have the support reactions how to then forces in each member the internal forces we have to determine, ok.

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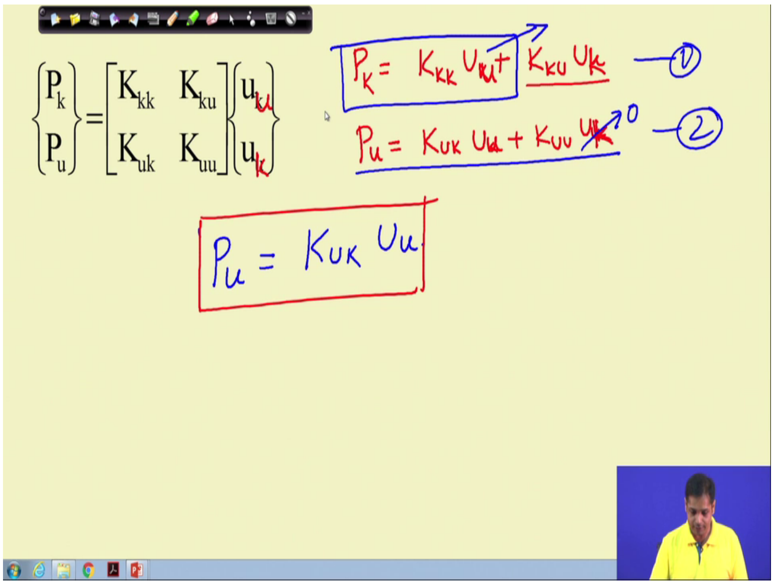

Now, let us start with this expression. Now, this is the expression and if you recall this expression was written like this we had P K is equal to K kk into u K plus K ku into u u and P u is equal to P uu is equal to K uK into u K plus K uu into u u, ok. Now, ah this is this is u this is K there is a mistake, ok. So, this should be u this should be K and this should be u this should be k right should be k, ok.

Now, this is anyway 0, so we use this expression we use this expression to find out the unknown reaction, ok. And this was x equation number 1 and I told you that how to use this equation number 2 we will be discussing ah in lecture 4. Now, that is what we will be doing now. Now, we can we will be using this expression to find out the unknown forces, ok.

Now, this u k is anyway 0, so we can straight away we can say that this is 0. So, this expression finally, becomes P u is equal to K uk, u u. This expression becomes this, right. Now, what is u k? u k is, if you go back to the expression u k will be this part will be this part, ok. So, this into this plus this into this is equal to this, ok. So, write it here now.

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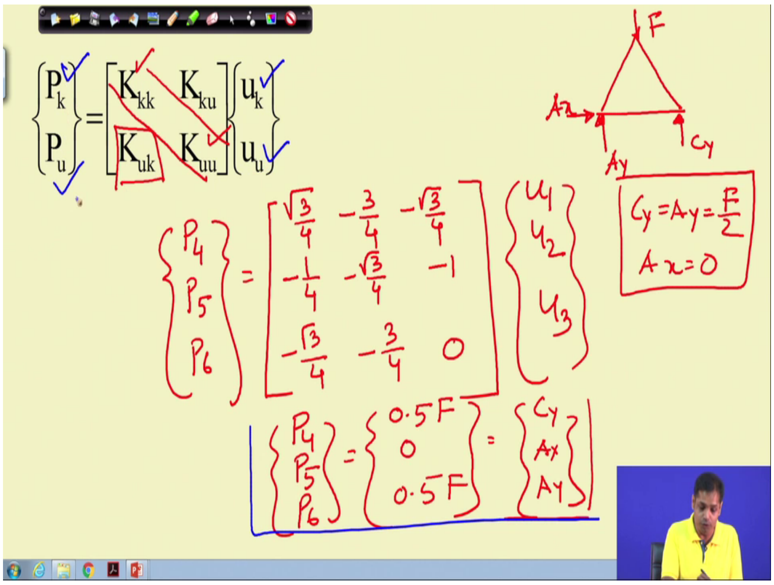

So, P, so we have expression this we have. Now, P u which is is equal to P 4, P 4, P 5, P 6 are unknown is equal to the stiffness matrix and then the displacement u 1, u 2 and u 3. And this matrix is root 3 by 4 you can check from the global stiffness matrix minus 3 by 4 and then minus root 3 by 4. Then minus 1 by 4 minus root 3 by 4 and then minus 1 and then minus root 3 by 4 and this becomes minus 3 by 4 and then become 0, ok.

Now, now, do not hear the diagonal one of the diagonal term is one of the diagonal term is nonzero ah that is fine because that does not ah the that is not against the against the property of stiffness matrix that you decide because this K the diagonal terms gives you by these 2 expression this one and this one. So, we are actually this one is K uk term which is off diagonal off diagonal matrix and essentially. So, naturally it can have negative entries, ok.

Now, then we know u 1, u 2, u 3, just now if we substitute u 1, u 2, u 3, u 1 as from the previous from the previous expression u 1, u 2, u 3, you check u 1, u 2, u 3, is this. Now, if I substitute that u 1, u 2, u 3, in that expression what we get is, we get u we get unknown forces and the solution will be P 4, P 3, P 4 are P 4, P 5, P 6 they are is equal to 0.5, 0, 0.5 or 0.5F, 0, 0.5F these are the force.

What are P 4, P 4, P 6? If you recall P 4 was Cy this was Ax and this was Ay and from the static equilibrium condition if it is subjected to force F and these are the your support reactions by applying the this is Cy, Ay and Ax. By applying the equilibrium equation we can easily say that we can solve this Ay a Cy is equal to a y is equal to F by 2 and a x is equal to 0 that is the solution you can, if you can directly apply equilibrium equation you get it. And then you are getting the same expression here.

You may ask that once you have once we can directly get the solution for by equilibrium equation what is the purpose of all these exercise. The purpose of all these exercise is not for the structure where you can easily solve using manual calculation. These exercise has to be done for basic weight for a structure which is in determine at the very large a very large structure where the manual calculations are really not feasible, ok.

For demonstration we have taken an example where that example can be solved by equilibrium equations and we are solving through the matrix method of structure energy so that the solution can be compared easily. So, this is the support reactions we can obtain from this. So, in this expression now, all these u k is known u unknown is determined already and then P K is known and P unknown is this. So, in this expression all these displacement vector, force vector everything we know.

So, what we know about the structure as of now, we know the displacement of all the joints and we know the reactions at all the reactions and the joints wherever we have supports. Now, recall when we analyze truss along with the reactions we also need member forces, ok. Now, next see how to determine the member force, ok.

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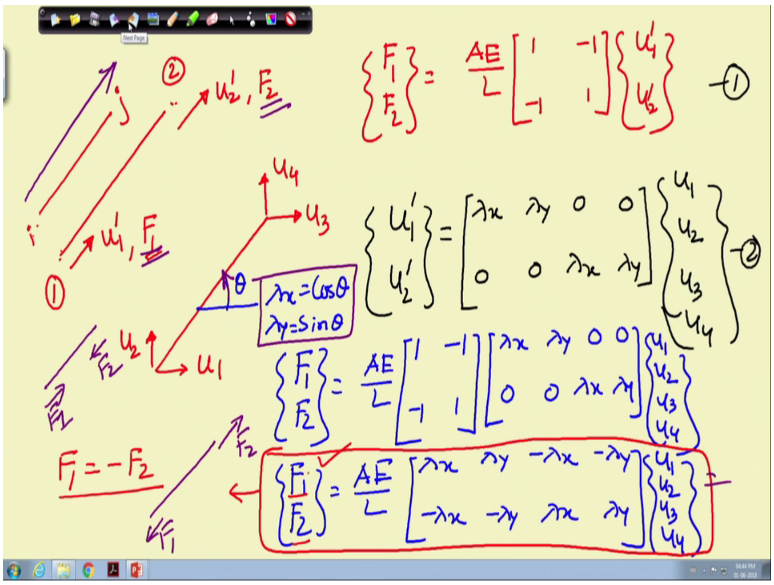

Now, ok; you see take if we recall if we have a if we have a truss member any arbitrary member like this and this is number 1 and this is number 2. And then we have degrees of freedom as u 1 dash and here degrees of freedom as u 2 dash and it is F 1 and it is F 2, then recall the first class we discuss these F 1 and F 2 they are related to u 1 dash and u 2 dash as this AE by L and then 1, minus 1, minus 1, 1 and this is u 1 dash u 1 dash and u 2 dash, that in that this expression already we know.

Now, we also know that again for the same structure the if we if the degrees of freedom are u 1, u 2 and then it is u 3 and u 4, and this angle is say theta, this angle is theta then we also know that that u 1 dash; that u 1 u 1 dash and u 2 dash they are related to u 1 they are related to u 1, u 2, u 3, u 4 as lambda x, lambda y, 0, 0, 0, 0, lambda x and lambda y right and then this is u 1, u 2, u 3, u 4 that is the expression we know, ok.

Now, if I substitute this expression, this expression say expression number 2 and this is expression number 1 if I substitute expression 2 into expression one what we get is this, we get F 1 F 2 that is equal to AE by L you can parallely try this. If we substitute this is minus this is 1, minus 1 and then minus 1, 1 and this become lambda x, lambda y, 0, 0, 0, 0, lambda x, lambda y and this is u 1, u 2, u 3, and u 4 where lambda x is equal to cos theta and lambda y is equal to sin theta, ok.

Now, if I if I do that what we get is F 1, F 2 that is equal to AE, this is just simple matrix multiplication AE by L this is lambda x, lambda y this is minus lambda x and minus lambda y and this is minus lambda x, minus lambda y then lambda x, lambda y. And then this is u 1, u 2, u 3, and u 4 this is important, this expression is important.

Now, if you observe this expression what you get is F 1 is equal to minus F 2, right. Either you can use F I can you compute F 1 or F 2, but they are opposite, they are opposite because which is again is consistent with the fact that the all the class members are 2 force members so the force that we have always along the along the longitudinal axis of the truss, ok. So, which says that this F 1, this F 1 and F 2 they will be equal and opposite and whether they are whether F 1 is positive or F 2 is positive depending on that we can say the member is either in compression or in tension.

For instance, if your numbering is like this means if your if you are starting from one and if your movement is like this direction. So, F 1 is from 1 to 2 and F 2 is the same direction. If F 1 is positive means F 1 is your that is the direction we considered as positive if we if your forces are in this direction and F 2 is in this direction suppose your F 1 is this direction and F, F 1 and F 2 is this direction the member is under compression. And if your F 1 is in this direction and F 2 is this direction we can say his member is under tension.

So, if F 1 is positive and F 2 is negative we say it is compression member if F 1 is negative and F 2 is negative we say it is tension member. Now, F 1 and F 2 its not arbitrary it depends on which is your starting know starting joint node and what is your end node because that is how you calculate your define angle theta angle theta is a measure in this direction, ok. So, this is now, this expression will be shortly we will be using to calculate force in the members. Let us do that, ok.

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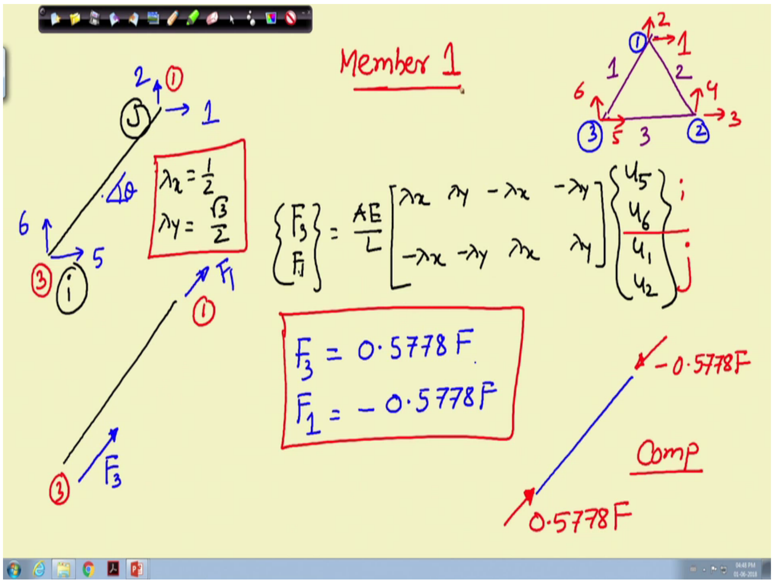

Now, we have 3 we have this was the class, we have member number 1, member number 2 and this was member number 3 and then your node numbering was in this it was degrees of freedom 1 2 3 4, it was 1, then 2, 3, 4, then 5 and 6 this is how we number the degrees of freedom, ok. Now, and this node number is use different color say and your numbering was this was your node number 1, this was our node number 2 and this was our node number 3, ok. Now, let us find out what is the forces in member 1 first.

The member 1 is the ah I am trying to write all these steps very explicitly, but really when you just for your understanding, but when you actually compute them you do not have to do there are way you can quickly do it ok. So, this is the member number 1, where your this is node number 3 for member 1, this is node number 3 and this is node number 1 this is how we defined, ok. And these are the degrees of freedom this is your 5, this is 6 and this is 1 and this is 2 right and then this is the, this is this angle is theta, this angle is theta.

And if you refer to our previous classes we for different members what are the lambda x and lambda y are given, in this case the lambda x is equal to lambda x is equal to half and lambda y is equal to root 3 by 2 that we have already, ok. Now, so in this case if I go back if I see the previous one this expression we will be using. Now, this expression we will be using.

Now, in the previous case you see it was node number 1, this was node number 2 that is why it is we wrote F 1 and F 2, but if it is node number 3 node number 4 we can write F 3 and F 4. Similarly for any arbitrary if it is i and if it is j then it is true for F i and F j, where the measure your the numbering from i to j. And this is again I am telling this is very important because that is how we measure the angle ok.

So, now, in this case your i is 3 and j is 1. So, expression will be F 3, F 3 and F 1 they are they will be related as AE by L this expression is lambda x, lambda y, then minus lambda x, minus lambda y and then minus lambda x, minus lambda y, lambda x and lambda y and then this will be your node number 3 your degrees of freedom at 5 6, this is u 5, then u 6, u 1 and u 2, ok. So, in this case actually this is our ith node and this is our jth node, ok. So, this is the force at ith node this is the force at jth node. Similarly these are the displacement these are the degrees of freedom for ith node these are the degrees of freedom for ith node these are the degrees of freedom for jth node, ok.

Now, so if I substitute the values of lambda and lambda x and lambda y in this what we get is we get F 3, F 3 is equal to 0.5778F and F 1 is equal to minus 0.5778F which is obvious because they are equal and opposite. Now, let us understand whether the member is under tension or compression how we, how we do that how we understand that. Draw this member once again.

How we this was this was the num this was 3 and this was 1, this was 3 ah this was 1. Now, how we define the forces? We define the force like this. So, this was F 3 and here it was F 1 right. Now, F 3 is positive and F 1 is negative it means your force in this member is this and these value is 0.5778F and this is minus 0.5778F. So, member is under compression, compression, ok.

You again I said when you actually do it using computer then you do not you do not really draw it and then check whether tension and compression just based on the sign we can say whether the member is under tension and compression. These exercise I am doing so that you can correlate you can understand how to identify a member compression or tension. So, this is for member 1, this was for member 1, member 1.

Let us do it for member 2. Member 2 is, member 2 is this member and so draw it.

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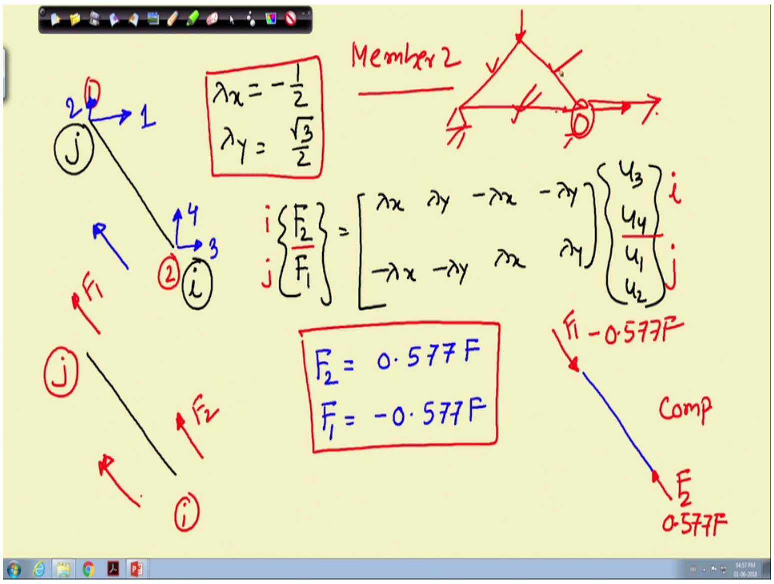

Member 2 was, is like this where if you recall this joint is 1 and this joints is 2 this joint is 1. So, this joint is this joint is 2, this joint is 1 and then also it is 3 4 1 2. So, it is 3 4, this is 3, this is 4, this is 1 and this is this is 2, ok. Now, what else? Ok, and also we again with lambda x in this case lambda y and these; this was numbering if we number it from like this from this direction means 2 to 1 this is your i, this point is i, this point is i and this point is j. If this point is i and j, means i to j then for this member lambda x is equal to minus half and lambda y is equal to root 3 by 2, that is already there if you check our previous lectures, ok.

Now, if it is then for this what we what we have is ith force, ith force is F 2 say F 2 and jth force is ith force is F 2 and jth force is F 1, F 1 that is equal to same expression lambda x, lambda y, minus lambda x, minus lambda y, minus lambda x, minus lambda y, lambda x and lambda y. And then here we have displacement at ith node and the displacement at jth node ith node degrees of freedom are u 3, u 4 and then jth node is u 1, u 1 and u 2 right. So, this is for ith node and this is for jth node. This is for ith node and this jth node.

Again if we substitute lambda x and lambda y in these expression we get expression for F 2, expression for F 2 is equal to F 2 as 0.577F and F 1 is equal to minus 0.577F that is the expression for F 1 and F 2.

Let us see where the member is tension and compression. If we if we draw this member once again it is it was a ith node it was a ith node and it was jth node. So, F 2, F 2 is in the ith node and you are moving in this direction i to j. So, your F 2 is, F 2 and F 1 is this, ok. So, in this case, so essentially the member is then the member is F 2 is this the forces are like this, forces are like this and F 1 is opposite direction F 1 is opposite direction this is F 1 this is F 2 both are same because is a 2 force member. So, this is 0.577F and similarly this is minus 0.577F. So, member is under compression, ok.

So, by intuition we can say that if you have a truss like this and you apply a force like this and actually this member and this member will be under compression. These joints will go in this direction. So, this member will be under tension. Let us see whether we get the same thing or not. So, this was for member number 2, member number 2. Let us do it for the last member, member 6, a member 3.

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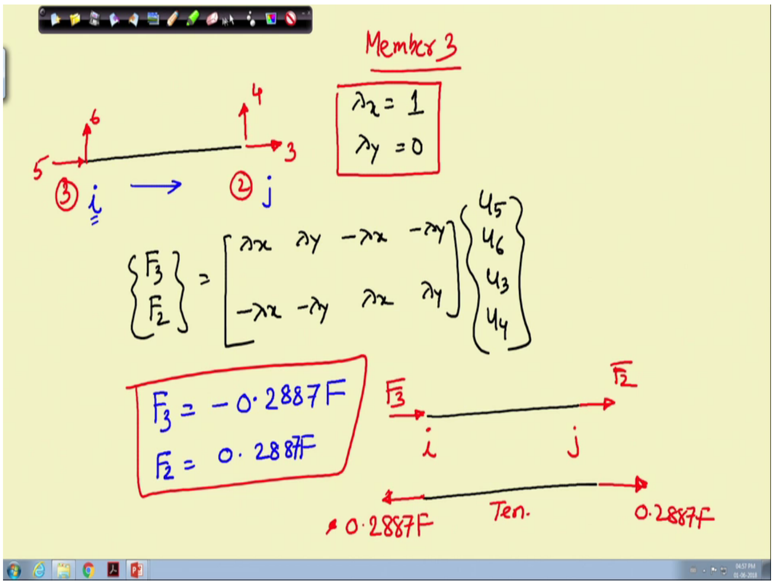

So, member 3, if you look at member 3 is this where your I ith I is it was the member 3 lambda x and lambda y defined as this. So, let us draw member 3. So, member 3 is horizontal member where this joint is this joint is 3, this joint is 3, this joint is 2 and these are the degrees of freedom, these are the degrees of freedom. This is, this is 3, this is 4 and this is this is 5 and this is 6 right.

And then for this member 3 lambda x and lambda y. Lambda x you have lambda x is equal to 1 and lambda y is equal to 0. And you get this lambda x and lambda t while computing lambda x and lambda 3 it is assumed that this is ith node, this is ith node and this is jth node that is why you get theta is equal to 0. And if you get if you take this as a ith node this as jth node and move from this to this direction you get theta is equal to pi. So, for this i is equal to and then you get line lambda x one you get lambda x is equal to minus 1, ok. So, we are moving in this direction ith point this jth point is.

So, now, write the expression for F 1 a the; then this case this F 3 the force at ith point and force at jth point that is equal to the same expression lambda x, lambda x, lambda y, minus lambda x, minus lambda y, minus lambda x, minus lambda y, then lambda x and lambda y. And the degrees of freedom will be since we are moving from the ith point degrees of freedom and jth point degrees of freedom ith point degrees of freedom u 5 and then u 6, u 3 and u 4, right.

Then you substitute lambda and lambda x and lambda y here if we do that then we get finally, F 3 is equal to F 3 is equal to minus 0.2887F and F 2 is equal to 0.2887F. So, they are opposite direction and this is opposite direction, ok.

Now, let us try to understand what is the what is the weather is the compression or tension, member force is this and then ith point is ith this is our ith point and this is our jth point. So, ith point force is in this direction and this is our this is F 3, F 3 and jth point force in this direction which is F 2, F 2. Now, look at this value F 3 is negative, F 2 is positive, so essentially in this case the forces will be, this is and the force here is this and this value is 0.2887F and this value is 0.2887F. So, this is under 10. ah

This is negative actually when you are showing in opposite direction you do not have to write negative. I wrote it in the previous slides, but when you are showing the direction so this negative is does not does not make any sense, ok. So, so this means it is under, tension it is under tension which is very obvious for this member for this member as this deflects this these joints goes this direction, so this member will be under tension and you get this member under tension.

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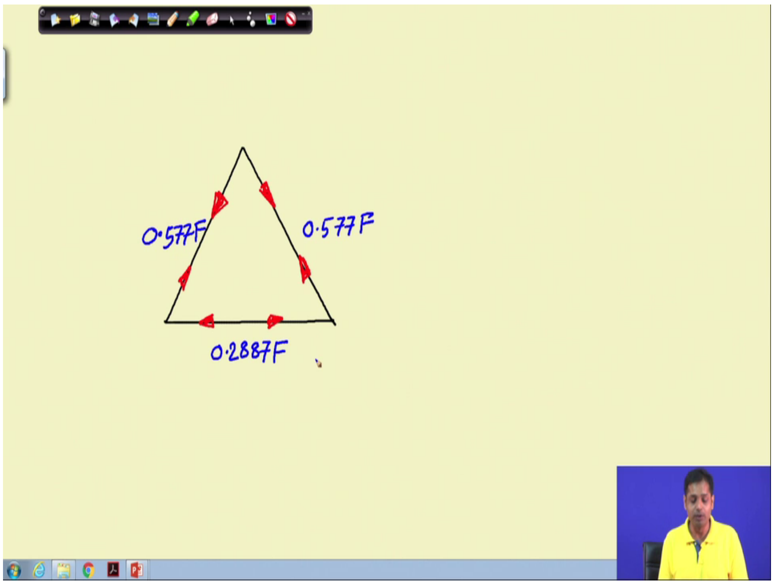

So, now, the member forces are we can write we can draw the member forces as this. This is the truss. We just now, saw this member is under compression, this member is under compression and this member is under tension right. And the values are this is this is 0.2887F, this is 0.577F and this is 0.577F, you may write in 5 decimal 2 decimal any decimal places you can check whether the equilibrium is being satisfied or not. So, this is the final solution this is the internal force member and then support reactions are the displacement we have already determined, ok.

Now, since you are actually doing some numerical calculation you may not get exactly the equilibrium because of the round off error how many decimal places you stored the value depending on that whether you get 0 or very small number. So, this is how we can determine the member forces. So, what we have done so far we took a truss and then we discuss how to write these stiffness matrices for each member and then we discussed how to write the global stiffness matrix by assembling the all the member stiffness matrices.

Then, we discuss once you have the global stiffness matrix how to apply the boundary condition and then consequently partition the matrix. And then we discuss how to determine unknown displacement, once we have determined unknown displacement we also discussed how to determine the unknown support reactions and then finally, unknown member forces, ok. So, this is the step we follow in matrix method of analysis of trusses, ok.

Now, one thing is very important here is that you will come across as you will probably appreciate once you actually solve some example doing for large problems how you numbering a system, how in what order you are numbering the nodes, how you are numbering the members that is very important. That will not discuss right now, let us first understand how to implement this, how to do this exercise is in computer then that time we will discuss how to give the numbering of different members and nodes, ok.

So, we will stop here today. Next class what you do is we will see some more example of some more example of um, some more examples. Some more members will not solve the here the purpose of all these exercise the first 4 lectures purpose has been to demonstrate entire calculation. In the next class also we will see some of some example couple of examples we will just briefly discuss these steps.

Not doing any calculation, we will see how to what would be the size of the global stiffness matrices, how to what are the known vector displacement vector known force field and then what would be the corresponding partition of the matrixes. Those we will discuss in the next class. Ok then, I stop here today. See you in the next class.

Thank you.