Matrix Method of Structural Analysis
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Lecture – 17
Matrix Method of Analysis of Trusses
(Contd.)

Hello everyone. In the last class we discussed how to determine the member stiffness matrix of a truss, ok. What we do today is we will see we will compute the member stiffness matrix through an example, compute the stiffness matrix for different members and then how those stiffness matrixes are need to be assembled to get the global stiffness matrix, ok.

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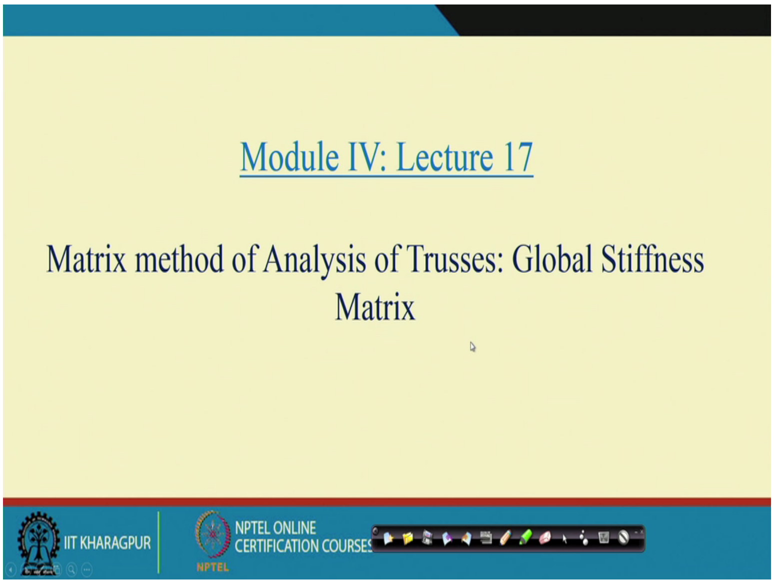

Now, so today's topic is global stiffness matrix.

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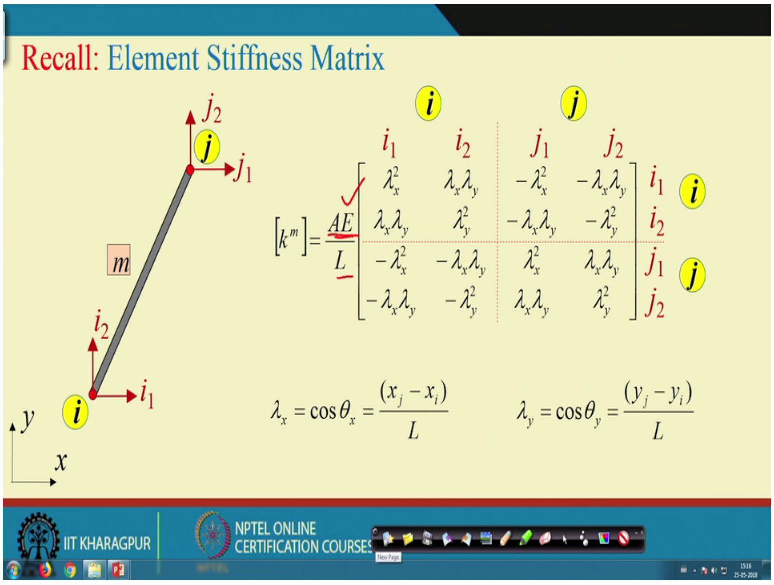

Recall, this is the last class we discussed if we take an element any arbitrary element of truss which as these are the degrees of freedom and then these are the degrees of freedom, corresponding degrees of freedom. And then the element then the element stiffness matrix with respect to which, which relates the global load vector which is with the global displacement degrees of freedom with respect to the deformation forces defining global coordinates as this and lambda x, lambda y at this, ok.

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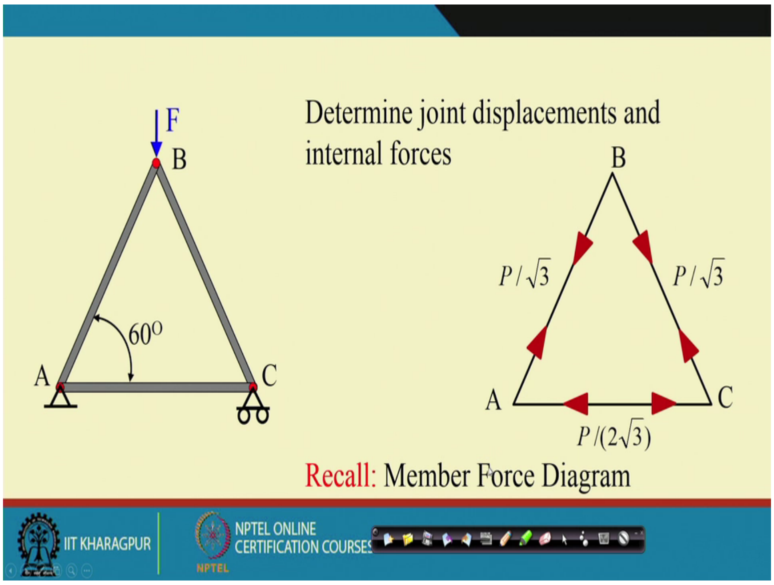

Now, so let us take this example that is the example we started our discussion.

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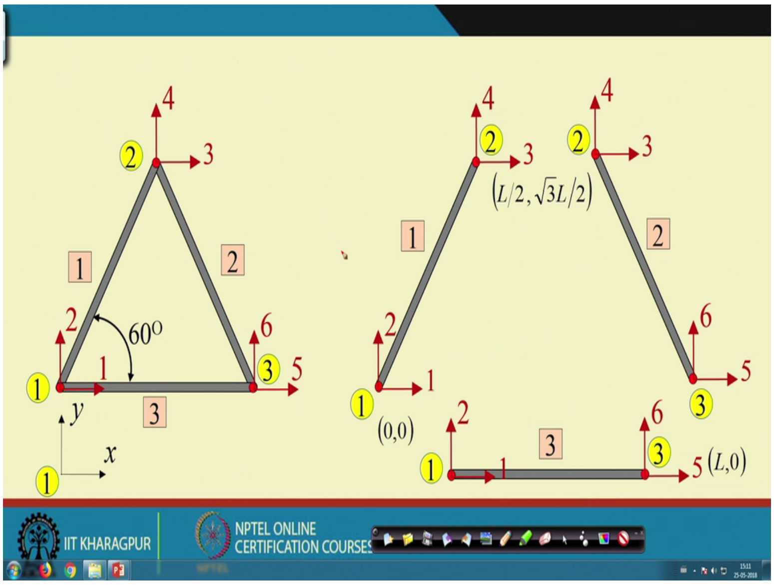

Now, if you recall the, these are the 3 members forces and corresponding degrees of freedom at each joint and then we have 3 members separately. And then this is the global coordinate system and then these are the joint coordinates if we this angle is 60 degree and these are the ok. Now, what we do is we will try to find out what is the stiffness matrix, global stiffness matrix for this for this problem, for this truss with 3 members, ok.

The reason why we chose a very simplified problem because all the calculations here I have to show you different steps so that the number of if the number of members are very large probably it will be difficult to show all the steps in a great detail. So, first for 3 members will be showing all the calculations everything manually, but really we do not do it manually for a large structure we will see how to do the similar operations using computer code, ok. Now, towards the end of this discussion, so this is the element stiffness matrix.

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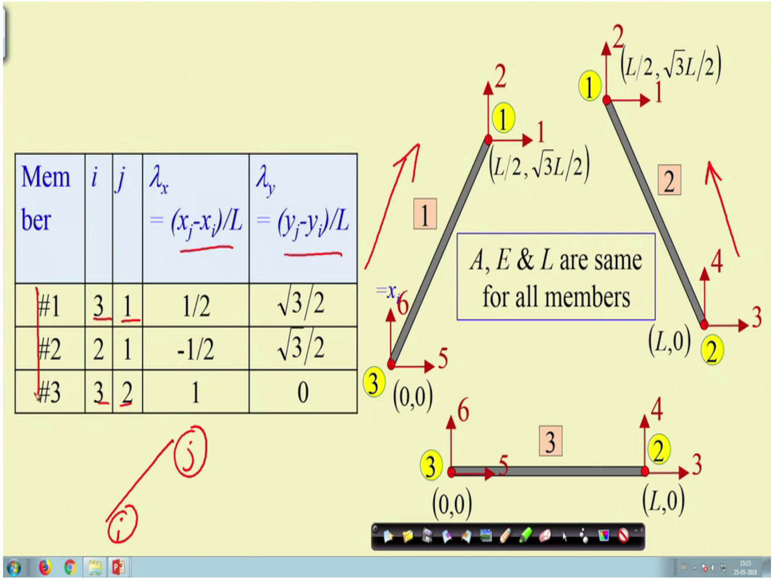

So, let us take. So, in element stiffness matrix what we need is we need lambda x and lambda y, lambda x is a, theta x is the, theta x is how this is with risk how it is well with the x axis and theta y is how it is how it is how it is how this is oriented with respect to vertical direction vertical direction this is these theta y, ok. Now, so but probably the expression that we derived that t that was we only took one theta and instead of theta y we define lambda y as sine theta so both are same, ok. Now, so for this member, we have to create a table where the member numbers different members numbers, we have the different members numbers and ij for different member number. You see, recall the last we have this is the is node and this is jth node, and then we have the stiffness, this is stiffness matrix for this.

Now, for depending on the deep member the i and j are different for different members. For instance for member number 1 i is node number 3 and j is node number 1. So, i is node number 3 and j is node number 1, so we are moving from this direction to this direction, ok. Now, for member 2 your i is say 2 and j is 1. So, we are moving from this to this direction. The reason why I am telling moving from this to this direction, the reason why and i and the identification of i and j not the identification the definition of i and j I could have taken i for this member is as this and j as this, but only thing is whatever i and whatever j we take we need to compute the angle accordingly that is it, ok.

So, similarly for this member i is 3 and j is 2, ok. So, if we do that now, lambda x which is defined as the, this which is essentially theta cos theta x and sine theta y or cos theta and sine theta this becomes for different I. Now, if we take ij set the opposite means if I take i is this and j is this only difference will be here sign of this sign of these values, ok. Now, so that flexibility we have.

But as far as the numbering is concerned this numbering comes concerned it is a 3 member, only 3 nodes we have the numbering is very straightforward. But for a larger structure where we have the many number of nodes many number of members the numbering is not very arbitrary I told you in the last class as well, numbering has to be done in such a way that the at the end of the matrix that you get the matrix gets the banded matrix that what is the banded matrix will discuss that, ok.

So, once we have come once we know what is the angle of orientation of different members then we can just substitute this lambda x and lambda y for different member to get these stiffness matrix, ok. Now, in this case an another important thing is for this problem we assume that we assume that area in a area Young’s modulus and length are same for all the members. But if they are not same then we have to calculate we have to consider for give for a given member when we calculate the stiffness matrix we have to consider the area Young’s modulus and length of that particular member. If area Young’s modulus then the same for all member it is same.

Usually will area and Young’s modulus in many problem that we come across area and Young’s modulus may be same or at least Young’s modulus will be same, in area and length are different for different members and that you have to that we have to take care of ok. So then, once we substitute that this then we get the stiffness matrix element stiffness, member stiffness matrices for different member as this, ok.

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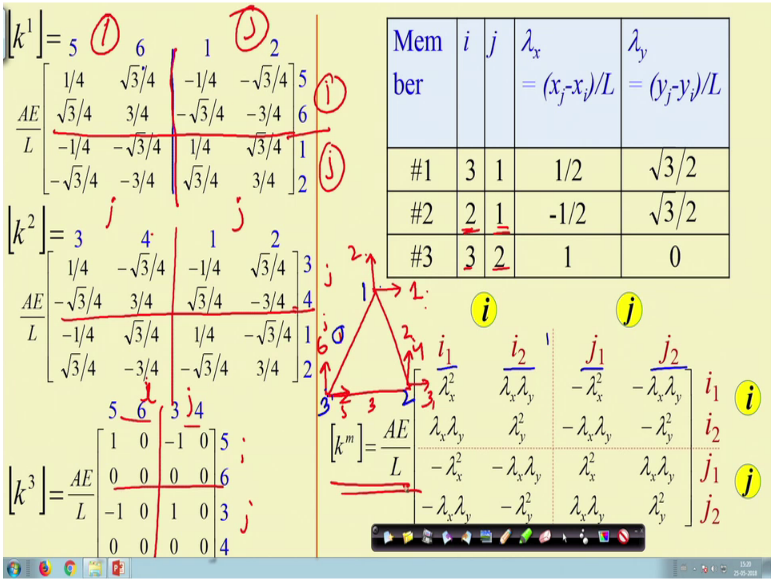

Now, if you recall that member 1 was if this is the if I draw this once again here, this was the member 1, this was member 2 and this was member 3. And the probably this was 1 degrees of freedom on1e and then 2, and then this is 3 4, this is 3 4, and this is probably 5, and 5 and 6. So, member number 1, member number 1, joint member number 1 which has joint which has which connects degrees of freedom 1 2 and 5 6, so member number 1 has 1 2 and 5 6. Similarly member number 2 where we have degrees of freedom 1 2 and 3 4, so we have degrees of freedom 1 2 and 3 4.

Now, again recall for member number 1, member number 1 i is equal to this point this was node number 1, this was node number 1, node number 2 and node number 3 so for this member we have i is this and j is this. So, when we write the stiffness matrix this block is for ith node, this block is for jth node, ok. And this block is for 2 degrees of freedom as the ith node, this is 2 degrees of freedom at jth node.

So, since Ii is 3 and j is one for member number 1, and the corresponding degrees of freedom at this point is 5 6 and then here at this joint is 1 2, so the elements will be here 5 6 and then 1 2. So, if I, if I this is so this is i for and this is j for first member, so this is i, this is i, and this is j for first member. Similarly, second member we took i is equal to 2 and j is equal to 1. So, i is equal to, i is equal to 2 and j is equal to 1 so it will be. So, this is i and this is j, i means degrees of freedom 3 4 degrees of freedom 3 4 j means degrees of freedom 1 2, degrees of freedom 1 2 and here also i and j. And similarly here it is 3 and it is 3 and 2 so degrees of freedom 5 6 and 3 4, so similarly degrees of freedom 5 6 and 3 4. So, this is i, this is j, this is i, then this is j for this member.

So, getting members once we have this expression, this expression, this expression getting stiffness matrix for different member is very straightforward. We calculate what is lambda i first define what is i and j for different members and form that calculate lambda x and lambda y and substitute lambda x and lambda y. And area length and Young's modulus for 4 different members in this expression and get the stiffness matrix for all the members, ok; Now, once we have the stiffness matrix for all the members the next step is we have to assemble the stiffness matrix; we have to connect this stiffness matrix. And how, it this is a very important step. Let us do that.

Now, you see another thing which is very obvious in this case. If we look at the element stiffness matrices then the size of the element stiffness matrices are 4 by 4 matrices because every element every member we have only 4 degrees of freedom to where it joints. Now, but these are for any member stiffness matrix.

Now, if we if we look at the index structure globally then in this problem at least we have 3 nodes at every node 6 degrees of 2 degrees of freedom so total 6 degrees of freedom. So, essentially the global stiffness matrix will be the size of the global stiffness matrix will be 6 by 6 in this case, but if you have different problems the size of the elements stiffness size of the global stiffness matrix will depend on the number of joints we have in a problem. But irrespective of anything the member stiffness matrix the size of the member stiffness matrix will always be 4 by 4, ok. Now, so let us let us find out.

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So, what we do is suppose these are the elements these are the stiffness matrix. Now, suppose this is our stiffness matrix or we do not know what is the value of this stiffness matrix yet, but what we do is we now identify it these are the, these are the, these are our node degrees of freedom 1 2 3 4 5 6. So, this is for essentially node number 1 and this is node number 2, node number 2 and node number 3, node number 1, 2 and 3, ok.

Now, so we have 3 joints, we have 3 joint, so this joint is for these joint is, these joint is joint number 1, these joint is joint number 2, and these joint is joint number 3. Similarly this is joint number 1 this is joint number 2, and this is joint number 3, ok. So, if we talk about this block, this block, this block tells you how this how the joint one number 1 the relation between joint number 1 how the force at joint number 1 and the force and the displacement and joint number 1 how they are related to each other that is this block.

Similarly, force at joint number 1 and displacement and joint number 2 related through this block. Similarly force at joint number 2 and displacement and joint number 1 related to this block and so on, ok. Now, first what we do is we first sub put all the values are 0. ok. This is a very important step because when we when we as when we do it first this is said initialization of the stiffness matrix, ok. We assume all the values of this stiff one what we know is we know the size of the stiffness matrix here is 6 by 6 and then all the components all the elements of the, this global stiffness matrix we make them 0. And then we see what joints are interacting with each other how the joint different joints are interacting with us each other and take that particular value and substitute in the global stiffness matrix, ok.

Now, so first take joint number 1 and joint number 1 You see joint number 1 and joint number 1 is this is joint number, here also we can write here also we can write. So, this is joint number 1, this is joint number 1 and 3 4, this is joint number 2, this is joint number 2 sorry, this is joint number 2, and this is joint number 2, this is joint number 2, this is joint number 2. And joint number 3 is 5 6, joint number 3 is 5 6, joint number 3, it is joint number 1, and joint number 1 and joint number 3 is 5 6, joint number 3 is 5 6, ok.

Now, so 1 1, 1 1, 1 1 we have this is this is 1 1, this block is 1 1, and then we have this block is 1 1 this block is 1 1, ok. Now, in when we assemble them then this value will be this block plus this block ok. And if we do that then this becomes so 1 by 4, this is 1 by 4, 1 by 4 here and 1 by 4 here, becomes half and then root 3 by 4 minus root 3 by 4 becomes 0, and then 3 by 4 3 by 4 becomes 3 by 2, and minus 3 by 4 3 by root 3 by 4 becomes 0.

Now, what does it, what does it physically means? Again go back to our second week when we discussed the basic concept of displacement method. If you remember what we did is we will then apply the equilibrium kind of once we have the forces and displacement relation for each member separately, and then we at the time of assembling them at the time of jointing them we applied the equilibrium equation at every joints is not it. And how this equilibrium equation we applied? We apply the force at this joint should be equal to the force at this force at these, force from one component and the force from force from another component when they are joining together the total force will be this plus this. It is essentially a it is essentially the same thing we are doing here, ok.

So, this is done so next see joint number the next joint is joining this. So, joint number 1 and joint number 2. Joint number 1 and joint number 2 is they are they joint number 1 here and there is no joint number 2 here joint number 1 here and joint number 2 here. So, let us use different color so joint number 1 and joint number 2 here and there is no joint number and joint number 2. So, joint number 1 and joint number 2 becomes 1, 1 and 2 become this. So, this becomes, this is this is minus 1 4, minus 1 by 4 root 3 by 4, root 3 by 4, root 3 by 4, root 3 by 4 and minus 3 by 4 and then minus 3 by 4, ok. There is no other joint number 1 and to joint 1 and 2 is interacting with each other only in this member, ok.

Now, let us joint number 1 and joint number 3. Joint number 1 and joint number 3 is we do not have any joint number 1 and joint number 3 here so joint number 1. And, so then what will happen? Your these becomes, this becomes this joint number 1 and joint number 3 where is joint number 2 joint number 1 joint number one here and then 3 4 5 6, oh yeah, it is it is not 3 it is that is why this is joining for this is joint number 3 right, ok.

So, joint number 1 and joint number 3 interact, interact here. So, this interaction becomes if we take these becomes joint number this is joint number 3 this is joint number 3 right. So, this becomes joint number 1 and joint number 3 become this. So, this is minus 1 by 4 minus 1 by 4 minus root 3 by 4 minus root 3 by 4, root 3 by 4, root 3 by 4 and then minus 3 by 4, minus 3 by 4. So, this gives us how joint number 1 is related to various other different other joints.

Let us now look at this block this block this block is how joint number how joint number 1 and joint number 2 is related. Joint number 1 and joint number 2 related here, so this becomes so this becomes joint number 1 and joint number 2 is this 2 and 1 is this. Again the same thing we can have we can go back yes, so it is essentially the same thing here so, this, this, this, this ok.

Now, similarly we can have joint number 2 and joint number 2 joint number 2, joint number 2 related here like this. You see joint number 2 and joint number 2 is here and joint number 2 with joint number 2 is these see this. So, it will be this, this plus this, so it will be 1 by 4 plus 1, means 5 by 4 then root 3 by root minus root 3 by 4 minus root 3 by 4 and then 3 by 4, 3 by 4, ok.

So, next is then joint number 2 with joint number 3. Joint number 2 and number 3 becomes this, joint number 2 and joint number 3 is this. And then we have joint number 1 and joint number 3, joint number 1 and 3 they are they relate here joint number 1 and joint number 3 so this is this becomes this. So, this becomes minus 1 by 4, minus 1 by 4, minus root 3 by 4, minus minus 3 by 4, minus 3 by 4 here and then minus root 3 by 4, minus root 3 by 4 here ok.

Now, next is your joint number 3 and joint number 2. So, joint number 3 and joint number 2, joint number joint number 3 and joint number 2 is this or this same value, so this is same. And finally, joint number 3 and joint number 3 joint number 3, joint number 3 is joint number 3 here we have joint number 3 interact with joint number 3 and then joint number 3 interact with joint number 3.

So, this become so this becomes 1 by 4, 1 by 4 1 by 4 plus 1 5 by 4, then root 3 by 4 and then 0 root 3 by 4, root 3 by 4, 0 root 3 by 4 and then finally, 3 by 4 and then 3 by 4. So, this then you will look at this all the blocks that we have we considered and then this is assembled in a global stiffness matrix form. So, this is our global stiffness matrix we have AE by L here, AE by L here, this is the global stiffness matrix, ok. Now, so once we have this global stiffness matrix. So, yes, so this becomes the global stiffness matrix ok.

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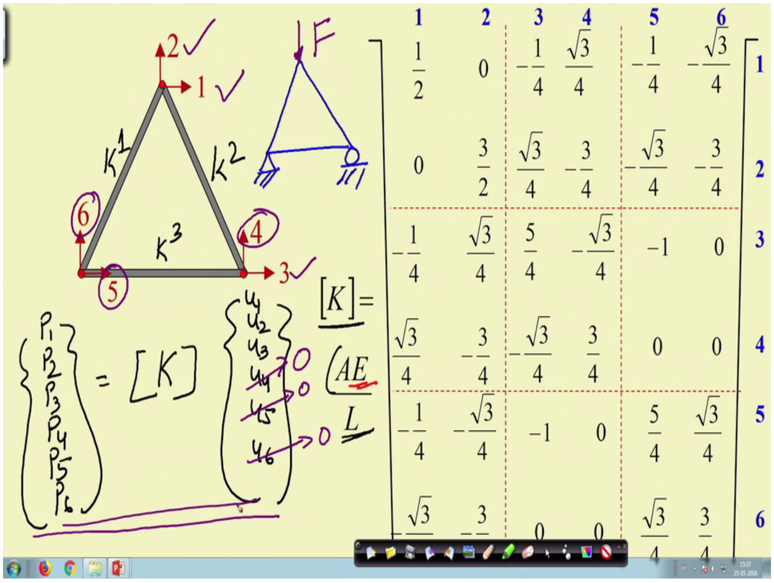

Now, once we have the same exercise we can do for any other cross member. Essence remains same, procedure remains same, but the assembling stage is the very important stage and where we can do it for manually, we can do it manually for this simple problem, but really we cannot do it for large problem. So, assembling and all everything we can do it using computer, using computer codes, but the essence remain same.

So, once we have these elements global stiffness matrix then what this global stiffness matrix does is this. So, we have total force says P 1, P 2, P 3, P 4, and then P 5, and then P 6, ok. And corresponding displacement we have or the degrees of freedom we have u

1, then we have u 2, u 3, u 4, u 5, and u 6. And this global stiffness matrix relates this. Element stiffness matrix like a deep member stiffness matrix K 1, K 2 and K 3; K 1 relate 1 2 5 6, K 2 1 2 3 4 and K 3 relates 3 4 and 5 6. But when we assemble them all then we get the global stiffness matrix which relates the forces and the displacement defined with respect to global coordinate system.

Now, once we look at the stiffness what are the information we need to calculate the stiffness matrix we need only the geometry, and the info information about the geometry because the length and the area is required and also we need information about the material property because the Young's modulus is required. So, this stiffness matrix does not depend on what are the load you are applying on the structure, what are the boundary conditions it depends only on the geometry and material of this structure.

So, based on that geometry and material we calculate the stiffness. So, this stiffness matrix it is the property of a given structure. So, if we take a tooth same structure subjected to different kinds of load their response may be different they responds differently because they are subject to different kinds of load. But the stiffness matrix if the geometry and material property configuration everything same the stiffness of mem of this structure remain same irrespective of the applied load we are giving on this structure, the at least the load that we are considering in this analysis, ok.

Now, next is once we have the stiffness matrix, the next is two another information we have to we have to give here. We have already given the information about the material property, we are given the information about the, we have given the information about the geometry. Now, other two information required information about the boundary condition and information about the forces. Next what we do is we will see how to apply these boundary conditions, ok. You see out of we have 6 degrees of freedom here, but actual structure actual structure means the structure that we started way the structure was like this if you recall. It where it was a hinge support and it was a roller support right and then it is subjected to a force something like this a force like this, ok.

So, naturally since it is in support u 5 and u 6, u 5 and u 6 this has to be 0 u 6 has to be 0 is not it. And then since it is roller support so u 4 has to be 0, u 4 has to be 0 so this is 0, this is 0 and this is 0 so that we know from the boundary conditions. Then essentially we have to solve for u 3, u 1 and u 2 so we have to solve for 3 unknown, but now, is essentially the matrix is 6 by 6 matrix in this case. But really we do not have to solve for entire 6 by 6 matrix we have to solve only for 3 unknown because other 3 unknowns are known ok, which is 0 here or it may be any other specified value at a given joint.

So, we will see next is how these boundary conditions to be enforced in this, this is enforced in this structure and once I write this the force displacement relation this is the mathematical representation of this structure. Now, in this mathematical representation is incomplete this representation because we have not, as of now we have not given the information about the boundary condition and the forces.

Next class we will see how to give these information in this expression and then and then how to solve for the rest of the unknown. So, I stopped here today another thing that you can try before coming for the next class, you remember in the last some time back I told you this stiffness matrix if you take this stiffness matrix the this 6 by 6 stiffness matrix and find out the find out the you try to find out the solution by solving these 6 these 6 equations. You will see that you cannot find out because this stiffness matrix is singular stiffness matrix.

Even in the truss, even in the element member stiffness matrix if you take separately and see: what is the determinant of the member stiffness matrix you will find that stiffness matrix is determinants are 0. So, this is and I told you that you put a storm of carry will discuss that point the interpretation of physical interpretation of that that point later. Well, this is the time to discuss that.

So, next class what you do is first we start our discussion why these stiffness matrix the singular stiffness matrix is singular, and then how to give the boundary conditions. And then once we give the boundary conditions what form of the stiffness matrix take and then subsequently how to solve them, ok.

Stop here today. See you in the next class.

Thank you.