Matrix Method of Structural Analysis
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Lecture – 16
Matrix Method of Analysis Trusses

Hello everyone, welcome to the second phase of the course and the first phase which was panning over first 3 weeks, we discussed some of the basic concepts which constitute the premise of this method, method of matrix method of structure analysis.

There, we discussed the underlying philosophy of the method, and the underlying philosophy is if we have a structure then we need to break this structure into small small pieces, and for each piece for instance if it is truss then divide the entire truss into small small each member. Then for each member similarly for beam and frames for each component we need to write the force displacement relation using the stiffness definition.

And then once we have the forces first plane relation for all the members then we need to assemble them to get the global system of equations and then we apply the boundary conditions and solve them. What we will be doing in the second phase which is starting today we will see how this is to be done for different structural components.

Today this week we will start with truss and then subsequent weeks we also see the similar exercise for beams and frames. So, the what essentially we will be discussing is the of course, how that method is to be a is to be translated for truss problem, and also some of the implementation, some of the difficulties or the some of the implementation issues that we may come across while applying this method, ok.

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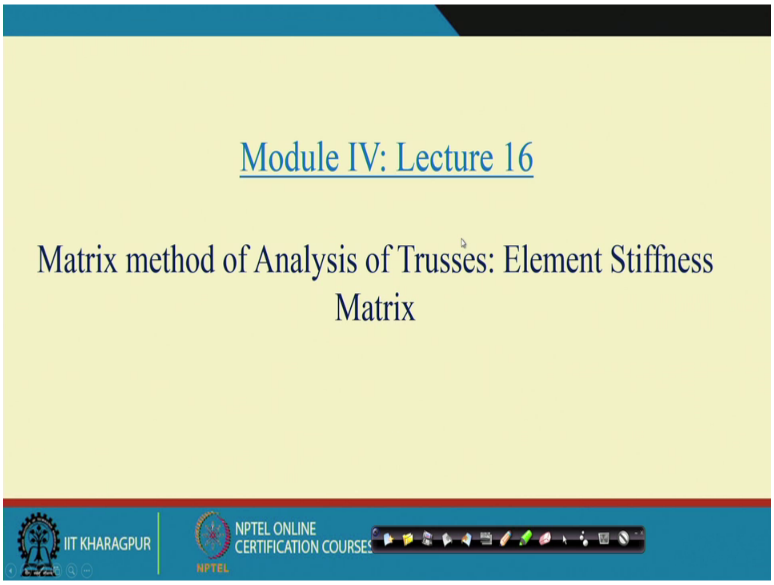

You see, so today is this is the 16th lecture we will start with application of matrix method analysis for truss. And today's topic is element stiffness matrix we will see how the equation once you have discretized this, once you have divided the entire structure into small small elements small small segments for each segment how to write the force displacement relation and that is we are going to do it for truss today,

See here it is latent element stiffness matrix you can write it is member stiffness matrix as well, ok.

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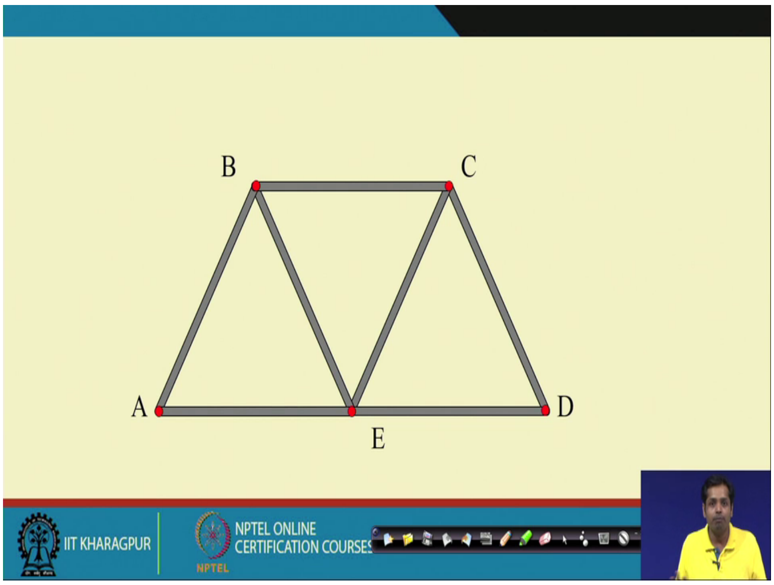

So, let us take a truss problem. Now, if we the first step is we remove the take only this truss, remove the loads, remove the boundary conditions, how to use the load and the boundary conditions we will see in the subsequent lectures. Now, this is the structure we have

Now, the first is the structure we need to break the structure into several elements in these cases several members, ok.

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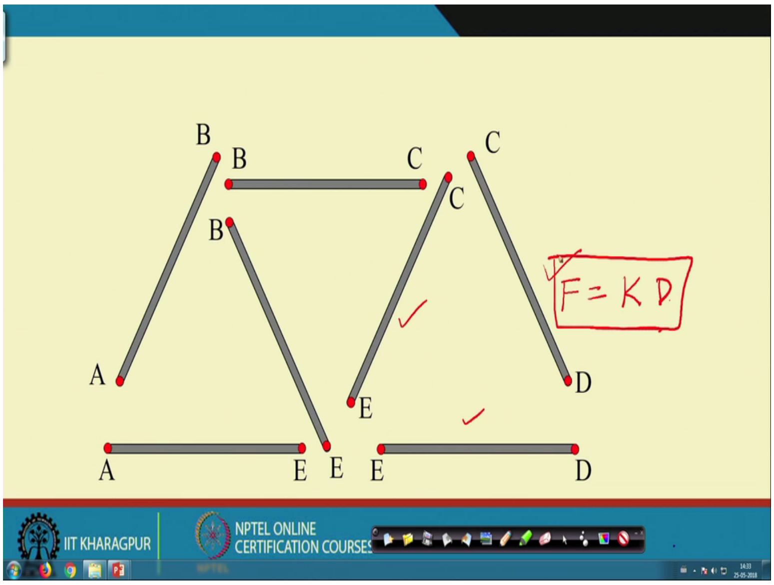

Now, then for each member we for each member, for each member we need to write the element for different members we need to write the force displacement relation means the force is equal to is equal to stiffness into displacement right. This relation we do it for every member and then assemble it and today what we see is we how to write this expression for a given member, ok.

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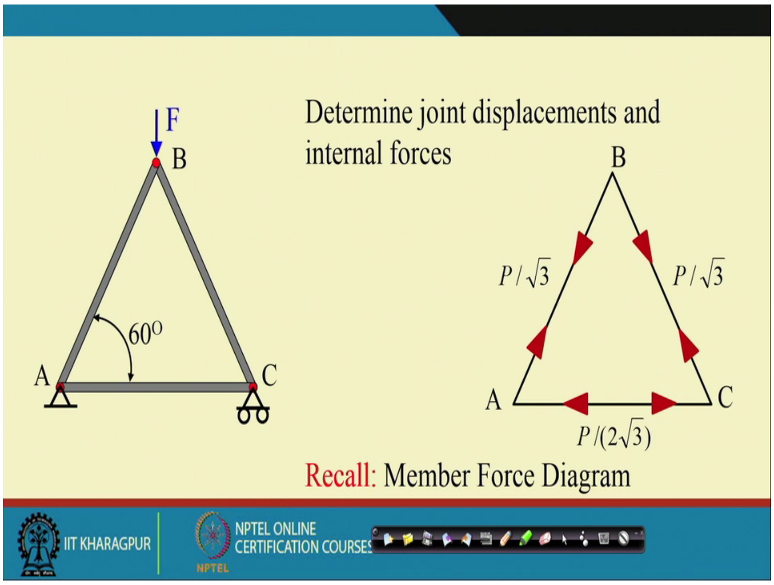

Now, let us take one example, this is an example this is a again a statically determinate structure and we can solve it and this is the solution of this problem. But let us not right now bother about this solution because we have not come to that stage where we get the solution, let us first write the element stiffness member stiffness matrix.

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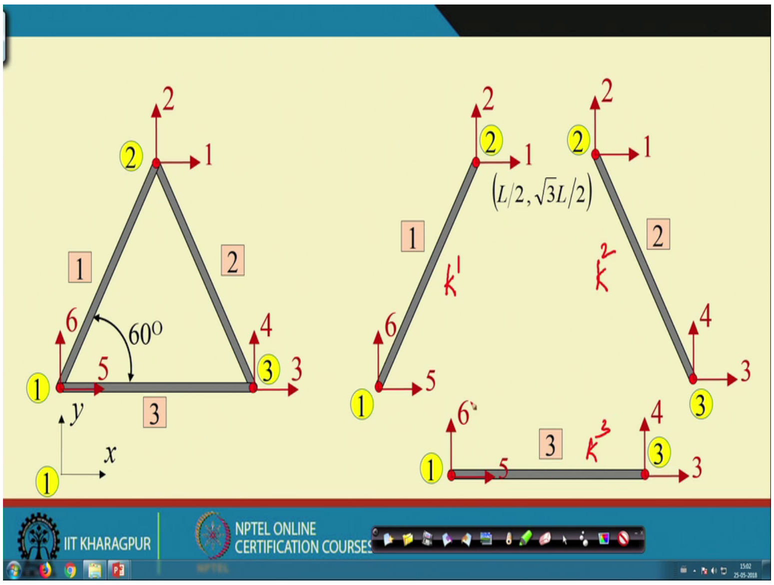

Now, see the first thing is remove the loads and the and the and the boundary conditions. Remove the load means it is just do not do not bother about loads and boundary condition right now. We will bring we will see how the external load and the boundary conditions to be taken into account that will seems a subsequent, subsequently, ok.

So, now, you see actually the if we look at the numbering of the member this is note this is point number joint number A, joint number B, joint number C. But this is not the way we identified joints in this method instead of writing ABC or XYZ what we do is we write say this is this is a first joint number 1, joint number 2, and joint number joint number 3 ok.

Now, now, once we have identified the joints and if you have a if you have say 4 n number of joints in a in a truss then it has to be number like this 1 to n. Now, here one point I just I am mentioning now, we will come to this point once again when we do the numbering, numbering is also not very it is not very arbitrary way we should number different joints. We will see that we will see that you have to number in a specific way. So, that the matrix is not the end at the end of the data matrix that you that you that we arrive that matrix becomes banded matrix. And what is banded matrix and all we will see that.

So, at this point keep that in mind the numbering of different joints is not arbitrary. Yes, you can number the way you want, but make sure there are certain requirements comp computational requirement that needs to be satisfied while you do this numbering, ok.

Now, once we have this. So, let us number the members also. So, this member number 1, member number 2 and member number 3. Now, once we have the member then let us suppose at this joint at this joint let us, at this joint to every joints we have 2 degrees of freedom, ok. So, suppose these 2 degrees of freedom is, so at this point we have one this one horizontal displacement say u here and one vertical displacement say it is ux and uy, but again we do not write the we do not identify the degrees of freedom by ux, uy, vx, and vy and so on, instead of that let us they degrees of freedom it is if you are using uth then it is u 1 and it is u 2. So, whenever we say that the first degrees of freedom in this case it is the displaced horizontal displacement of this joint. Second degrees of freedom of this structure is the horizontal vertical displacement at this joint.

So, similarly so we have we have say it is 3 4 and it is 5 6, the total 6 degrees of freedom we have each degree, each joints 2 degrees of freedom, ok. Now, each degrees of freedom similar to nodes or points and the member each degrees of freedom we need to give an id and this is 1 2 3 4 at the different ids for the different degrees of freedom.

Now, once we have that the next is we need to take 3 members we have here 3 members separately. So, this is member number 1, member number 2 and member number 3. Now, see member number 2 we have 2 at this joint it is degrees of freedom 1 and 2 at this joint degrees of freedom 5 and 6. Again for 2 point common point 2 is the common node between member number 1 and main member number 2. So, naturally degrees of freedom at node at point at this point would be 1 and this is will be 2. So, this is how the degrees of freedom for different member, ok.

Now, we have to write the stiffness matrix or the force displacement relation for each separate member, ok. Now, so if we take this length is the length of each is L, this is L, this is L, this is L so we can find out what is the coordinates of each joints as well, ok. These points are important we will be, using this point we can in this case it is a 60 degree, but it could be any arbitrary angle say theta. So, these, these, these coordinates of these 2 points are equal to find out this angle theta and the length of this member ok.

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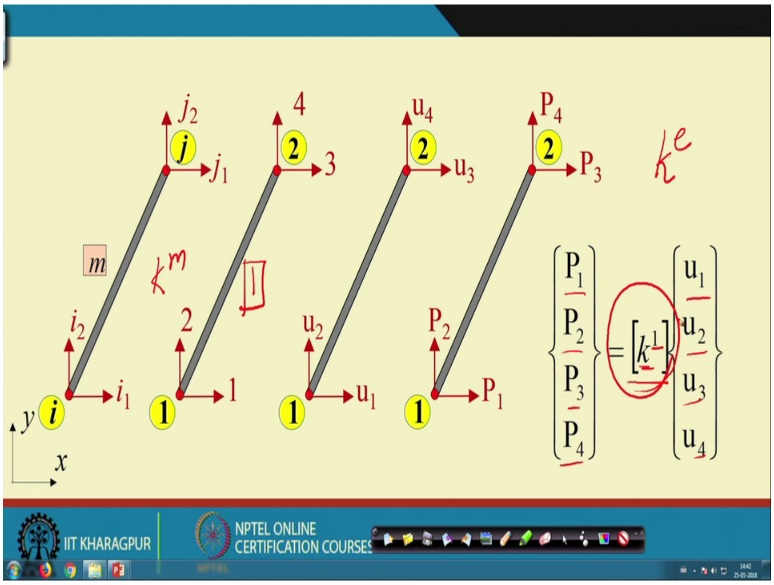

Now, next is suppose take and any arbitrary as I said our objective is now, is to find out the stiff find out the force displacement relation for each member. So, let us, let us find out a force displacement relation for any member which is arbitrary oriented and once we have the force displacement relation for that or the stiffness matrix for that member, then depending on the degrees of freedom and depending on the coordinates of the coordinates of the coordinates of the points we can have the similar equations for different members.

So, let us take any member arbitrary member say member m and which the this is node i and this is node j and the degrees of freedom at node i is i 1, i 2 and this is j 1 and j 2, ok. It is you know depending on i 1 and i 2, j 1 and j 2 and depending on the value of m and depending on the coordinate of this ith member and jth member we can have different member in different orientation.

Now, take us for writing, but the ease in right for writing let us say this member is 1 and 2 and the degrees of freedom is 1 2 here and 3 4 here, ok. So, we will find out what is the force displacement relation for this member. So, for so in this case we take i is equal to 1, j is equal to 2 and i 1, i 2 is equal to 1 2, j 1 j 2 is equal to 3 4, ok. So, that we do not have to write i 1, i 2, j 1, j 2 again and again, writing will be easier in terms of 2 3, 1 2 3 4, ok

So, now, suppose as I said these are the degrees of freedom u 1, u 2, u 3, u 4 are the corresponding degrees of freedom. And then the forces at members force in 2 different direction say it is P 1, P 1, P 2, P 3 and P 4; P 1, P 2, P 3 and P 4, ok. So, when we when whenever we say P 4 it automatically means P 4 is the force in vertical direction at joint 2, ok.

Now, what we need to find out? So, this is your displacement u 1, u 2, u 3, u 4 and the associated forces are P 1, P 2, P 3, P 4. So, what we are interested now is to find out a relation between the force and the displacement. So, here we have 4 degrees of freedom and the 4 corresponding 4 forces. So, these are the forces and corresponding associated displacements are this, and how these forces and the displacement are related to each other. And so, they are related to each other to a stiffness matrix k. You remember k is a supers superscript 1, this k 1 means this is the node number this is suppose this is element number 1 or member number 1, ok. Any arbitrary any arbitrary element any arbitrary member we can write the stiffness matrix is say k e, where is the id of the element in this case this is k m ok.

Whenever we say k m it means the stiffness matrix the member stiffness matrix for the mth member. So, k 1 is the stiffness matrix of the first member number 1, ok. And another thing is usually as you can use your notation, but usually we use a small k for element stiffness matrix, element stiffness matrix, ok.

Now, today what we do is we need to we need to see how to determine this k 1 or k for any arbitrary member.

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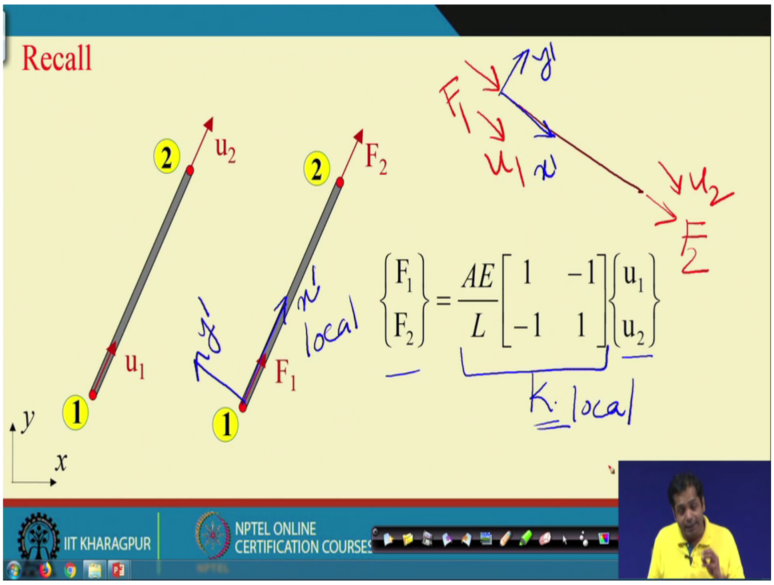

Now, if you recall in the second class, second week we discussed this if we have a truss, truss is a two force member. So, essentially you if you if you take any member we have a force along the longitudinal axis of the member. Now, you see there are, now, let me introduce two coordinate system, one is the local coordinate system and one is the global coordinate system.

You see local coordinate system is the coordinate system along for instance if we take F 1 and F 2. If we assume a coordinate system like this if we assume a coordinate system which is x is in this direction this is your x direction and this is your y direction, say it is y dash and x dash because x and y you have already used for something else. So, the local coordinate system local coordinate system ok. So, F 1 and F 2 these 2 forces are with respect to local coordinate system, ok.

Now, if we have F 1 and F 2 two forces defining with respect to local coordinate system and the associated displacements are u 1 and u 2 again in the local coordinate system, then if you will you recall we discussed in the second way that F 1 and F 2 and u 1 and u

2 are related with this, and this is the stiffness matrix this is the stiffness matrix k. And this stiffness matrix is the local stiffness matrix, this is local stiffness matrix, ok.

If we have a different member for instance, if we have a member like, if you have a member like this, if you have a member like this, ok. And then in this member if we have a if we have the coordinate system like this for instance this is this is this is x direction, this is x dash direction and then this is y dash direction. And then corresponding forces are say this is the corresponding forces this is F, F 2 and this is F 1, F 1 and similarly we have u 2 and u 1, u 1 here, u 1 here. So, u 1, u 2 and F 1 if F 2 again will be related by the same stiffness matrix, there will be no change in the stiffness matrix and this is stiffness matrix is with respect to local coordinate, locals coordinate system.

But you know what at the, when we when we at the end of the day we will not be dealing with local stiffness matrices local, force displacement relation we have to we have to add assemble all the force displacement relation to get the global system of equation. And therefore, we need to write the form of the element stiffness matrix the k, we instead of this is with respect to local coordinate system, but instead of local coordinate system we need to transfer them we from local coordinate system to global coordinate system, ok.

So, once we have the global coordinate system in this element stiffness matrix for global coordinate system then irrespective of the orientation of the orientation of the truss member always, ok. If we when we when we say that that force displacement relation with respect to local coordinate system and global coordinate system it essentially means when we say the local stiffness matrix it relates the forces and the displacement defined in local coordinate system. When we say global stiffness matrix it means or the stiffness matrix with respect the global coordinate it means that it relates the force and displacement defining global coordinate system.

Now, at the end of the day we need to solve the entire structure right. So, entire structure all the forces and the displacement is are which are defined with respect to a cord fixed coordinate system and therefore, it is very important that we transfer this element stiffness matrix which is written in local coordinate system to global coordinate system.

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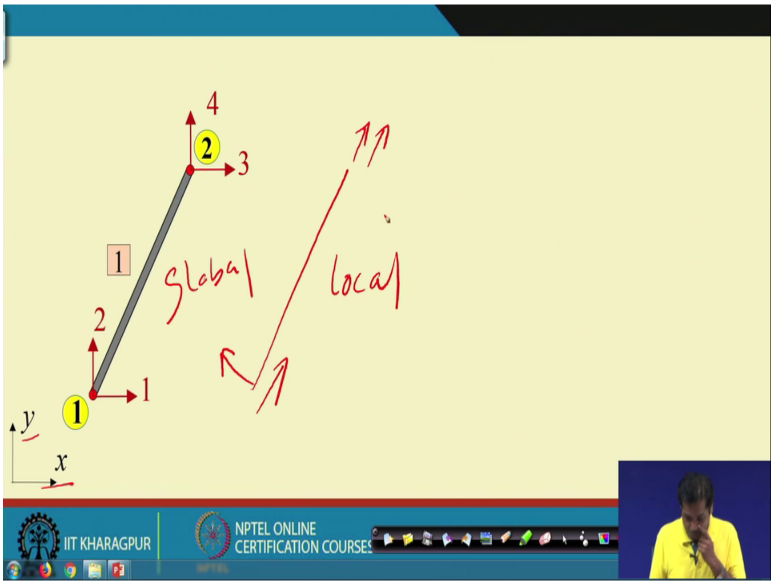

So, when you do that means now, we have to write the we have to write, you remember last in the in the previous slide just now, we saw your equation was defined with respect to this and this coordinate system and where we have the force one this. But now, this is local and then this is global with respect to global coordinate, where the displacements the degrees of freedom in the forces are defined with respect to global coordinate system which is x and y here.

And that is the reason why at every joints you have two, the two displacement, one is global x direction, global y direction, global x direction, global y direction and similarly force is also a global x, global y and global x, global y direction, ok.

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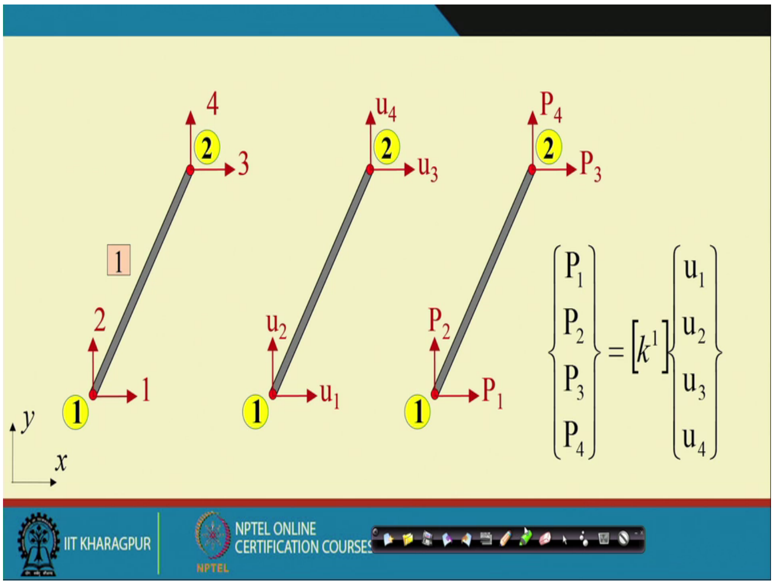

Now, let us do this. Let us um. So, this is the thing that we need to find out and that we discuss this we have to do now, ok. So, let us do let us do that.

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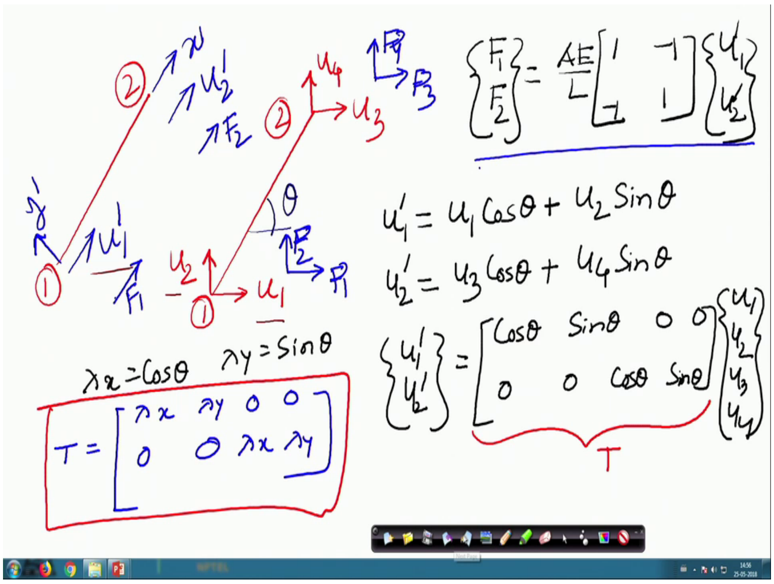

You see suppose take any. So, what we know is if we have say take first take a member and in this member we have this is the coordinate system where we have the x dash and this is y dash, ok. And then we have defined here is u this is say u 1, u 1 dash and this is u 2 dash. This is your node number say this is node number 1 and this is point numb joint number 2, ok.

Now, now, we have the same thing here we have this is u 1 and then this is u 2 then u 3 and u 4, this is node number 1, this is number 2, ok. And suppose this angle is theta suppose this angle is theta, ok, and what we know is what we know is that F 1, F 1 and F 2 the this is equal to ae by L 1 minus 1 minus 1 1 and this is u 1 dash u 1 dash and u 2 dash, right.

Now, you see you can we write now, u 1 this though they are written in terms of different coordinate system u 1, u 2 and u 1 dash they are they are defined here in terms of different coordinate system, but they have a relation, right. Because essentially they both tell us what is the movement or what is the displacement of joint 1. So, we can find out a relation between u 1, u 2 and u 1 dash and that relation is very straightforward we can just apply the we can just take the components of u 1 and u 2 along this direction. And if we do that we can write u 1 dash is equal to u 1 cos theta cos theta plus, plus u 2 sin theta. And similarly u 2 dash is equal to u 3 cos theta plus u 4 sin theta.

And now, this we can write as u 1 dash u 2 dash which is this is equal to cos theta, sin theta, 0, 0, 0, 0, cos theta, sin theta and then this is u 1, u 2, u 3 and u 4 ok. Now, suppose cos theta is equal to let us take lambda x is equal to cos theta and lambda y is equal to sin theta and then we can write these expression these expression as this is called, this is say this is called this is called T. T is the, T for transformation what it transform it is a transformation it gives it relates the displacement defined in one coordinate system to displacement define it another coordinate system. So, this is transform that you how the transformation of displacement can take place ok, that is why it is called transformation matrix.

Now, so T can be written as once we have this then. So, T we can write. So, T is equal to, so T is equal to lambda x, lambda y, 0, and 0, 0, 0, lambda x, and lambda y ok. This is very important, ok, we will come to this expression once again or rather several times ok.

So, what we have done so far is we have displacement in one local coordinate system and the displacement defining degrees of freedom rather defining global coordinate system, then we have seen how these 2 displacement can be transformed, ok.

Now, let us do the same exercise for forces as well and if we do that then forces also can be written. So, let us take, let us take let us let us take we can go we can we can we can suppose now, the forces will be here it is say F 1, it is F 1, F 1, and this is F 2 and similarly here it is F 3 and F 4 right. And here it is say sorry instead of F 1 you write P 1 because that is the that is the that is how we define forces and in local coordinate system it is F 1 that is how we defined and it is F 2, ok.

Now, this gives you a relation between u 1 dash u 2 dash and u 1, u 2, u 3, u 4 what we do the similar exercise which relate F 1 and F 2 with P 1, P 2, P 3, P 4. Now, if we do that if we do that then you check that you can write like this P 1 is equal to P 1 is equal to F 1 cos theta and P 2 P 2 is equal to F 1 sin theta. Similarly P 3 is equal to it is just the taking the component of forces F 2 cos theta and P 3, P 4 is equal to F 2 sin theta, F 2 sin theta.

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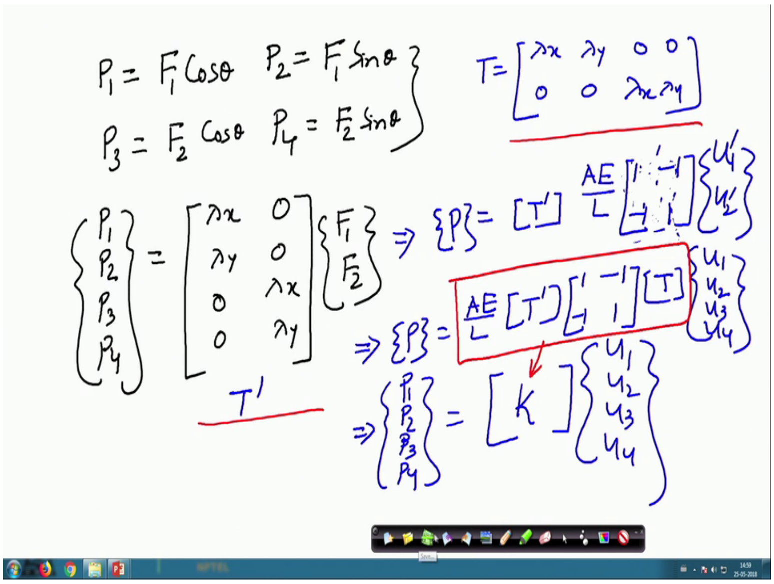

Now, you recall if I write P 1, P 2, P 3, P 4; P 1, P 2, P 3 and P 4 and that become if we substitute cos theta as lambda x and sin theta as lambda y. Then this become lambda x 0, then lambda y, 0 and then 0, lambda x, 0, lambda y and then F 1; F 1 F 2, ok. So, so this directly comes from these two equation, ok.

So, now, again this is if you recall T was T T was like this T was lambda x, lambda y, 0, 0, 0, 0, lambda x and lambda y this was transformation T. So, naturally this becomes T dash, ok. Now, P is related to F 1, F 2, F 1, F 1, P 1, P 2, P 3, P 4 related to F 1 F 2 through T dash, ok.

Now, next if recall that F 1 F 2 is related to u 1 dash u 2 dash with this. So, if I substitute in the in the in this expression if I substitute F 1, if you substitute F 1 F 1 F 2 from this what we have is what we have is, so this can be written as now, so, this can be written as say this is P is a vector which consists of P 1, P 2, P 3, P 4 is equal to this is say T dash which is transformation of T. And then F 1 F 2 F 1 F 2 was AE by L, AE by L into u 1 dash and u 2 dash ok, which is directly from this expression, a 1, a 1 not only that we have another term here which is which I forgot to write. Let us erase it. It should be, yes it should be 1, minus 1, minus 1, 1 and then u 1 dash and u 2 dash, ok.

Now, once again you remember last u 1 and u 2 dash u 1 and u 2 dash is related to u 1, u 2, u 3, u 4 by this T. So, what we can write is we can substitute u 1, u 2, u 3; u 1 dash u 2 dash as so this becomes P which is this and then we can take AE by L out and then transformation T dash and then 1, minus 1, minus 1, 1 and then this become T into u 1, u 2, u 3, u 4, right.

So, essentially this becomes P is equal to your P 1, P 2, P 3, P 4. So, let us write P 1, P 2, P 3; P 1, P 2, P 3 and P 4 this is something k into u 1, u 2, u 3, u 4 and that k this k is this the entire thing is this k, entire thing is this k ok. So, this is this k.

Now, if I substitute T dash T dash and T, T from this expression and T dash from these, these T and T dash are transpose and do this operation matrix operation and then we get an expression like this. We get an expression like this. We get an expression like this, ok.

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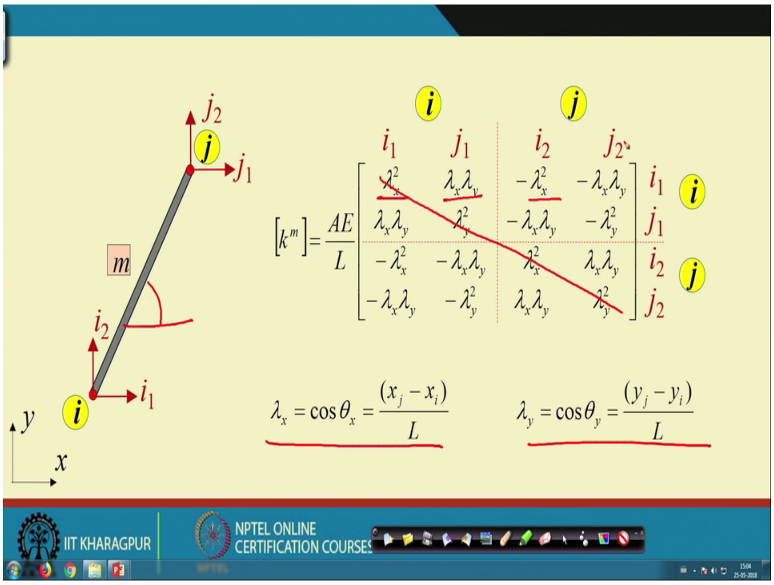

So, for any arbitrary for any arbitrary member any arbitrary member if the it was 1 and 2 if you recall in our in our the expression that we derived it was 1, it was 1, it was 2 and it was 3 and it was 4, and it is it was 1 and it was 2 something like this. So, it is 1, it is 2, it is 3, it is 4, 1 2 3 4 and this is, this is 2, and this is 1 this is 2, ok. So, if we do this we have the expression for T dash and the and if we substitute that then we get k m is equal to or element or member stiffness matrix for any arbitrary member m member stiffness matrix is equal to this, ok.

Now, this i and j depending on the depending on the, depending on the, depending on the your member i j will be and the i j and these degrees of freedom will be different. For instance, if we have if we have for instance for if we go back to; for instance for this case for if we consider this member your i is to, your i is 1 and j is 2 and degrees of freedom are this is 5 6 1 2 and for this i is 2, j is 3 and degrees of freedom is 1 2 3 4.

So, similarly, so the stiffness matrix what we get k 1 this relates degrees of freedom 5 6 1 2 with the forces 5 6 1 2. Similarly we get k 2 here which is degrees of freedom for member 2 which relate displacement u 1, u 2, u 3, u 4 with the forces P 1, P 2, P 3, P 4. Similarly stiffness matrix k 3 that we get here that that relates forces u 3 P 3, P 4, P 5, P 6 with the displacement u 3, u 4, u 5 and u 6. And if we just write a general form of the stiffness matrix then that general form will be that general form is this, that general form is this, where lambda x is the lambda x and lambda y are this is the angle if we take this angle theta, ok.

Now, this is the stiffness matrix for a given member this is also called member stiffness matrix, ok. And now, you see there are stiffness matrix in this case has to be 4 by 4 matrix because we have 4 degrees of freedom and this was 4 forces. So, this gives you the stiffness matrix 4 by 4

Now, let us just before we before we stop let us try to understand what the interpretation of each element in the stiffness matrix for instance you see lambda this is this means these relates i 1 to i 1, ok. Means it is the relation between force in this direction and the displacement in this direction. And suppose these lambda x this relates i 1 and j 1 this relates force in i 1 direction and displacement in displacement for j 1. Similarly, similarly if it is similarly if we take this they need relates also corresponding force and corresponding displacement.

Another important observation that you can see from the stiffness matrix the diagonal terms all the diagonal terms are positive and the stiffness matrix is symmetries. What is the interpretation physical interpretation of stiffness matrix symmetric you can recall structural analysis one you study reciprocal theorem it is very similar to that, ok.

And what is the physical interpretation of all these diagonal term being positive, and why whether the diagonal terms can be negative or not, what happens, what is the properties of the of the stiffness matrix, whether the stiffness matter is singular or not or is there any other properties the stiffness matrix stiffness matrix has any element stiffness matrix has. Those things we will be discussing towards the end of this course when we have when we discuss various implementation issues, ok.

So, I will stop here today next class what we do is next class once we have the element stiffness matrix or the member stiffness matrix. Next class we will see with one example. We compute the member stiffness matrix for different members and then how to assemble all these stiffness matrices to get the global stiffness matrix, ok. Stop here today. See you in the next class.

Thank you.