Matrix Method of Structural Analysis
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Lecture – 12
Matrix Algebra Review (Contd.)

Welcome this is lecture 2 of module 3. Now as in the previous class, we have reviewed some of the elements of Matrix Algebra; now in this lecture also we will continue with that.

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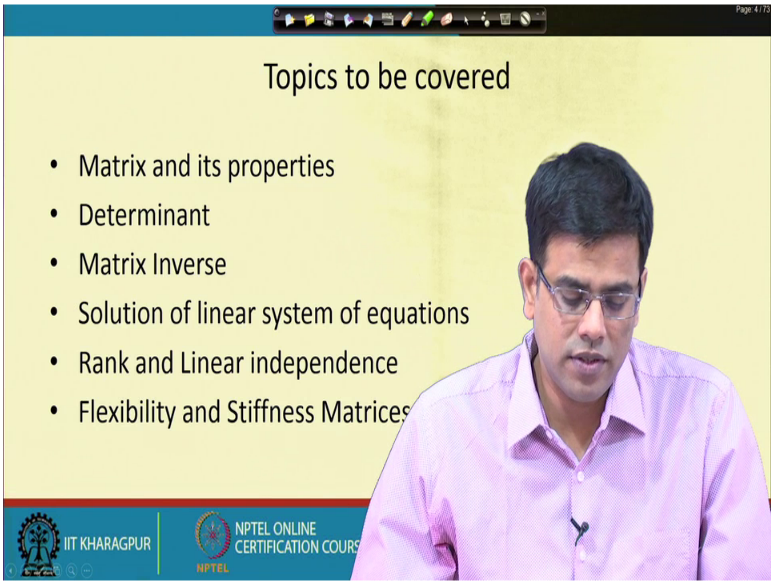

So, here we have in the last class we have discussed matrix and its properties, determinants matrix inverse partially we have discussed matrix inverse.

Now, in this lecture I will actually discuss it in more detail in the matrix inverse.

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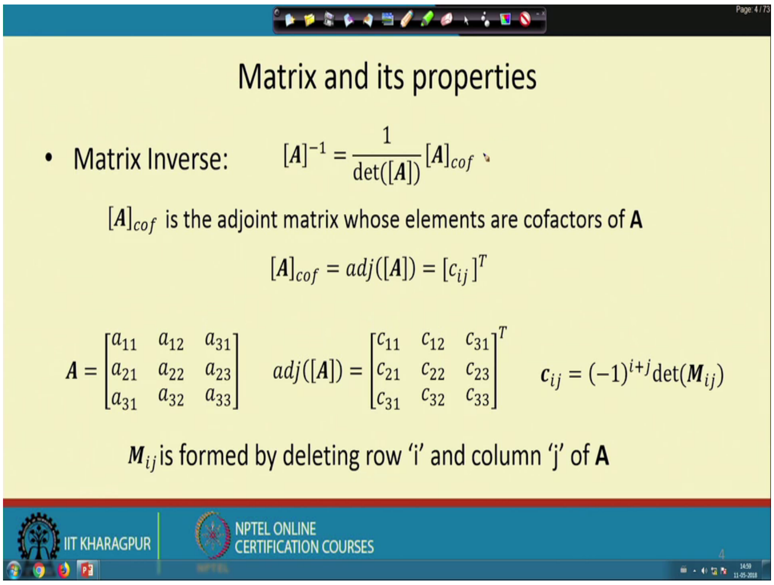

Now we have finished there that the matrix inverse is actually can be written as the cofactor matrix and the determinant divided by the determinant. Now the cofactor matrix is actually also called the adjoint matrix which is the transpose of the cofactors of the matrix A.

Now, in this cofactor matrix, we compute the elements of the cofactor matrix is actually it through the cij is actually the components of the cofactor matrix. So, it is calculated as the principle as the minors determinants of the minors. So, now, if we have a 3 by 3 matrix say this is a 11, a 2, a 3 3 and then adjoint matrix or the cofactor A cof that matrix is actually the c 11 to c 3 3 transpose.

So, now how to calculate the c 1 1 and c 2 2 and so on? Is through this formula which is minus 1 to the power i plus j and then determinant of the minors. So, how do again the how to calculate the minors we have also discussed in the last class. So, minors are formed by deleting the row i and column j of A. Now um we will go through with an example of doing.

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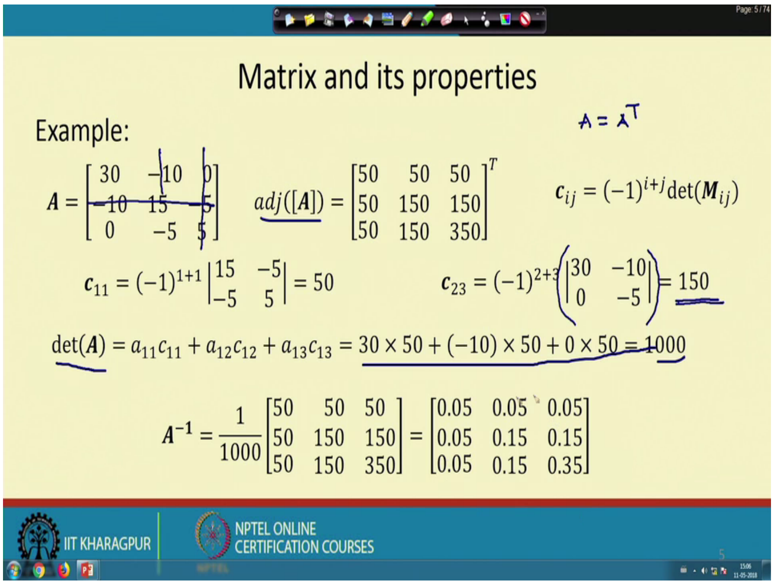

Suppose this is a matrix A we want to invert. So, what to do we have to write first the adjoint matrix, the adjoint matrix will be of something of this form because we first calculate the cofactors. So, for instance the c 1 1 is my first cofactor. So, with this formula I calculate c 1 1 which is minus 1 to the power 1 plus 1 and then deleting first row and first column. If we delete the first row and the first column, then this portion of the matrix is left. So, this portion actually I take the determinant. So, which is a very simple 15 into 5 minus minus of minus 5 into minus 5; so, which turns out to be the 50.

So, now if you have c 2 3 for instance, we want to calculate c 2 3 then it is the deleting the second row and the third column. So, if I delete the second second row and the third column. So, this will be 30 minus 10 and minus 0 and minus 5. So, taking determinant of this quantity will give me my required cofactors c 2 3.

So, this turns out to be the minus plus 150. Now in this way we can calculate all the cofactors of matrix A. So, if you look carefully I have already calculated these things these turns out to be this matrix. Now adjoint matrix is essentially the transpose of this cofactor matrix. Now if you look this matrix A since the matrix A is symmetric, then this cofactor matrix will also come as symmetric and transpose of that is also the same matrix because we defined already the symmetric matrix is A equal to A transpose.

So, then this is my; the adjoint matrix. Now once I calculate the determinant of A which is again in the last class we have discussed which is the a 1 1 c 1 1 a 1 2 c 1 2 and so on.

So, if we calculate these one this turns out to be the 1000. So, from the previous formula A inverse is 1 by determinant of A and then cofactor matrix transpose, which is actually the same matrix here; so, adjoint of A.

Now, a if you put the 1 by 1000 inside or multiply with the 1 by 1000 to each element of this matrix, then it turns out to be the this matrix. So, if you look now carefully this matrix is the inverse of A and since the A is a symmetric matrix then this A inverse is also a symmetric matrix. So, this is a one corollary or the one information, we can draw if the matrix is symmetric then its inverse is also symmetric.

Now, this procedure this inverse is actually we will break if this determinant comes to 0. Now division by 0 we all know that it is infinity. So, this inverse; that means, the inverse cannot be computed. So, inverse will be inverse cannot be computed. So,; that means, we will say that matrix is a singular matrix. So, the definition of singular matrix is that, that if it inverse doesnt exist or cannot be computed then the matrix is called singular matrix and it can only happen here is that if this quantity or the det of A goes to 0 right.

So, next we will see how to compute inverse through the other methods.

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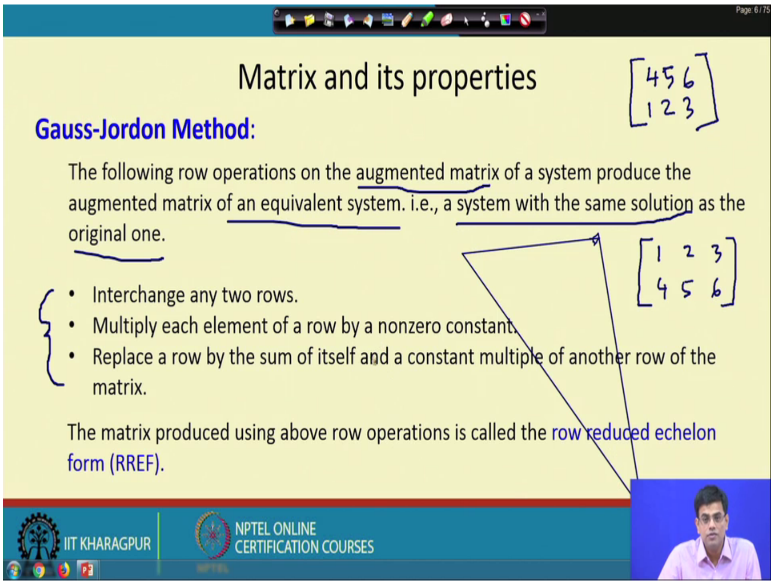

For instance the Gauss Jordon method; this Gauss Jordon method is actually very efficient method of finding out inverse or finding out the solution of the linear system of equation. So, first let us before discussing the Gauss Jordon method, let us now know some of the things how we will proceed.

So, the following row operations on the augmented matrix, now I will explain what is augmented matrix of a system produced produce the augmented matrix of an equivalent system; that is a system with the same solution as the original one. So, now, how to calculate this thing we will see through an example.

Now, let us the study these operations. So, interchange any rules so; that means, I can if I have a matrix of something like this, then I can interchange it say for instance 1 2 3 4 5 6 this is a matrix. So, if I now change these matrix or if I interchange that; that means, 4 5 6 1 2 3; that means, I interchange or swap the row 2 to row 1. So, interchange of row is allowed.

Now, multiply each element of a row by a nonzero constant. So, I can even multiply a row with a constant and then subtract it from a row or add it to a row. So, these also will not change my augmented system. So, and then replace a row by the sum of itself and a constant multiple of other row of the matrix. So, these operations if I do; that means, this operation will not change my matrix or the this augmented system is actually the equivalent of the original system.

Now, the matrix produce using above row operation is called the row reduced echelon form. This is very important term row reduced echelon form because when we will study the rank and rank of a matrix and nullity of the matrix we will we will again come back to this. So, I will see we will see that how this row reduced matrix can be obtained.

So, let us start with an example that, suppose I want to solve these 3 system of equation.

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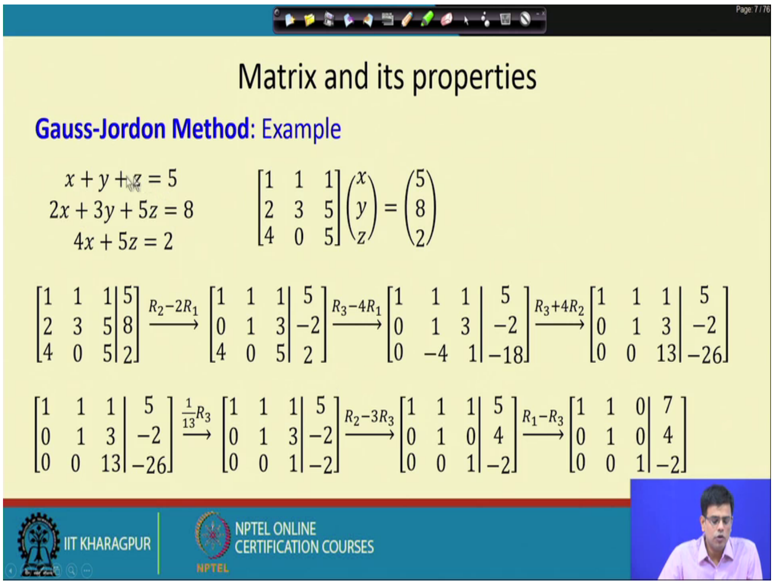

So, this can be represented in a matrix form in this that we have learned in the first class. So, this can be written in a matrix form a this 1 1 1 and so on these are the coefficient of the variables x y and z. Now this is the right hand side 5 8 and 2 also if we do this represents a matrix vector equation.

Now, if I want to solve this system through the Gauss Jordon method, then what I do is I write this matrix original matrix and then augment the right hand side of the vector with this matrix, and then I will start doing the row operation on it which you have learned in the previous slide. So, if I now suppose I do R 2 minus 2 R 1; that means, I am subtracting two times of row 1 with the R 2. So, if you multiply two with the row 1 and subtract with the R 2 then what will be what will be my R 2; that means, row 1 row 2; that means, 2 times of 1. So, 2 minus 2 is essentially 0 and then similarly 3 minus 2 times of 1. So, it it comes to 1.

So, similarly if I do these operations my augmented matrix becomes this. Now from here to here I do these row operations and comes to here. Now look carefully the other rows have not change only this row has changed. So, a it has to be carefully noted that only this row is changed due to this operation. Now similarly if I perform R 3 minus 4 R 1, then the row 3 will change while row 3 row 2 and row 1 will be remain same. So, this comes to this.

Now, similarly if I do another operation R 3 plus 4 R 2 then this will be this. And then again with this again I actually do divide by row 3 by 13. So, if I divide row 3 with the 13 then my final matrix will be this. And now see till now these row 1 row 2 will be unchanged; now similarly I do another operation and so on if I perform this thing, then my final thing is this, but you see still there is an 1. So, we probably need to do another operation on this and which comes to this.

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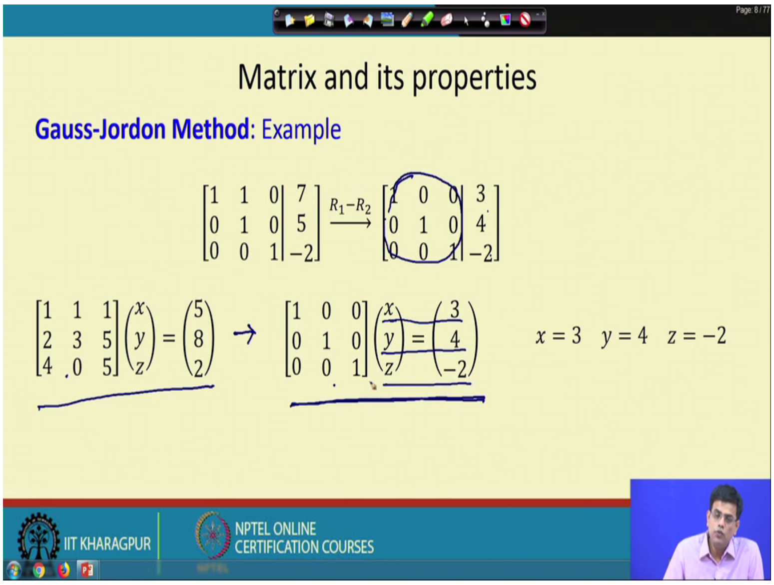

So, if hm I now R 1 minus R 2 if I do, then it goes. So, you see the my original matrix was this original augmented matrix was in the this form this matrix with the right hand side 5 8 2. Now what I did essentially I make the original matrix as my identity matrix as my identity matrix. Now due to for making this identity matrix my right hand side vectors have changed.

So, if I write this system of equation to now another system of equation, which I obtained through the row operation I get this system of equation. What this rule says is that the solution of this system of equation is same as solution of this system of equation; that means, the row operation is the result of row operations are equivalent; that means, there is no change in the solution now solving these kind of system is very easy.

Now, if you look carefully that its solving this means it is x equals to 3, x y equals to 4, z equals to minus 2. So, this is a solution of the original my equation. Now since this is a diagonal matrix. So, solution is just the x equals to 3, y equals to 4 and z equals to minus

2. So, by this process what should what will be our objective? Our objective is to make the original matrix to a diagonal matrix so, that our solution comes very easily.

So, and these will be done through a hm row operations, which will nost change the original system. Now if you if we can extend this for a another class of problem I will show, which is which we can extend. So, this is in a nutshell that this Gauss Jordon method can also be used through the for the solution of the system of equation so, and how to do that we have just seen now.

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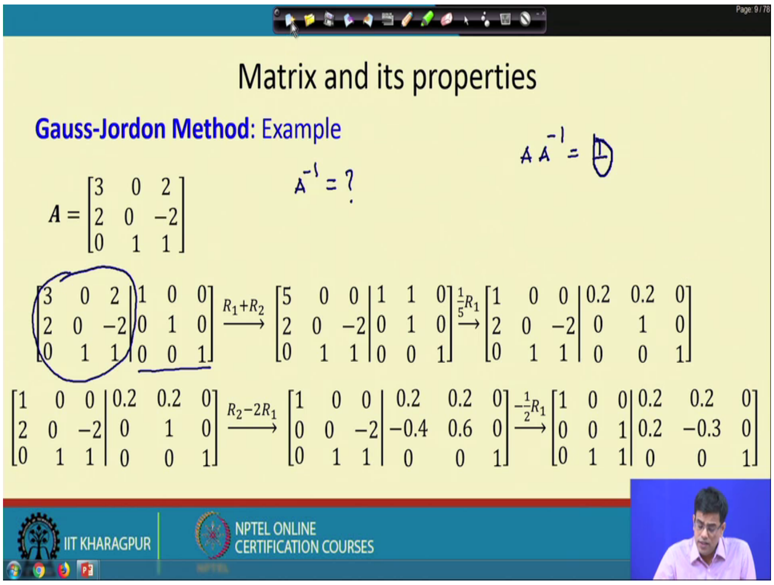

Now, if we can use this for a for a total matrix, now what here we want to do is just we want to the invert the matrix. So, what should be the A inverse. Now if we know from the inverse formula that is A A inverse is actually the identity matrix. So, identity matrix will be of order 3 here because A is a 3 by 3 matrix. So, if you now see the right hand side for me is the matrix, but my solution will also be a matrix if you look from this from the previous example we have seen.

Now, similarly as in the previous example, I write in the original matrix and then augment my right hand side which is the identity matrix here, this identity matrix I write it here. Now similar to the other example I start doing the row operation. So, what will be the my objective here? So, my objective here will be to make this original matrix as identity matrix, and due to that this matrix will be changed. So, first I do the row 1 plus row 2 and then divide one fifth with the row 1 and similarly I do R 2 minus 2 R 1 and minus half R 1 and so on.

So, if we do this, then this comes to a form which is similar to the which is the not identity matrix, but like this matrix. So, we need to do little more.

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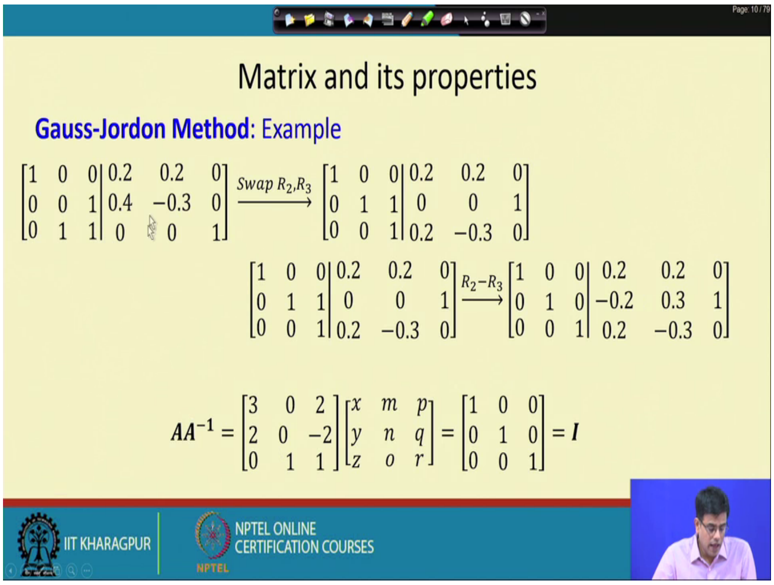

So, we will see this. So, it was my original matrix and then this was the changed augment, change matrix change right hand side matrix now what I do you see there is a 0 in the diagonal term of the original matrix.

So, now, if I swap the row 2 and row row 3, a then I will I can obtain a diagonal term nonzero. So, these.

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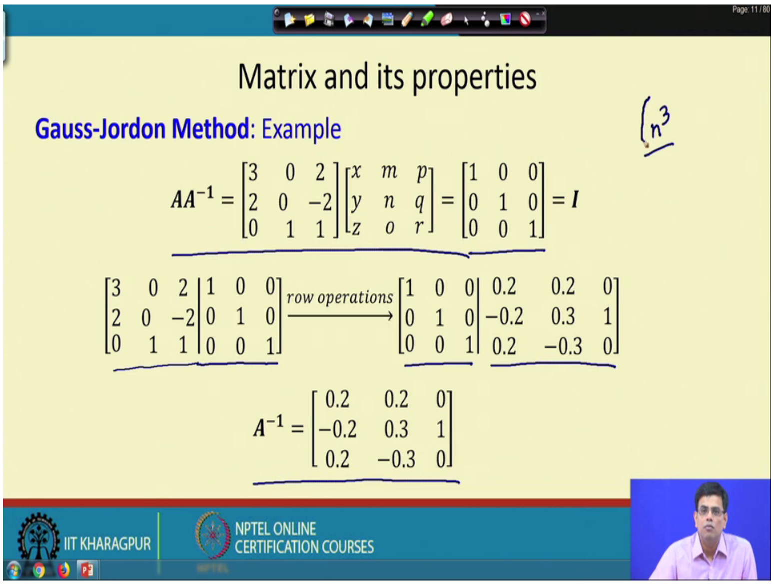

So, that is why I swapped this row 3 and row 2 and then my final matrix becomes this a final matrix becomes this my final matrix becomes this now. So, if we have looked carefully that a inverse is high I have written it in equation format. So, where my x y z m n o p q r my unknown. So, if I now I have a I through this Gauss Jordon method, I want to find out x y z m n o and p q r what should be the value and this value is essentially the component of A inverse.

So,. So, this is a basic Gauss Jordon method and which is very helpful for computing the inverse now. So, what was what we have done through this is, essentially a this matrix this matrix I augment with the identity matrix, because my right hand side was my was an identity matrix. Now through the row operation I made this these matrix as my identity matrix and due to that this right hand side matrix will be changed and these change matrix will represent the inverse because this equation suggests that my inverse is these matrix.

So, this is one of the way of calculating matrix inverse and if you compute the what are the row operations and all those things, you can also compute the cost of matrix inverse, which is typically n cube operations. So, even though we solve matrix equation sometimes by matrix inverse, but generally solution of matrix solution of simultaneous linear equation is costly by matrix inverse so that we will see when we will study the

Gauss Elimination. So, this completes the second lecture of module 3, we will in the next lecture we will again continue with the matrix algebra and its review.

Thank you.