Matrix Method of Structural Analysis
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Lecture - 11
Matrix Algebra Review

Welcome. I am Bishwanath Banerjee, I am a faculty member of Department of Civil Engineering, IIT Kharagpur. I will be second instructor for this course matrix analysis of Matrix Structural Analysis. So, we have module 3 today and it is the first lecture for the module 3. So, here we will review the basic matrix algebra, because the objective one of the objective for this course is to do large scale structural analysis, which requires knowledge of solution of matrix vector equation.

So, for that we need knowledge of structural for little bit knowledge of matrix analysis, all of you have this basic knowledge, but in this module we will basically review this some of these matrix analysis. And then we will move to the conventional structural analysis where we will use the some of this matrix algebra.

So, in this course we will be covering mainly matrix.

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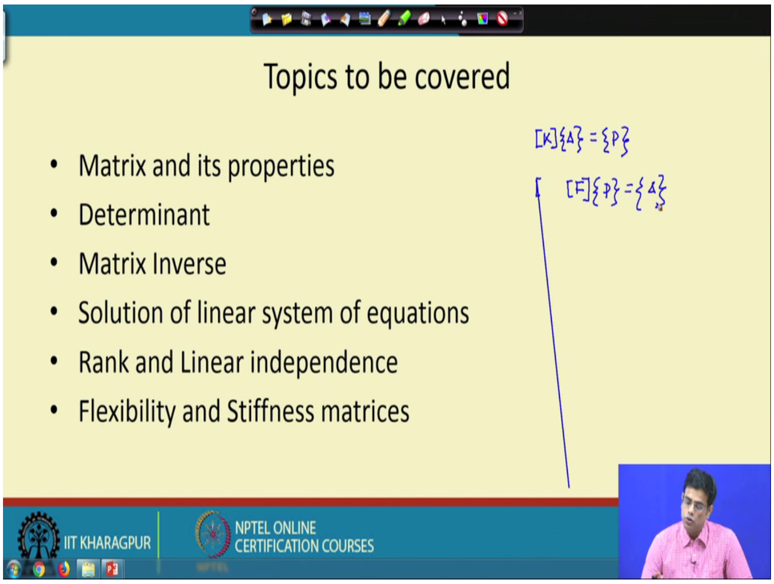

And it is in this module we will be covering mainly matrix and its properties, determinants, matrix inverse, solution of linear system of equations, rank and linear independence. And then finally, we will consider the flexibility and stiffness matrices and its properties, which are very important for solving the large scale structures; that means, where the degrees of freedoms are large.

So, probably by this time you have learnt what is stiffness of a stiffness of a structural component, what is flexibility of a structural component all those things you have learnt. So, finally, it boils down to a matrix which is popularly known as stiffness matrix or flexibility matrix, and then by which we solve those structures some methods probably you have harder, force method or flexibility method or compatibility method and the stiffness method or the displacement based method.

So, basically in a structure this kind of equation will encounter will be giving details for this kind of equation. So, one of them is stiffness method, where the equation will be like this k delta equals to f where k is the stiffness matrix, delta is the unknown displacement vector; displacement vector may contain the rotation also and then the known force vector.

So, this is popularly known as stiffness method or direct stiffness method; and another method we will also discuss in later part of this course is the force method. Where we will use the flexibility matrix if, and then the unknown force and with the known displacement. So, this is also known as the force method. So, to solve these equations finally, we need some knowledge of how to solve this equation for instance probably you have already learned the Gaussian elimination LU decomposition or any other method like URD composition.

So, we will just briefly review these things before we jump into these equations and how these equations are formed.

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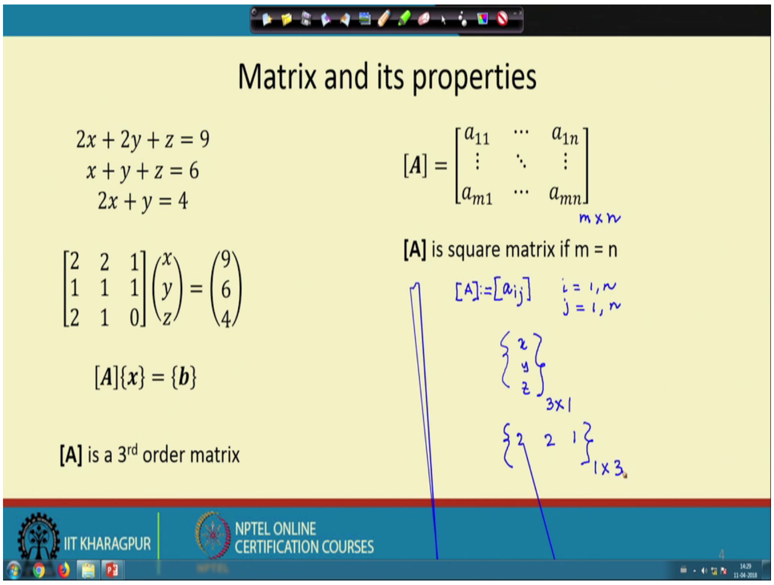

So, to start with we will just how matrix comes to us, actually our main objective is to solve some equations. So, for instance this is a 3 simultaneous linear equation, which can be represented in a matrix form that is the matrix A and matrix A and unknown vector as x and right hand side or the force vector is b.

So, if you look carefully that the coefficient of these equations are arranged in a row and column wise so that it forms a matrix. So, here it is there are 3 equations so, this is a third order matrix A, and so it can be generalized; if you have n number of equations it can be generalized as nth order matrix, but matrix can be rectangular also. So, if I write any matrix A in terms of its component a 1 2 a 1n and n then 21 and so on and n 12 n mn.

So, this is a rectangular matrix of dimension m n, which means m rows and n columns right. So, now, we will be mostly dealing with the square matrix, where m is equals to n.

So, for instance the matrix A I we are sometimes write in this form the a ij ok, the matrix A sometimes we write in this form that its component form right which means i runs from 1 to n, and j runs from 1 to m or n. But we will be mostly dealing with the square matrix so, doing bother about rectangular matrix here.

So, this is a basic notation will follow and sometimes we will also write the row vector and column vector. For instance if I write this is this is this vector x is a column vector.

Column vector means if we if we have x y z, this I am writing as a column vector. If you look carefully the dimension of this matrix is 3 cross 1.

But if I want to write a row vector right; so for instance here the row vector for this matrix is 2 2 1 right this is a row vector. Now if you look carefully its dimension is 1 cross 3. So, this is just a preliminary different definition, and then we will now determine some of the matrix properties are different types of matrix for instance the diagonal matrix.

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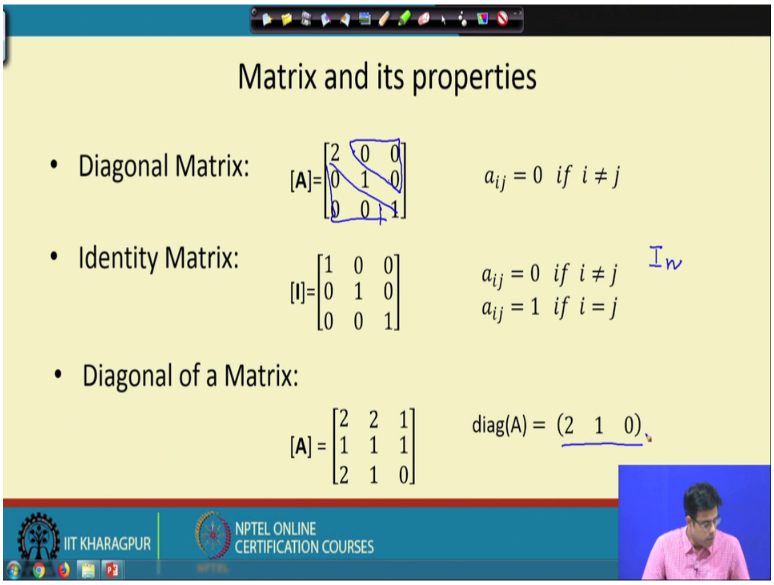

Diagonal matrix means it is all diagonal elements all are some of the diagonal elements are nonzero, but off diagonal elements are 0; that means, if a ij equals to 0, if I know not equals to z. So, if you look carefully this is the third order matrix, where first row in a first row all of diagonal elements; that means these elements are 0. So, these elements are 0 all of diagonal elements are 0.

So, similarly we will define identity matrix where all diagonal elements are one and off diagonal elements are 0. So, if I write for instance identity matrix of order n; that means, its nth order matrix, whose diagonal elements are all 1. So, this is the basic definition of diagonal and identity matrix.

Sometimes we also represent diagonals of a matrix. So, this has to be looked carefully. So, diagonals of A here is a 3 cross 3 matrix. So, diagonals of A are the diagonal elements of A that is 2 1 and 0, which we write sometimes diagonals of A like this and this is a matrix a this is a vector, which contains the all diagonal elements.

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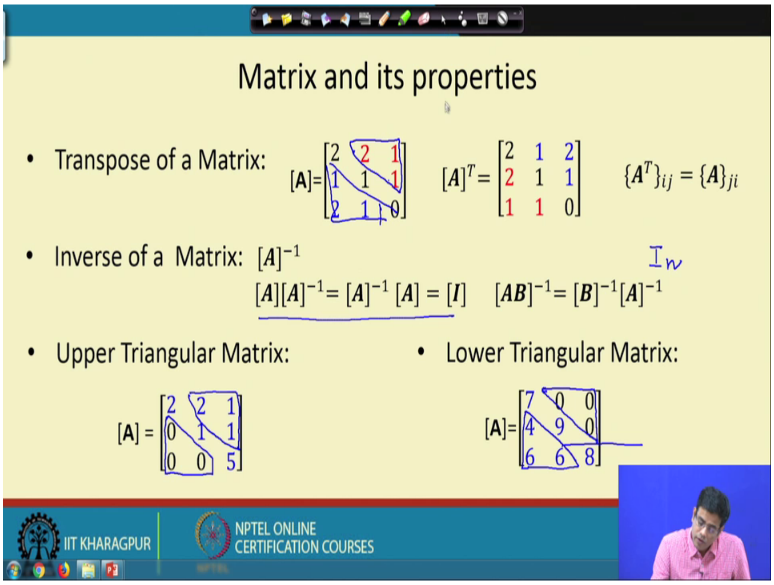

So, now the transpose of a matrix, the transpose of a matrix means its if you just flip the row and columns of the matrix; that means, here this is a 3 cross 3 matrix is you see, these blue letters goes up and red letters go red numbers goes down in a transpose.

So, the by definition we can say that A transpose ij is A j i; that means, the off diagonal elements are changed. So, inverse of a matrix all of you have heard inverse of a matrix. So, inverse means there exists a matrix for which if you pre multiply or post multiply with the matrix to be inversed; if you pre multiply or post multiply we pre multiply and post multiply it will be identity.

So, this also you have learned in your first year course probably, and then one of the properties of the inverses is AB universe is; that means, the product of 2 matrix and then taking its inverse is B inverse A inverse. So, these properties can be validated for the 3 cross 3 matrix or it can be proved also, but we will not go into details of this proof because we will just use it here.

Now, another type of matrix will be very important for us is upper triangular matrix; that means, the matrices which has only upper parts are nonzero and the lower part are 0; that means, the diagonal elements are nonzero and the upper part of the upper of diagonal elements these parts are nonzero. But carefully if you do the lower part of the matrix its 0.

Similarly, we can define a lower triangular matrix, where the lower part of the upper part of the matrix is 0 upper off diagonal elements of the upper part are 0, and the off diagonal elements in the lower part is nonzero. So, then we will go for symmetric matrix.

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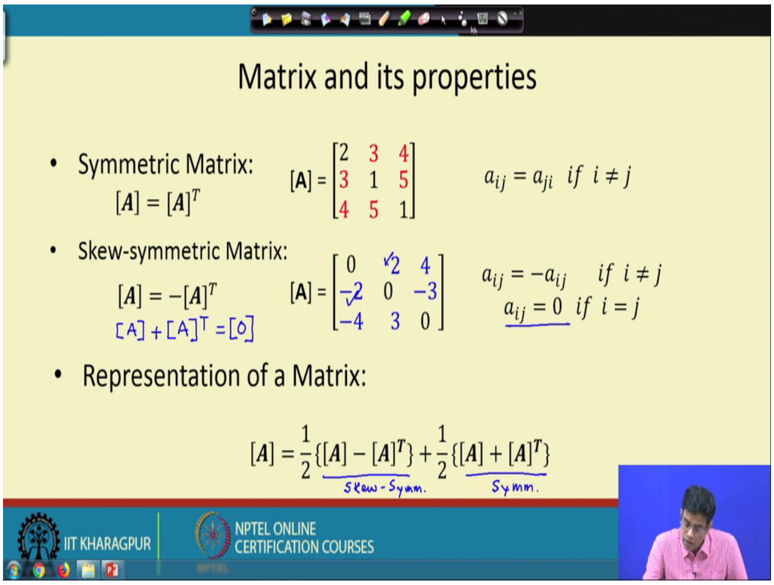

So, since we have defined transpose, we can now define symmetric matrix if a matrix A transpose is A then we say the matrix is symmetric matrix; that means, the off diagonal elements are same; that means, a ij if a ji if a ij is equal to a ji then we see that matrixes symmetric matrix.

Now, similarly skew symmetric matrix; skew symmetric matrix is if a A transpose A is equals to minus of A transpose then we say this is a symmetric matrix. Now as a consequence of this if you write this equation carefully, then you will see that A plus A transpose is a 0 matrix. So, if you write this equation, if you take this my A transpose this side then A plus A transpose is 0, which finally, results that all diagonal elements has to be 0 right.

So, the skew symmetric matrix or sometimes also known as anti symmetric matrix is if a ij is minus of a ij, when i not equals to j; that means, the off diagonal elements of the matrices are negative, but they are same in magnitude right only sign flips. So, here for instance this 1 2 element is just negative of 2 1 element, similarly all other of diagonal elements.

So, any matrix can also be represented any matrix can be represented in terms of a skew symmetric part and a symmetric part of the matrix. This proof is also very similar, but will not as we discussed earlier will not give you discuss the proofs mainly here we will use the results. So, this is a skew part of the matrix, skew part of the matrix of skew symmetry part and this is the symmetric part.

So, this form sometimes we will use that a matrix can be decomposed into a skew symmetric matrix a symmetric matrix.

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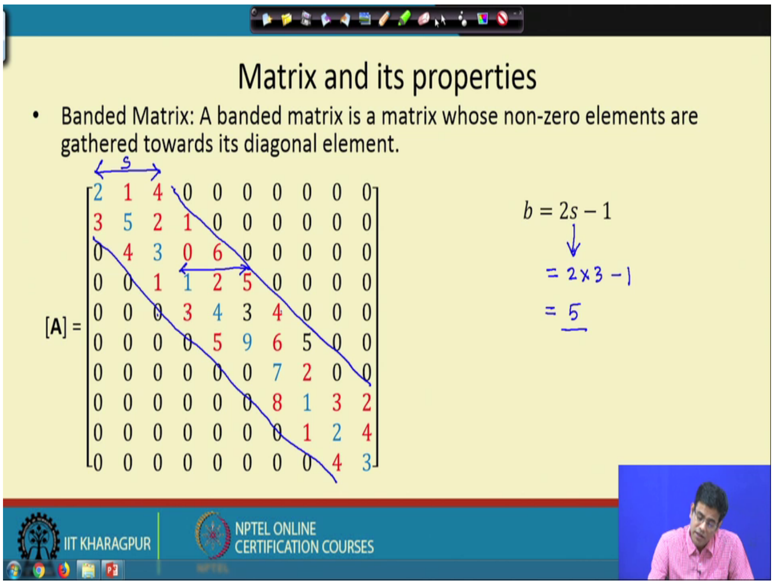

Now similarly we can define banded matrix for instance. The banded matrix means where the all nonzero elements of the matrices are arranged or gathered towards its main diagonal element. For instance look this matrix a where the all nonzero elements are located near the diagonal element. If I draw a line here so, it is looked like this.

So, you can see that this matrix clearly shows a band, which actually helps us in solving this matrix, that we will learn later part of this course. So, sometimes we need to feed the bandwidth of the matrix so that our solver can be effectively utilized; for instance if you want to calculate the band; so band width.

So, here the s is the largest number of columns from the diagonal which will have nonzero element. For instance if you look this matrix this is the so, this is the s ok. So, this s how I calculate this s? If you look carefully all diagonal element, maximum number of nonzero elements are only 2. So, this could this is my the bandwidth of the; this is my s. So my finally, bandwidth will be 2 into 3 minus 1 that is 5.

So, how I get the 5? The 5 I am getting in this way that maximum I can have this from the diagonal, I can have 2 and from this diagonal I can have two more non zeros here. So, this is how we calculate the bandwidth of the matrix.

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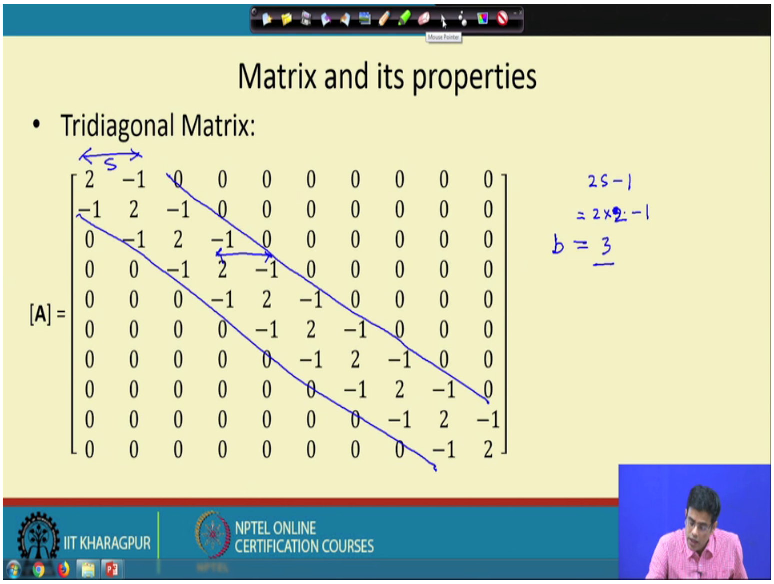

So, another matrix sometimes is useful is tridiagonal element tridiagonal matrix. This matrix is occurring in terms of when you do finite difference calculation, this type of matrix appears. So, sometimes it is also known as the finite difference operator. So, if you look carefully this tridiagonal matrix; that means it has a bandwidth of it has a bandwidth of 3, because how I calculate 2 s minus 1. So, s is 2, so 2 into 3 2 into 2 minus 1.

So, why 2 because s is here s is my 2, but it should be remembered that this s may not be only in the first row this is the maximum number. So, it can appear in any row; even though for this course this matrix this is same, this has to be the same. So, here also if you see this is the s is the 2. So, this bandwidth of this matrix becomes 3.

So, this also has a specific advantage when we learn lu decomposition of a matrix to solve a matrix vector equation, this type of matrix properties helps us in decompose in decomposing the matrix or sometimes in other factorizing the matrix. So, this property sometimes we will use. So, it is always better to know these properties.

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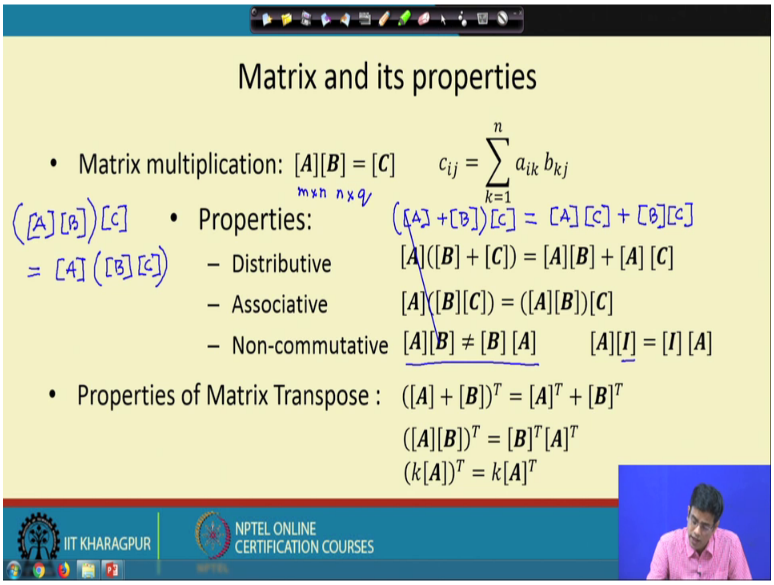

So,. So, now, I think all of you know matrix addition and matrix subtraction. So, matrix multiplication also you know, but I will just review once this is a matrix AB multiplied with C. So, AB can be multiplied a A can be multiplied with B if m has if m has A has m cross n rows and ma m rows and n columns and if B has n rows and q columns, then only it can be multiplied otherwise it cannot be multiplied right.

So, and this if I write in initial notation it looks like this. So, where k we runs from the number of columns ok. Now, if we look these properties of matrix multiplication, sometimes it is very important and also it is important to know these properties. This property is the first properties it is distributed what does what does this distributive means? That means, if I add first two matrices and then pre multiply the matrices another matrix A, then that will be equals to the post multiplying the matrix B with A and matrix C with A this property is known as distributive property.

In the same sense if I write this property. So, in a different way for instance if I want to write A plus B first, and then C I want to post multiply then this product will be matrix A then matrix C plus. So, this is the distributive property; then matrix the second property is the associativity. So, this also if I just pre multiply matrix A with the product of matrix B and C, then that will be equals to the if I post multiply with A and then the product I post multiply with C.

Similarly this can be if I want to write AB for instance, matrix A then matrix B with matrix C. So, this can I can write in this way this property is known as associativity right. Another property is non commutative property. In general matrix products are not commutative what does this commutative means?

This A if you if you post multiply with a post multiply B with A that will not be equals to pre multiplying B with A, which is actually the important property that matrix properties are non-commutative. But there are certain exception for instance if the matrix B is an identity matrix, which is I we have defined identity matrix. That means, all diagonal elements are 1 then this will be satisfying the commutative property.

That means, if I post multiply I with A and then if I pre multiply I we take those 2 matrices will be equal, but this is an exception, in general this is valid. Now there are some properties of matrix transpose also for instance the A plus B if you want to transpose, which is equals to A transpose B transpose.

If you multiply 2 matrices and then transpose it then it will be B transpose plus A transpose. If you want to multiply a scalar with matrix A then you can take out the if you multiply a scalar with matrix A. And then you can take out, the if you multiply scalar with matrix a and then transpose it that will be equivalent if you take out the scalar outside and then transpose the matrix and then multiply this scalar with this matrix A.

So, these properties not a very simple one and we need to remember these properties.

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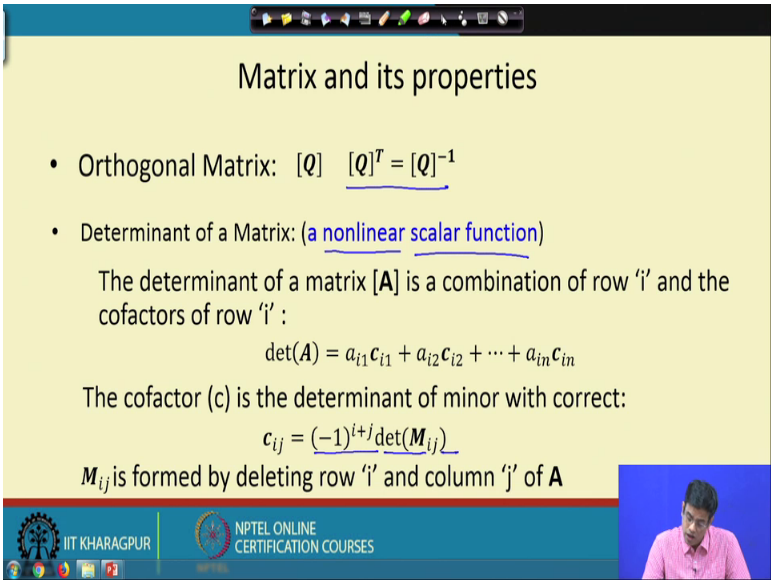

Now, another matrix probably we need is a orthogonal matrix; is orthogonal matrix can be defined as the matrix whose columns are orthonormal ok. So, we will discuss what is orthonormal and all those things will not discuss here, but it is important to know here for this course that an orthogonal matrix is that for which its transpose is actually its inverse.

So, this is the property for the orthogonal matrix. Now determinant, determinant is a scalar is a non-linear scalar function. So, remember one thing this is a non-linear scalar function. So, when we say determinant we always say the determinant, is determinant is something which is a scalar value. So, determinant can be represented as the A i 1 c i 1 c i 1 plus c i 2; that means, the determinant of a matrix is a combination of row i and the cofactors of row i.

So, we will see what is cofactor; the cofactor of a matrix is the determinant of the minors with proper sign; that means, the cofactors are essentially determinants. So, if I can compute the minors of matrix a which is a M i j and its sign if we take carefully, then we say this is the cofactor of i ii j th cofactor of matrix a.

So, how these miners are formed? These miners are formed by deleting i-th row and j-th column of the matrix.

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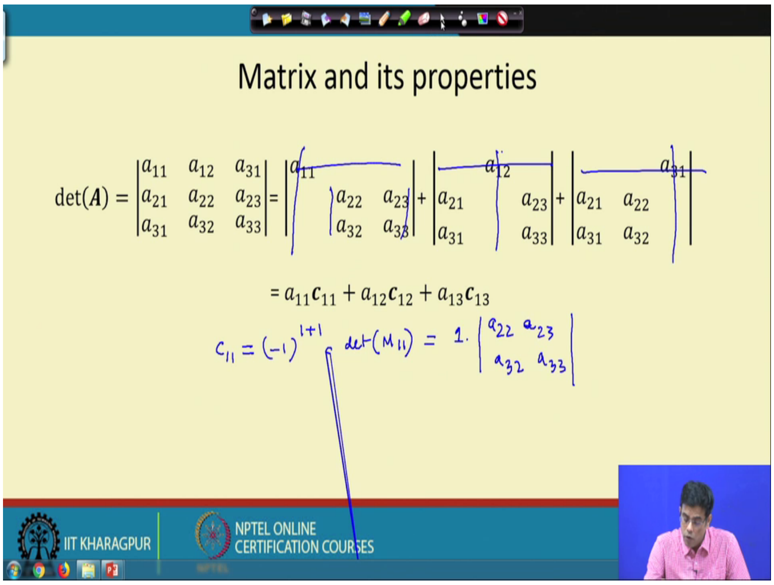

So, we will see once for instance this is a matrix 3 by 3 matrix mu you want to calculate the determinant.

So, if I want to write this, this is the formula for which we have discussed just now. So, the cofactors of c 11 is essentially the c 11 is essentially the minus 1 1plus 1 determinant determinant m 1 1. So, what is m 11? Tm 11 is actually. So, this will be plus 1 right the m 11 is the determinant of removing first row and first column; that means, if I remove this row this column then this is the case.

So, it will be a 22, a 23, a 32, a 33 right similarly for the others. So, this determinant is simple a 22 minus a 33 and a 22 into a 23 3 minus a 23 into a 32.

So, similarly for the other case they are deleting the second row and the second column and the first row, and first row and the third column. So, we will get the determinant of the matrix now.

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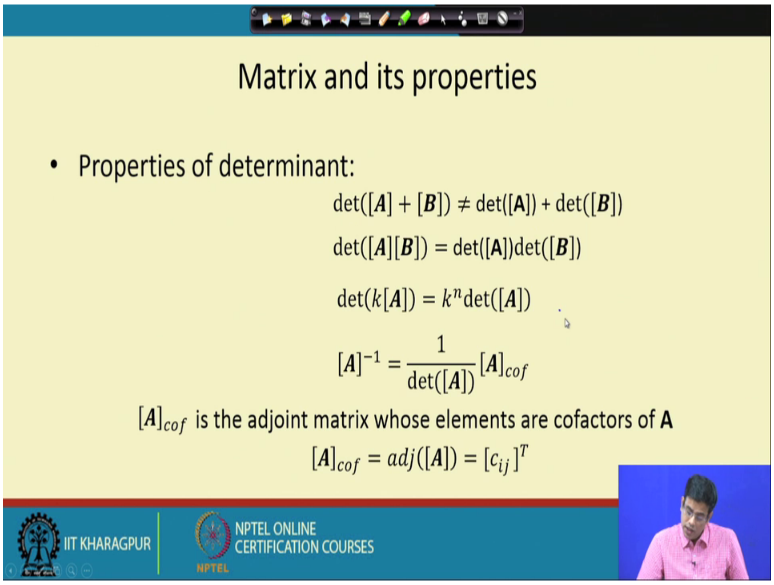

So, determiners property; so, as I have said determinant is a non-linear function. So, its summation on we just det A plus det B and then product of 2 matrices and taking its determinant is the taking individually its determinant.

So, det of A AB is det of A into det of B. So, if you want to pre multiply a scalar that is det of k into A. So, it will be k to the power n det of A. So, this n is the order of matrix; another property is the cofactor property of the determinant that inverse of the in calculation of the inverse. And inverse is essentially 1 by det of A A of cofactor. So, this e cough is the adjoint matrix. So, whose elements are cofactors of A. So, A cofactor is a generally known as the adjoint of A adjoint of A and then c ij is a cofactor and then its transpose.

So, this is we will see in a detailed manner in the next lecture. So, this cofactor matrix is also known as the adjoint matrix, which is the transpose of the cofactors of a matrix A. So, we will see this in the next lecture in detail manner and we will compute the inverse of a matrix.

Thank you.