Matrix Method of Structural Analysis

Prof. Amit Shaw
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture - 10
Review of Structural Analysis - I (Contd.)

Hello everyone. Welcome to the last lectures of Review of Structure Analysis one this is essentially a concluding lecture of this review. You see the last class what we did is we tried to write displacement force, displacement relation for a truss member. And then we also told we also mentioned that the idea, the purpose was not to write the expression rather the purpose was to make a very important point, we will be doing the exercise in detail in this up sequent weeks.

But the points that I wanted to make is you see, when we if you recall sometime back, this we discussed the stiffness and flexibility does not carry any meaning, until and unless we explain what are the forces and of the displacement are associated with that dis associated with this stiffness. Because stiffness essentially relate force and displacement, then naturally what are the forces and what are the displacement components we have in structure, that definition is very important right.

Now, if you have a for the first example in the previous class, where the where we have just the we consider the displacement, as the net displacement a net, net changing length of a member, then we had just one force and this strain displace the this load displacement relation becomes a scalar equation, then if we instead of instead of taking the net changing length for a member.

If we have to find out, if we have two displacement components, one displacement with a displacement at one end and, the displacement in the other end of the truss member, then you have two displacement components and then, two forces the force at 1 and at the force at, the other end the then the two displacement relation, two displacement components are related to two forces through a through stiffness, which is not a scalar which is a matrix 2 by 2 matrix right.

So, this becomes matrix equation that is called stiffness matrix and that, stiffness matrixes essentially the building blocks of matrix method of structure analysis. Now,

now but again then that that is not the end to it because, again we can go further and change the definition of the displacement, instead definition may not the proper, word change the direction of the displacement, the definition of the coordinate axis, instead of now taking displacement along the longitudinal direction you can take displacement along vertical and horizontal direction.

If we do so, then at every points we have two displacement one at horizontal and one at vertical, the two joint two joints means four displacements components and, then naturally four force components, the forces in horizontal and vertical displacement at both the ends.

And then four displacement components relate to force 4 force components to a stiffness matrix, which is essentially a 4 by 4 matrix ok. Now, then what is this size of this matrix that depends on, what is what are the displacement components, you take and what are the corresponding force components we take. But the one thing is very important in this stiffness matrix and, a I will just mention that that property today, but without giving really any physical interpretation of that property.

Because let us first go to third week and review, some of the concept of matrix methods and, then when we come to formulate the formulate the formulate the a formulate the method for truss and, then subsequently beams and frames, then will again come back to this property of the stiffness matrix irrespective of whether, it is 2 by 2 matrix or 4 by 4 matrix irrespective of what displacement components, you take and what force components, you take this property is a very important property and that remains same.

And we will come to that property once again, in a subsequent way. Now, you see so, if you recall if you recall, one of the prop if you recall this stiffness matrix for a truss for a truss.

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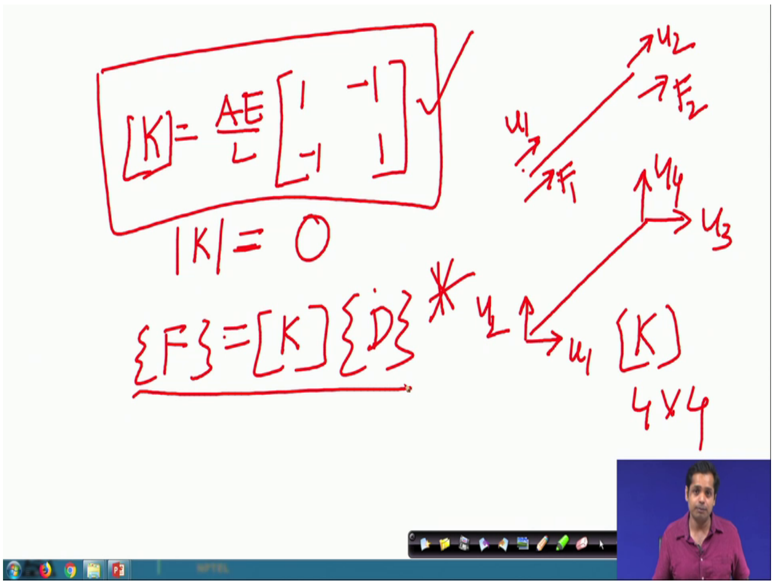

This stiffness matrix was K is equal to A by L 1 minus 1 minus 1 and then 1 ok. And this is and this is your for a truss like this like this, where we have it is u 1 and, here you have u 2 and corresponding force 1 and force 2 ok.

So, these stiffness matrix relates force 1 and force 2, this is force F 1 and then this is direction force 2. So, this relates force 1 and force 2 with u 1 and u 2 right. Now, what I was telling just now is now instead of instead of that, if we take 2 displacement components here, say u 1, this is u 2 and then this is u 3 and, then u 4 and then correspondingly you have a F 1, F 2, F 3, F 4. So, 4 displacement components 4 force components, then your stiffness matrix K will be a 4 cross 4 matrix ok.

Now, in this case you have 2 displacement 2 forces 2 by 2 matrix. Now, see there are other important properties of stiffness matrix, those properties we will not discuss now, but just a property which is very visible, when we write this expression is you see what is the determinant of K the determinant of K is equal to if you take that is equal to 0. Now, determinant of K is equal to 0 means, we have a force which is related to displacement like this. Suppose, we know the forces we have to find out the displacement right.

Now, this is the linear equations at the system linear equation. And then these linear equation, but if the determinant is 0, if the determinant of this matrix is 0 then really we cannot have the solution of this matrix solution of this linear set of equation right, but we have not done anything wrong while deriving this expression. So, we cannot question these expression these expression is absolutely right given the assumptions of the structure which is very reasonable assumption.

So, these relation is fine, there is the absolutely wrong nothing wrong in this relation, but then if this relation is right, then does it mean that we cannot really find out, for a given force we cannot find out displacement because, it is evident as of now is it evident, that if this stiffness it if the determinant is 0 then, how do I find out the solution.

Now, I put a star mark here that is a very important very important thing, we will come back again, why it is 0 and what is the physical interpretation of the determinant of stiffness matrix being 0 and, then what is the mathematical interpretation of this determinant of stiffness matrix being 0. And, what happens what needs to be done mathematically and also physically there, they both are consistent, what needs to be done so, that we can have a solution of this equation ok.

Now, and you can if you do that and, it is not particularly for truss a it is true for a whether it is beam frame or any structural any structural components, or even if it is true if we move beyond matrix method of structural analysis as I said, the more general method is finite element method even in finite element method will see, that this stiffness matrix is determinant it the singular, it is not the determinant of the stiffness matrix is 0. And it tells the very important story and that story we will see will in this upcoming weeks ok.

Let us first before we come to that point, let us first review the concept of matrix algebra in the next week now. So, if we do it for if we do it for beam, say for instance if we take a beam problem take a beam problem.

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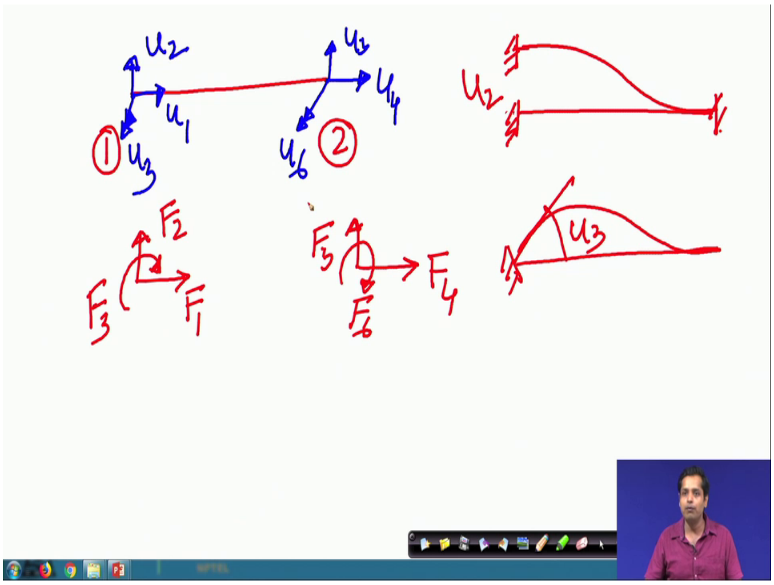

Now, suppose this is node number 1, this is point number 1 and this is point number take draw this is point number 1 and this is point number 2 ok. Now, at every points we have 3 a degrees of freedom, if for a plain beam if it is tra if it is space if it is space frame, then at every points we have 6 degrees of freedom, now what are the degrees of freedom.

Now, if we consider all the forces means, if we consider the force in this direction means, let us take different color means suppose it is u 1 is the displacement in horizontal direction, then we have displacement in vertical direction and, then we have rotation of point a. So, we have rotation of point a which is u 3 ok. Similarly at this point also we have this is u 4 and, then u 5 u 5 and then we have rotation at joints say u 6 ok.

So, total 3 com total 6 d 6 degrees of freedom we have. So, similarly we have 6 force components, force F 1 in this direction and F 2 is let us write the force components also. So, F 1 this is F 1 and this is F F 2 and then moment in this direction. Similarly we have F this is F 3 and, then we have F 4 and then F F 5 and then moment F 6. So, we have 6 components of forces and 6 components of displacement, the 6 components of displacement forces are related to a stiffness matrix naturally in these case the size of the stiffness matrix will 6 by 6.

But, now again if we assume that, there is no axial shortening of a member so, there is the axial shortening is neglected, if we neglect u 1 and u 4, there is no u 1 and u 4 if you do not eh do not the axial deformation, if you neglect axial deformation, then we are left with then we have only 2 displacement add do 2 degrees of freedom here, one is the displacement in these direction and, the rotation at this point and, then again displacement transfer direction and rotation at point.

So, essentially then we have a 4 displacement components. And, correspondingly 4 force components this size of the stiffness matrix become 4 by 4, but again irrespective of whether it is 4 by 4, 2 by 2 anything the similarity property that just now we discussed this, stiff this the determinacy of the stiffness matrix is 0 that remain same ok.

Now, just to know what a just to give you give you some idea about in what way, consider let us individual displacement and, then see what happens. So, if I take say for instance say for instance say a u 3, if I take u 3 if I take say u 2, u 2 means your displacement your displacement becomes displacement becomes this. So, this is the structure and, and then this is the structure and your only displacement is u.

So, all other displacements are then we can find out, what is the corresponding forces what are the corresponding. So, it is u 2 this is u 2 so, just now like the truss problem, we consider individual degrees of freedom and, wrote the how this individual degrees of freedom is related to corresponding forces. And, then we calculated what are the total forces at each node and get the stiffness matrix.

Similar exercise you can do for beam as well. So, this is we have to take individual degrees of freedom individual mode all the modes separately and, then we combine them together. Similarly if we have if we take u 3, then u 3 becomes if this is the original configuration and, then u 3 is the rotation at this end rotation at this end. So, rotation at this end so, this is u 3 yes this is u 3.

So, we write force displacement equation for u 3 and, then again we can do it same thing for u 4 and u u 4 u 5 and u 6 for all and, then we have the force displacement relation we combine them to get the stiffness matrix, but since we will be doing that in details I am not discussing right now. So, let us now summarize what we have discussed so far and then move beyond.

So, displacement comp a the stiffness definition is the building block of matrix method of structure analysis, we have to write the force. So, we have for first we need to decompose the entire structure, into small small part for each part we calculate the write the force displacement relation, through this stiffness definition and, then we assemble. And every part this relation the force displacement relation, every parts this stiffness matrix, this stiffness is essentially a matrix and, what is the size of that matrix depends on the what are the degrees of freedom where considering ok.

Now, once we have written the force displacement relation for all members. And, the next thing is we have to assemble all this members, to get the global individual the individual stiffness matrix. Sometimes it is also called members stiffness matrix, member stiffness matrix because, it gives you the how the members forces are related with each other, with the displacement in a particular member.

Once we have a member stiffness matrix, then we have to assemble all the member stiffness matrices and, get the global stiffness matrix of this structure and, when we calculate global stiffness matrix of this structure. We have to consider the orientation of different members and how this connectivity different members a all together.

Once we have the global differences, then we have to solve it, but then when we solve it some of the difficulties we may face not the difficulties some of the interesting observation, you will see a while solving those matrices. And those observations we will discuss those interpretation of those observation will discuss, when we face it. And all this exercise everything needs to be done through matrix operation right. And therefore, it is very important to review the matrix operation and, that we will be doing in the in the next week. Now, once we have once we formulate the probable once, we formulate the entire the method for different structural components for say truss for beam and for frames and, then demonstrate that formulation through a small examples.

Next you see there are two important motivation for going looking beyond matrix, looking beyond structural analysis 1. The first motivation is way first motivation is that we need to solve very largest very large structure, where are different members, large number member of large number of joints and different kinds of forces.

So, scalability is an important factor and, then another important thing is the ability of the method to get translated into computational code ok. So, whatever formulation we see we learned here, this formulation really we do not do it manually, we have to write at the end of the day we have to write at computed code to do this operations ok, to assemble to write this stiffness matrix. So, different member identify the orientation of different members, write the stiffness matrix, for different members and assemble it and then solve it.

Suppose you have for any truss, suppose you have 100 or say 500 joints and, each joints you have 2 degrees of freedom. So, 1000 by 1000 stiffness matrix you have. So, is not you can have a very large number of stiffness matrix which cannot be cannot be solve manually. So, computer has to do it, you have to write a computer code. So, what we will do towards the end of this course will spend one week to discuss various implementation issues, when we implement that method into computer codes definitely we will give some example how to write it, how to what will be the structure, in what way we need to store different variables, we call it stiffness matrix, but does it mean that we have to when we when we really store that matrix in a computer program.

We have to store that in a matrix form, or there are different ways, we can store those variables to get to optimize the operation to get the code faster. So now, those are the implemented implementation, implementation and translation into computer codes, various issues related to that we will be discussing towards the end of this course, ok.

So I stop here today. Next class, next week the review of matrix of algebra will be taken by Professor Vishwanath Banerjee. I will come again on forth week, where we will formulate the matrix method of matrix method of analysis for truss. See you.

Thank you.