Matrix Method of Structural Analysis
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Lecture – 01
Introduction

Hello everyone, welcome to the online course on Matrix Method of Structural Analysis. This course is being offered as NPTEL NPTEL online certification course. So, let us begin by acknowledging NPTEL for this great initiative and also a center of education and technology IIT Kharagpur for making this arrangement ok. To start with let me introduce the team here, we have umm there are 2 instructors for this course ah.

My name is Amith Shaw; I am a faculty department of civil engineering IIT Kharagpur. And along with me Professor Biswanath Banerjee, who is a faculty in the same department; we will be taking some of the lectures. Well, if you have enrolled for this course; then, you have access to the discussion forum and throughout this course you can post your queries in the discussion forum. But if you have not enrolled in this course you are just attending this course, then you can or those even after this course.

If over even after that as well if you have any clarification, anything to discuss then you can always write to you are most welcome to write to us, but please make sure whenever you write to us umm the subject of this email should be NOC hyphen MSA, this will be help us to umm this will help us to identify these emails among all the emails that we get regularly ah.

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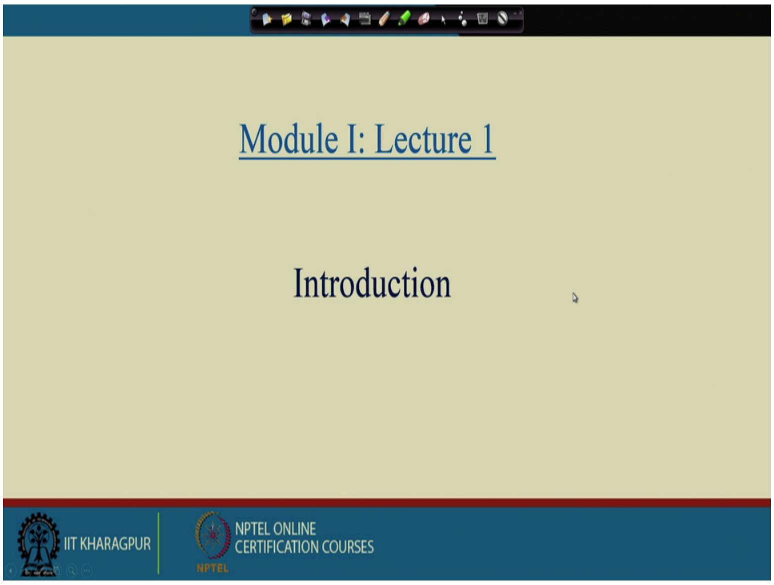

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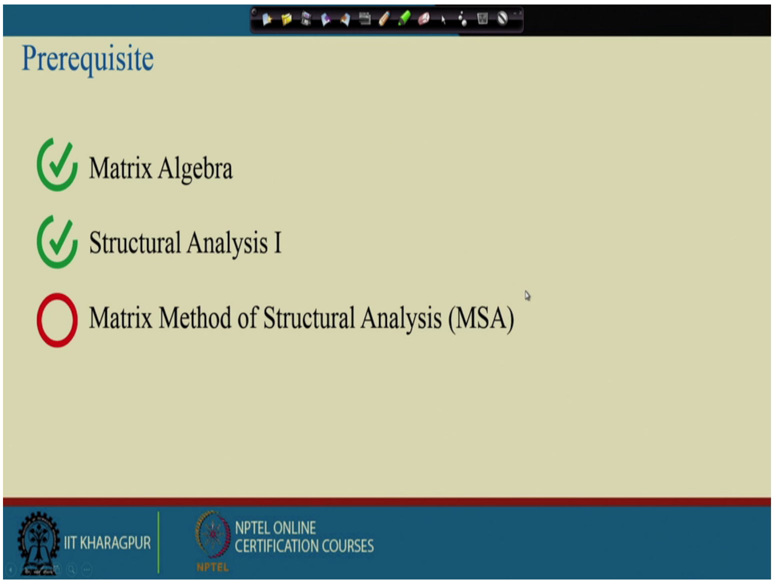

So, this is the first class on introduction. See this is an intermediate level course, so there are 2 important prerequisites for this course, one is matrix algebra and I am sure that all of you have all of you are familiar with the matrix algebra, if not then tell algebra some of the concepts of matrix operations, then properties of matrices and solution of system of linear equation many algebraic equations and as this is an intermediate level course the second level course.

So, it is it is assumed that you have already done the first level course on structural analysis, which is structural analysis I. And for the completeness of this of this course we will be spending some time initial few weeks to revisit some of the basic concept of matrix algebra and structural analysis. But do not wait for that these please go back to your ah go back to your basic go back to any structural analysis book on matrix engineering mathematics book, to brush up some of the concepts umm basic concepts in these 2 subjects ok.

You see if I have to this course is for umm 20 hours, as spanning over 8 weeks. Throughout this as we go through this journey we will we learn what is the matrix method of structural analysis and how the matrix method of structural analysis, can be applied to different kinds of structure? But since it is a first class let us try to understand, what is the motivation for this method. Why the knowledge of structural analysis one that we already have are not sufficient to analyze the structures that we come across in everyday life.

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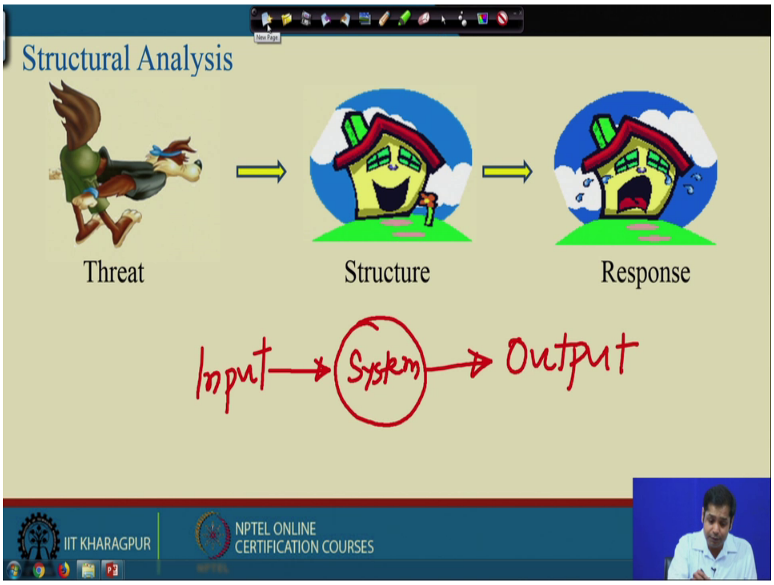

So, in this lecture, we will try to find out a good reason to learn matrix method of structural analysis right. You see if I have to define what is structural analysis, then probably in a in one slide if I have to define then this will be definition. If we have a structure which is subjected to different kinds of threat and then we want to understand how the structure may respond when they are subjected to a different kinds of threat.

And in fact, it is not only for structural analysis; if you if you look at carefully we will see that any engineering analysis is based on these philosophy, we have a system we have a system any system and in this system we there is some input, input and we want to understand what to do with the corresponding output right. Now, here input here the system is structures different kinds of structures and input is threat and the output is the response of the structure.

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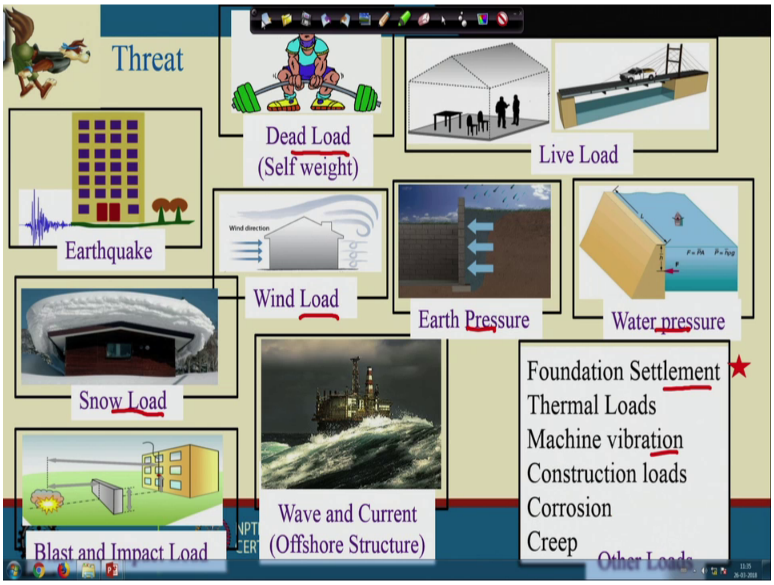

Now what are the different kinds of threat are possible? You see if you these are the different kinds of threats that is structure ah may be subjected to we can have dead load which is the self weight of the structure, live load on the structure, then you can have earthquake loads, then wind load earth pressure, water pressure, if it is offshore structure then wave and current loads are important.

We can have blast load and impact load depending on the environment we can have snow load as well and other than that you can have support settlement, you can have thermal loads, we can have machine vibration and so on there are different kinds of load. So, it depends on what is your objective? For which purpose you are building the structure? What is the purpose that the structure is going to serve depending on that you have to consider the load right.

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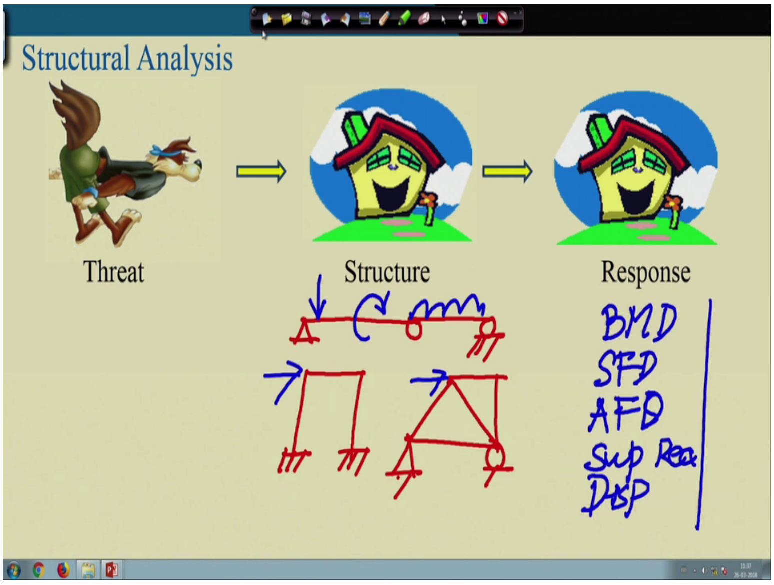

Now, now you see once we have once we have this load, then the then the what are the structures that you have so, far so, far studied in structural analysis one, you have studied how to solve say for instance a umm ah beam problem a beam problem, it could be it could be a say simply supported beam or continuous beam you have studied how to solve a the frame problem, say then you have studied say if you have a truss then you have a truss.

So, these it is these are some kind of structure that umm you have studied how to analyze these kind of structures ok. Now, the response what response you have studied so far we now if these are the structures subjected to ah the threat, again as I say there could be there are different kinds of threat, but at the end of the day you can see the threat can be idealized as some concentrated load for instance in these with these could be the are say these structure are subjected to uniformly distributed load, or concentrated load, or you can have umm you can have a concentrated movement, you can have a different kinds of load see these are different kinds of threats ok.

And these the, these are some structures and what we have studied what response we have studied. So, far we have seen how to calculate say bending moment diagram if it is beam then what will be bending moment diagram is at any particular section what is the bending moment? And at any particular section what is the shear force from the shear force diagram.

And then we also have seen how to find out axial force diagram at any at any such axial force diagram at any section what are the axial forces then internal forces in the member and in addition, we have we have seen support reaction how to find out support reaction, support reaction and then umm, then we have seen how to find out say displacement. So, these are some responses that we have you have studied in structural analysis I ok.

Now, you see this analysis is then see there are there is there are some steps followed in any engineering analysis. So, if a structural analysis and what are those steps those steps are. Say see first we have a physical system and that physical system is the actual real system that we want to study, but you know because of because of many reason because of the complexity associated with the system complicated boundary conditions and there are some uncertainty that, we do not know we cannot take the physical system as it is.

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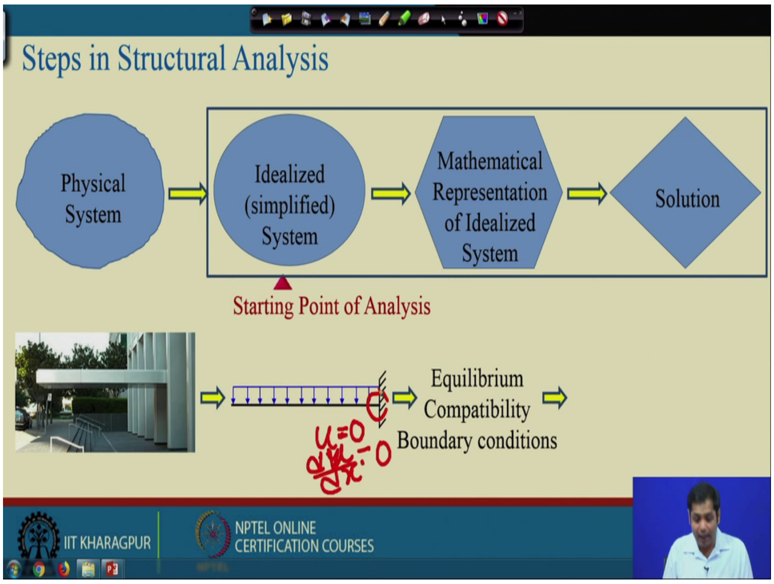

So, what we need to do is first we need to idealize the physical system. So, first step we do we idealize the physical system, simplify the physical system, but your idealization should be based on the physics of the system the idealization should be based on based on the what kind of analysis you are going to do with the system and you are at the end of the day giving all these given a particular objective your idealized system should be as close as possible to the real system the physical system.

Now, once you have idealize the system makes to present the system, idealized system through some mathematical equation that is the mathematical representation of the idealized system and that mathematical representation could be in the form of say algebraic equation, differential equation, integral equation or combinations of all ah. Now once we have the mathematical representation of this idealized system, makes the solution of this solution of this of this mathematical model.

Now, please remember one thing is to be noted here at every steps right from physical system to idealized system and then idealized system to mathematical model and then solution of the mathematical at every steps we assume something, we make certain assumptions and these assumptions are very important because these are the assumptions which are the limitations of this flow as well because the because suppose if certain..

If any theory or any methodology is any methodology based on certain assumptions it is very important to understand those assumptions because the limitations of that methodology are the assumptions. The cases where those assumptions are not valid those cases these method cannot be cannot be applied. So, it is very important to understand those assumptions now just to give you an example.

So, this is the starting point of an any analysis the class the frame the beam that I just that I the that I shown in the la in the previous slide those are all idealized system not the actual system right. Now, let us give you an example take your physical system this is a cantilever portion cantilever slab at the at the front at the front gate of the structure the entire structure is not shown for the shown here because of the space.

Now suppose I want to analyze this cantilever portion this structure, now what would be the idealize this is the physical system the idealized system will be this entire thing can be idealize the cantilever beam which is subjected to uniformly distributed load and uniformly distributed load is the self weight of this beam. Now, once you have idealized the system the mathematical representation, if you recall the mathematical representation that you used here in solving these kind of structures in your structural analysis I course that equilibrium equation and then compatibility equation and of course, the boundary conditions ok.

Now, now if you if you use this if you now once what is equilibrium equation, these equilibrium equation says that the internal forces and external forces they should be ah they should balance each other right, compatibility boundary conditions here is what for instance at this point, take a different color at this point at this point your displacement is 0, displacement is 0, u is equal to 0, u is u is equal to 0 and the slope is equal to 0.

So, dy du dx is equal to 0 these are the these are some these are the boundary condition you have here the slope is 0 and moment is 0 right, and then equilibrium if you set it if we if we use this equilibrium condition equilibrium conditions if you remember, summation of forces is equal to 0 and summation of moment is equal to is equal to 0 right.

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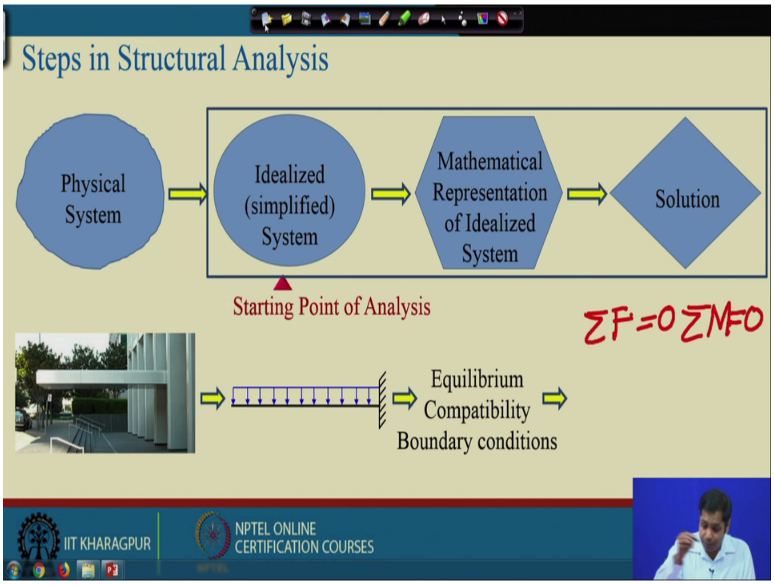

Now these are the equilibrium conditions now if we apply this equilibrium conditions, now if we apply this equilibrium conditions, then ah then umm then the bending moment this bending moment diagram for this beam will be something like this is the bending moment diagram and the shear force diagram will be linear something like this and the deflected shape will be something like this.

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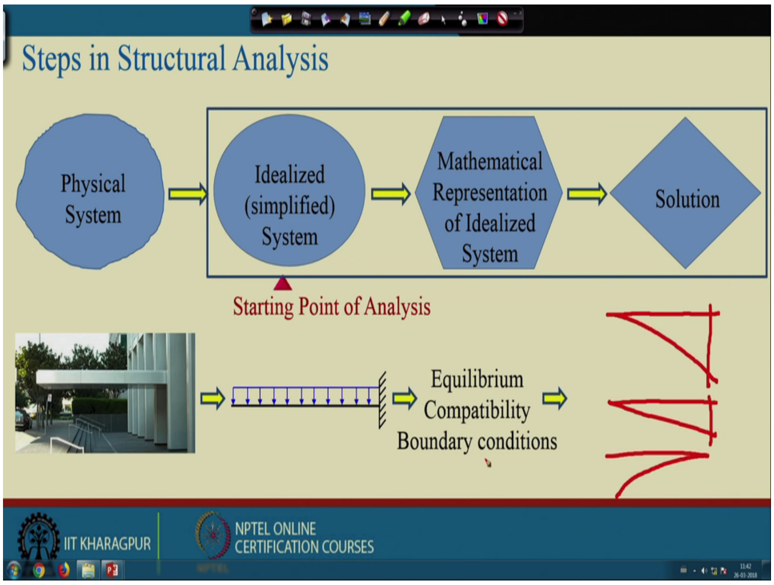

So, you will already know with the knowledge in the now with the knowledge of structural analysis I, how to get this solution? So, these are the solution of equilibrium equation compatibility equation and boundary conditions ok. Now Ah you see so that we do for in that for all the structure, now let us. So this is the starting point of analysis now you see a let us find out.

Now before as I say this seems this is the first lecture the idea of this lecture is to find out the good reason to learn matrix method of structural analysis, just the flow that I have show that is shown here, that this flow this flow we know how to do this how to solve the structure with the with the knowledge of structural analysis I right. Now, let us find out why this knowledge is not sufficient to solve to deal with structure that we come across most of the time.

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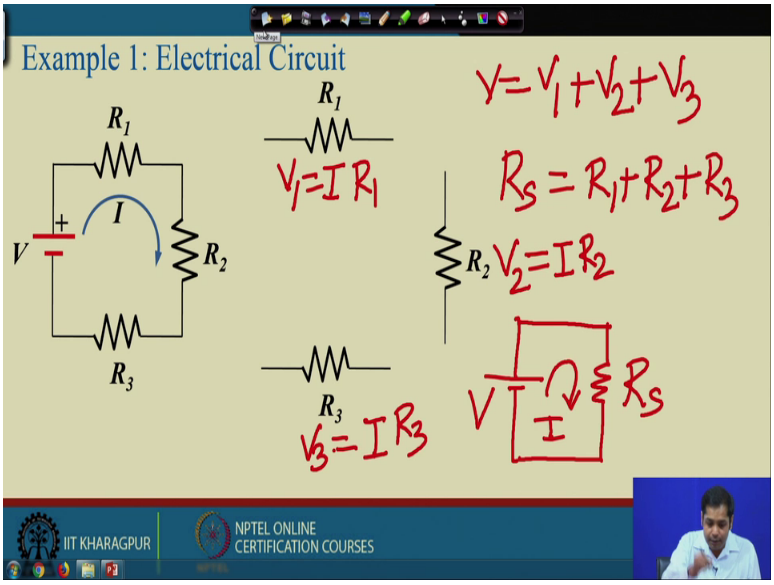

Now ba instead of directly coming ah coming on to the on to the on to the let on to the motivation, let us try to understand the motivation from some examples. Now you see. take one example this example we are going to take few example, that examples are are you have you have seen this kind of example in your in your school physics or your first engineering mechanics or even some of the examples that you have not you have not learnt in your engineering curriculum, but you can relate to those examples because you have seen them many times.

Suppose these are electrical circuit right, now what we want to do is as. So, these are 3 registers 1 R 1, R 2 and R 3 and then we are going to find out what is the this is the current is I, then we have to find out what is the voltage output V ok. Now, you see what we if you recall how you solve these example we have 3 resistors, this is 3 resistor this is 3 resistor R 1 R 2 R 3. Now then what we can do is we can write here that V 1 is equal to for this V 1 is equal to I 1 I plus I R 1 this is the relation right from Ohms law you can get this relation.

Similarly here we get V 2 is equal to I R 2 and similarly here V 3 is equal to I R R 3 right. So, these gives you the voltage drop across the circuit this gives you voltage drop across this resistor and these gives you voltage drop across these resistor. Now the total voltage drop is V 1 plus V 2 plus V 3 which should be equal to V then we have V is equal to V 1 V 1 plus V 2 plus V 3 and if we substitute V 1 V 2 V 3 these gives us these gives us the effective resistance R s or resultant will be R 1 you have you know this. So, I am not writing the entire steps here R 3 ok.

So, this these can be the these can be can be can be can have a system like this, where this is R s and this is I and this is V ok, and then we have V is equal to I into R s when R s is equal to R 1 plus R 2 plus R R 3. So, there is as an example there is there is there is nothing to see new here you have already seen it, but my objective here is not to not to not to solve this electrical circuit problem my object will be something else and that will be clear shortly.

Now, before we come before we before we come to that point let us see one more example, this example is again you have seen in your first year mechanics I believe that is the spring network. Ah we have 3 springs are connected k 1, k 2, k 3 and they are connected in series and, if there is force f being applied at the end now what we want to know k 1, k 2, k 3 are the stiffness spring stiffness of respective springs ok.

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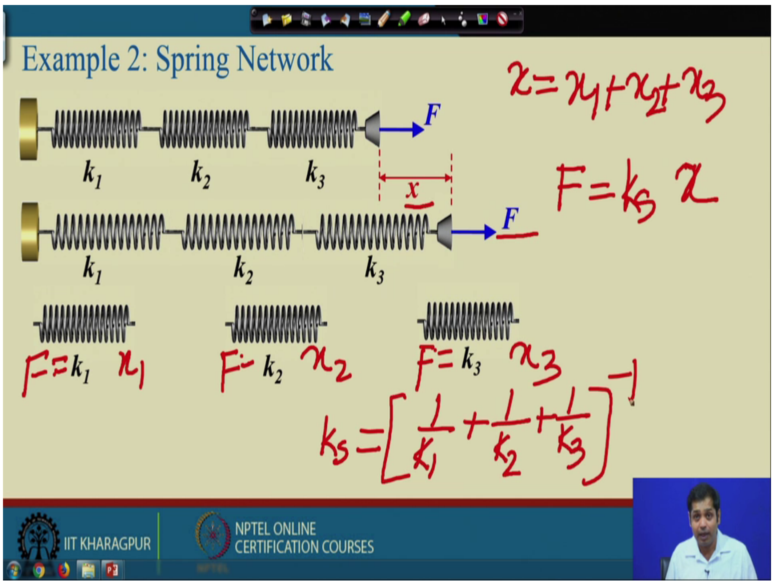

Now if we apply a force f like this suppose the spring take a deform spl take a form this, and the deformation and the total deformation becomes total deformation becomes total deformation becomes x. Now, let us find out let us try to find out a relation between this x relation between this x and the and f right. Now what we can do is before first we take and if you this also we have seen in your first mechanics we take these 3 springs separately spring 1, spring 2, and spring 3 and then at any point if we take the force will be will remain same F.

So, suppose x 1 x 1 is the displacement here x 2 is the displacement here and x 3 is the displacement here, then we know here that F is equal to F is equal to k 1, x 1, and similarly F is equal to here k 2, x 2 and here F is equal to F is equal to k 3 x 3. We know that and total x total x will be x 1 plus x 1 plus x 2 plus x 3 and if you substitute that and for this spring if I have to find out a relation between F and x we have that F is equal to k s into x where k s is the effective or the resultant stiffness of the spring system.

And k is related to k 1 k 2 k 3 as this k s as and you 1 by k 1 plus 1 by k 2 plus 1 by k 3 again please note and inverse. Please note here idea is has not been to solve this problem again my objective was my objective is to make a point and again before coming to that point. Let us these are the examples that probably you can relate to these examples if you are an engineering student let me give you some of some other examples you see.

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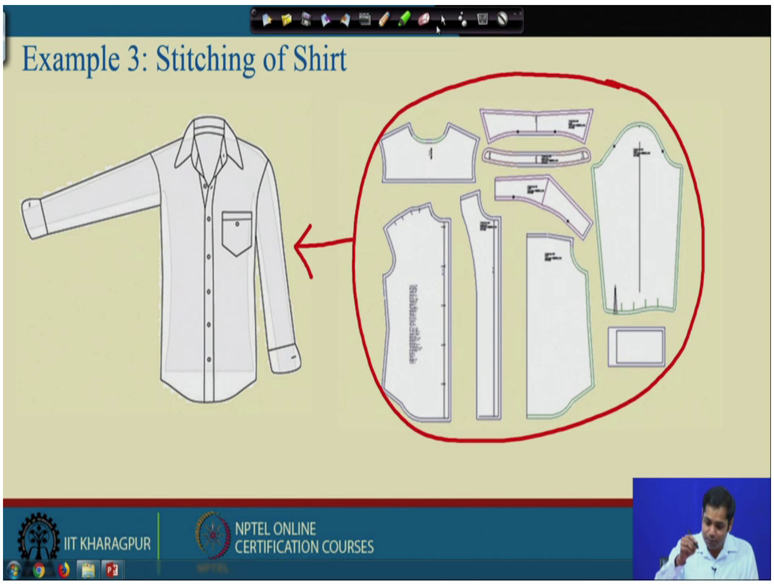

So, this is an example your own suppose I want to make a shirt this is the thing that if you go to any tailor shop they do, this is the final product and this final product in order to make this final product what tailor does is a tailor umm makes few cut pieces like this ok. And this cut pieces are not randomly random cut pieces this cut pieces are with proper measurement right, and then once we have this cut pieces then what tailor does they stitch this all these cut pieces together and get this get this the entire shirt.

So, all these stitches together to get this right. Now, again before I make the point let me see one more example and the example is you see. Suppose this is a typical passing pattern in football. So, take one more example this is the last example, before you make the point this is a typical passing pattern in football you see the objective is to objective is to ah to from this position from this position from position a objective is to pass the ball to different fielders in such a way that we can make a we can make the goal ok.

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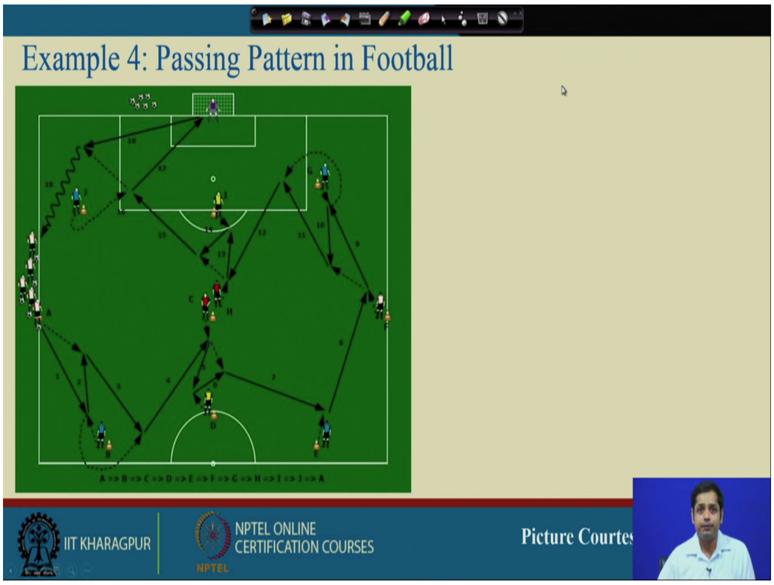

Now, you see all these passing patterns are shown and the movements are shown by arrow one is dotted arrow passing pattern, and the bold arrow are the movements of player. Now you see though if we follow these and the all the all these all these passings are a number 1 2 3 4 and so on, and here it is given these a b c are the a b c d are the different players and it is shown that how the ball should go from 1 player to in which order the ball should go from 1 player to other player and this is the this is the orders and this is the way the ball should go.

Now, in order the our objective is to get the to make the goal, but what we did is what is done here is instead of making the goal just directly a path is followed right, and if we follow this path if we if we if we if we add the entire path then the objective can be accomplished right. Now, let us see the example it is all these examples and if there are plenty of examples ah in nature and a everyday we follow the similar approach.

And what is the common threat between all these examples all the examples are from different fields they have different execution they have different objective, but there is a common threat in all these examples and what is that common threat? The common threat is you see remember in the electric circuit problem, there are 3 registers we broke the entire circuit and wrote the equations for each resistor separately and then we the conditions what we used is condition used is this total voltage will be V 1 plus V 2 plus V 3 right, and that condition when we apply that gives you resultant resist result resultant resistance as R s is equal to R 1 plus R 2 plus R 3.

Similarly, for the stiffness problem spring problem we broke the entire problem into 3 parts 3 spring parts wrote equation for each spring, when we what are the equations F is equal to k 1 x 1 for spring 1 F is equal to k 2 x 2 for spring 2 and F is equal to k 3 x 3 for spring 3 right. Now, and then these are separate equations for separate speeds then we wrote an equation wrote a condition and what is that condition the total deformation will be x 1 plus x 2 plus x 3 and that condition gives us what is the effective resistance of the entire spring.

In the case of shirt problem the entire thing we have several cut pieces and then those cut pieces are stitched together to get the final shirt and here also the entire objective entire objective is broken into small small steps. What are those steps? Here the steps are passing a ball from one fielder to another fielder. Now if we follow the all the steps if you can combine all the steps then we get the final objective the common threat in all these example is this if we have a problem.

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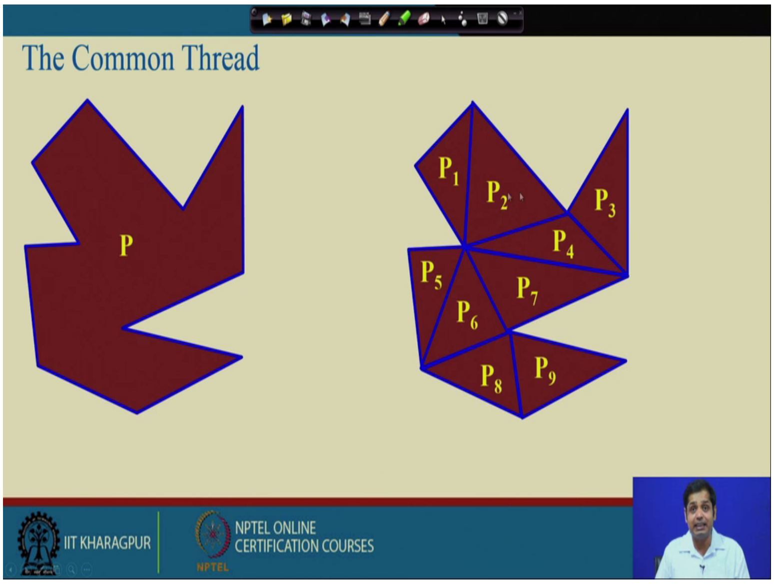

Which is very difficult to comprehend as a whole because of a the complexity of the problem because of the complicated boundary conditions or because of many uncertainties that probably we know we do not know about the problem. So, this is the problem and this is an this is a representation of the problem. This problem to be the electrical circuit problem, it could be spring problem, it could be the passing pattern problem, it could be stitching of shirt problem, it could be any other problem, that we come across in real life.

Now, these problem we do not solve we do not address our approach is what approach is not to analyze the entire problem entire problem together, instead the approach is we divide the entire problem into small small sub problems say P 1 P 2 P 3 and. So, on and then now you see the entire probe the initially this shape was complicated shape suppose in this case the problem is to find out the area of this shape this is a very complicated shape. So, what we do is we divide the entire area into small small areas small small triangular areas now for each triangle we know how to determine the area right.

Now, then so this is the this is the division of the decomposition of the problem this is also called descritization of the problem, then these are the different sub problems we write equations for each sub problems. Now these sub problems these problems could be when it is when it is the problem of a electrical circuit. So, these P 1 P 2 P 3 they were the each resistors separately when it was spring problem P 1 P 2 P 3 while the springs individual springs.

When it was by stitching a cut piece is stitching, stitching a shirt problem then this P 2 P 3 were the different cut pieces, and when it was passing pattern making a goal then P 1 P 2 P 3 by the passing a ball from one fielder to another fielder and. So, these P 1 P 2 we divide the entire problem into small small sub problems such that for each sub problems we know the equations, we know how to represent those sub problems through mathematical model.

We know how to represent each resistor V is equal to I R we know how to represent we know how to what is the relation between stiffness and the displacement of individual springs. So, we can write equations for each sub problem and then once we write the equation for each sub problem what we did we in the case of electrical resistor problem we write V is equal to V 1 plus V 2 plus V 3 in the case of spring problem it was it was x is equal to x 1 plus x 2 plus x 3 means, we give a condition to all this we give a condition which says how this difference sub problems are connected with each other how this difference sub problems are related to each other right.

So, in somehow we in somehow if we can relate these individual sub problems with each other through again an equation and then you solve that equation. So, these constitute the premise of matrix method of structure analysis not only that ah the advance structure at the after matrix method of structural analysis the advanced more advanced course in finite element advance course finite element method the premise is exactly that.

So, as a philosophy the matrix method of structural analysis the philosophy; that means, that philosophy we have to use many times many times not only engineers anyone, if you carefully think in day to day life also in some way we follow this philosophy, break the problem into small small pieces, small small sub problems, such that for each sub problems we know the information write equations for each sub problems and write a conditions how the sub problems are related to each other and then that condition gives you the solution of the entire problem.

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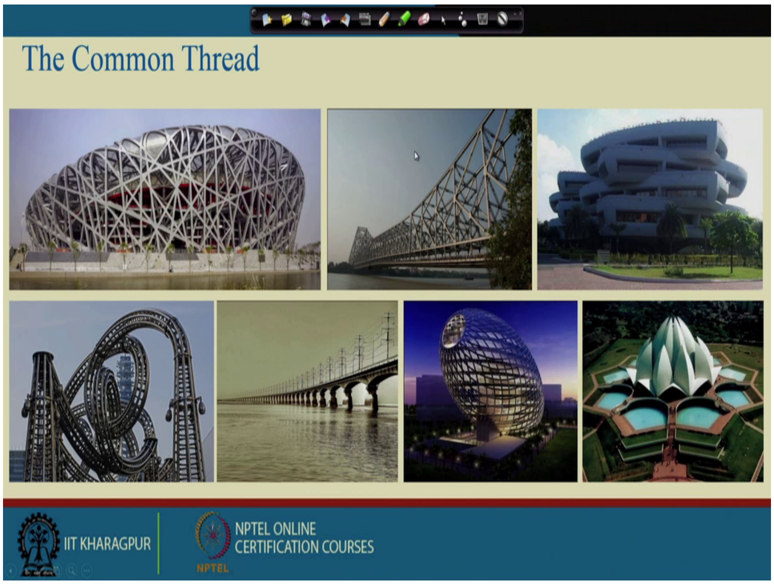
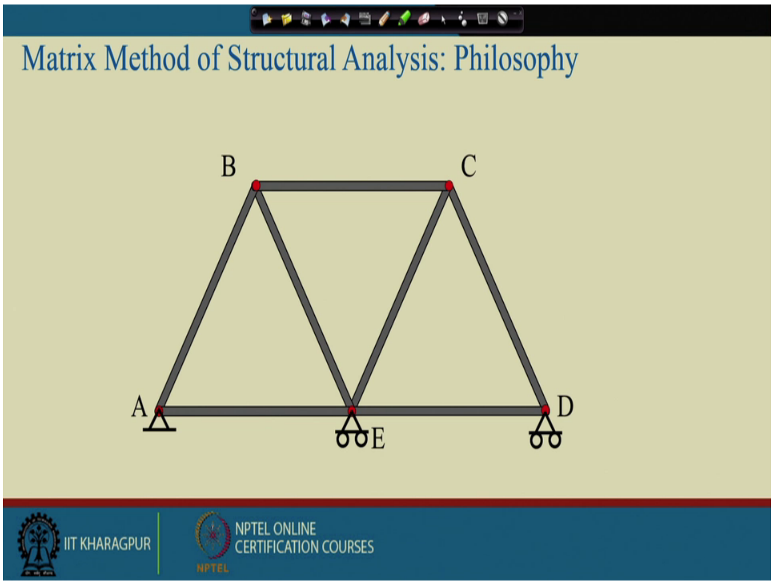


So, that is the philosophy of matrix method, now you see that philosophy why this is required because if you see the real structure the real structure, these are some examples of some famous structures. I have not given the names of the structure because I believe these structures are so famous their names are not required. Now if you look at the structure they are not just a beam they are just not a portal frame or a simple plain truss then they are they are a members oriented in a different configurations.

There are thousands of members connected with each other those kind of structures cannot be solved, a just with the method we studied in structural analysis one. So, what we have to do with this problems if you want to analyze this problem, what we have to do is we have to follow the philosophy that now just, now we have seen we need to break the problem into small small sub problems and then assemble all the sub problems to solve them ok. (Refer Slide Time: 31:17)



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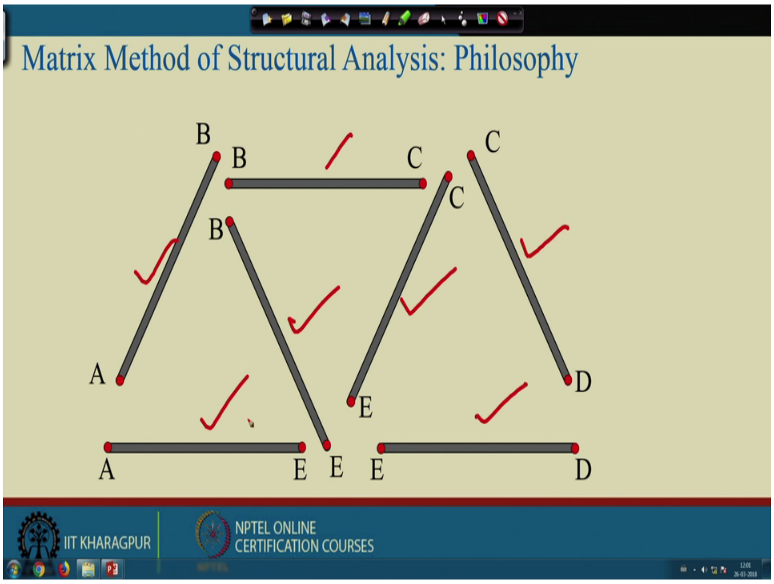

So if we have a truss like this what we have to do is this is the philosophies matrix method of structural analysis. We break the entire thing into small small problems you see small small problems and then write equations for you write equations for this, write equations for this, write equation for every every elements you write equations and then write a conditions which tells you how this elements are connected with each other how this elements are related to each other that writing that step is called the assembling.

So, we assemble these information from the each from each sub problem we assemble them to get the information about the entire system that is the basic philosophy of matrix method of structural analysis and that is exactly we are going to learn in this course. How it is to be done? How to decompose into small problems? How the equations for this some problems to be written? And how the assembling to be done and finally, once we had assembled it then how the assembled equations to be solved? Ok.

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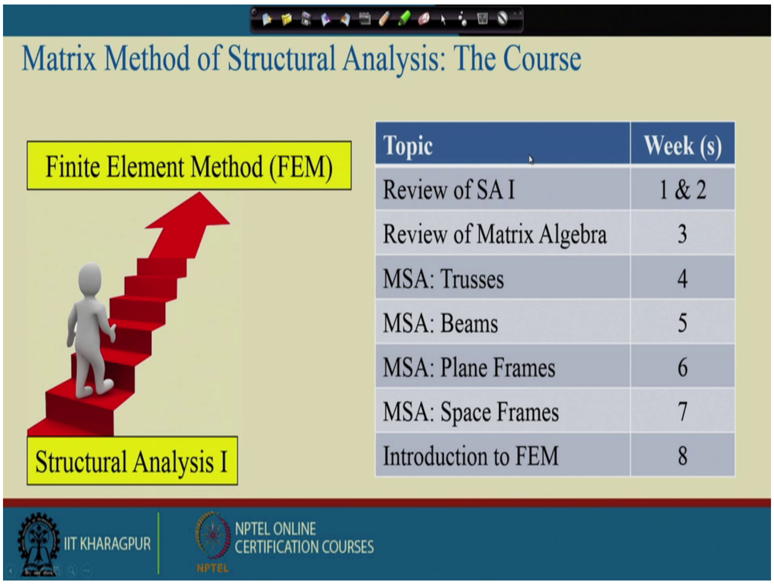

And everything we will do in using the concept of ah matrix theory that is why the name is matrix method of structural analysis ok. So, this course is this umm structure structural analysis is the foundation for this course. So, you already know the structural analysis I, then this course is a journey which takes you from structural analysis I, and at the end this course will prepare you for more advance course, more sophisticated, more elegant course, more general course, more general method is called finite element method.

Which also does the same thing breaking the entire problem into pieces and assemble them, but in a more mathematically sound when more elegant when more sophisticated with So, this is the core this is the journey which will take you from structural analysis I, to ma finite element method and this is the week wise the lectures the as I say it the first few first 3 weeks the first 2 weeks will be review of structural analysis I, and then review of matrix algebra some of the basic matrix operations matrix theory on the third week and then from fourth week.

We will start formulating these matrix method of structural analysis and that formulation will be demonstrated through classes in the fourth week and then through beams in fifth week and the planes frame sixth weeks space frame seventh weeks and then finally, when we know how to apply this matrix method of structural analysis for different problems, ah then one week will be spend to discuss how these concepts can be taken further to formulate a method, which is more elegant more general and that is on the eighth week we will have some brief introduction to finite element methods ok.

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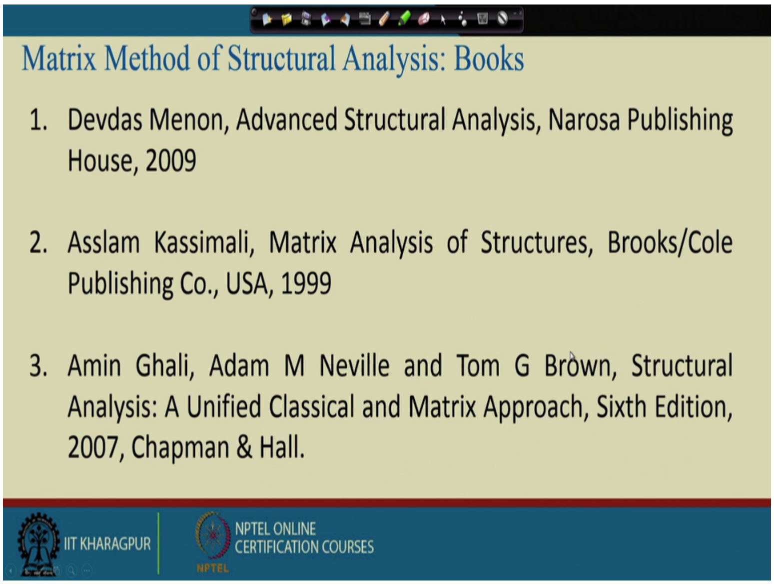

As far the books are concerned there are again many books these are the books that you can follow, but in this course we will not follow any particular book, we will get w will solve examples from different books some of the examples will be report listed in this books some of the pro problems will be from somewhere else, but for reference you can see these books ok.

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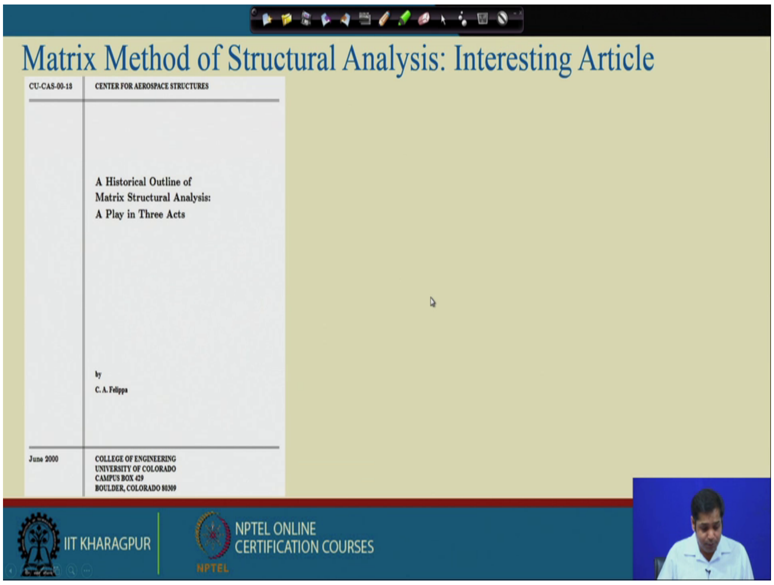

And there just before we before we stop you see when you are studying any theory any method. It is very important a learning from the history of that method is an integral part of a integral part of your ah of your learning process because you we cannot comprehend a course we cannot comprehend a subject, we cannot appreciate probably a subject in a comprehensive way if you do not know how the structure evolved over time.

So, if you just Google the history of structural anal history of matrix method of structural analysis there are many articles you get, but one interesting article I suggest all of you must read is the article which was published in June 2000, ah it was the article published in college of engineering university of Colorado that is the historical outline of matrix structural analysis a plane 3 acts, they the entire history of matrix structural analysis is told as a story ah in 3 acts.

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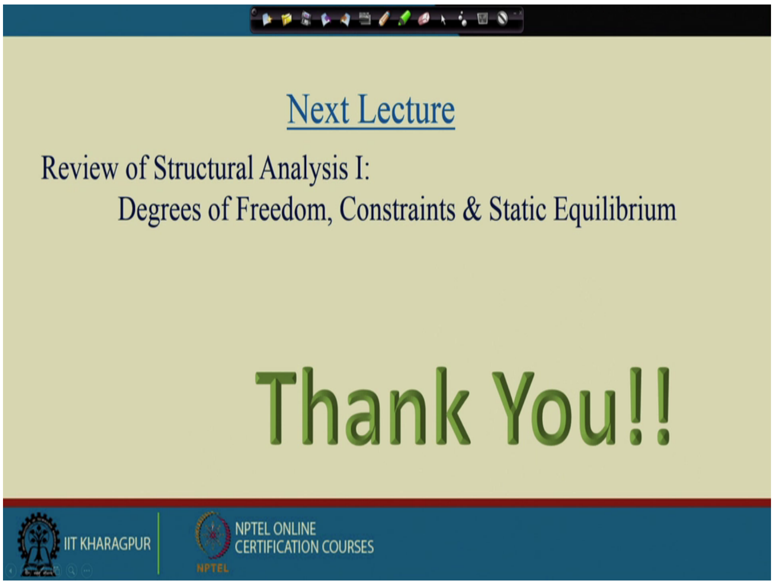

So, it is a Google it is an interesting read please go through that if you are interested, we stop here today. So, next class we will start we will be seeing some of the basic concept of structural analysis we will start with degrees of freedom constraint and static equilibrium see you in the next week.

Thank you.