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Lecture – 19 Natural Purification: Practice Problems

Welcome friends. So, as we said in the previous class that we are going to take up some Practice Problems on Natural Purification aspects or the estimation of BOD, DO, COD in the system.

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Practice Problem: Dilution				
> The sewage of a town is to be discharged into a river stream. The quantity of sewage produced per day is 9				
million liters, and its BOD is 350 mg/L. If the discharge indue river is 250 L/s and BOD is 6 mg/L, find out the				
BOD of the diluted water.				
H = 9 MD A 24 ×60×60 > 3				
Dome Boo matter				
(Re+Ar), Roog				
60,50,40,R = 250,451, $6. =$				
250 + 22 RoDa=6 mg Rio - Boyrd + Rionade				
RUD Stork RUDM = Rettle				
" 				
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So, let us discuss some of the problems; how these are solved. So, the first problem is on dilution which says that sewage of a town is to be discharged into a river stream. The quantity of sewage produced per day is 9 million liters and its BOD is 350 milligram per liter. So, we have been given the discharge of the sewage.

So, flow rate of the sewage is given as 9 million liters per day 9 MLD and the BOD of the sewage is also given as 350 milligram per liter. Now if the discharge in the river is 250 liters per second; so, we have a discharge in the river as 250 liters per second and the BOD is 6 milligram per liter; BOD of river is 6 milligram per liter, then find out the BOD of the diluted water ok.

So, you have a river where there is a sewage stream from the town is getting discharged and as we discussed the concept of dilution based on the basic mass balance principle. So, the concentration let us say this is your sewage; so flow in the sewage and BOD in the sewage this is your river. So, flow in the river is Q R and BOD in the river is let us say BOD R. So, at this mixing point post downstream the total flow in the river will be Q R plus Q S as this thing and say the BOD is; let us say BOD of mixed or ok. So, Q M is equal to Q R plus Q S and we need to determine the mixed BOD.

So, BOD of the mixed will be equal to BOD of the sewage into flow of the sewage plus BOD of the river into flow in the river divided by flow in the mixed flow; so, which is your Q R plus Q S. However, one needs to see that the units are appropriate this is milligram per liter fine, this is milligram per liter fine; this is liter per second and this is ml million liters per day. So, we need to convert this either to million liters per day or we need to convert this as liters per second right.

So, if you want to convert this to let us say million liter per day; how we are going to convert that flow in the river? So, we have 250 liters per second so; that means you multiply it with 60 we get 250 into 60 liters per minute. You further multiply it with 60 you get per hour and you further multiply it with 24; you get per day. So, this much liters per day and you divide it by 6 or you further multiply it with the 10 to the power minus 6, you get million liters per day.

So, that way we can estimate or we can convert similarly this into the liter per second. So, we have 9 into 10 to the power 6 liters per day and there is 24 hours in a day. So, we let us divide it by 24 and then we this becomes per hour, then into 60 this become per minute and further into 60 this becomes per second.

So, 9 into 10 to the power 6 divided by 24 into 60 to 60; so, either we use this and this or we use this and this ok. So, we and this is available; so we can multiply that and simply get the factor.

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Practice Problem: Dilution				
> The sewage of a town is to be discharged into a river stream. The quantity of sewage produced per day is 9				
million liters, and its BOD is 350 mg/L. If the discharge in he river is 250 L/s and BOD is 6 mg/L, find out the BOD of the diluted water.				
• Sewage Discharge $Q_s = \frac{(9 \times 10^s)}{24 \times 60 \times 60} = 104.167 l/s$				
• BOD of sewage = $C_1 = 350 mg/L_2$				
• Discharge of the river = $Q_s = 2001/s$				
• BOD of the river = $C_s = 6mg/L$				
• BOD of the diluted mixture = $C = \frac{C_s Q_s + C_s Q_s}{Q_s + Q_s} + \frac{(350 \times 104.167) + (6 \times 200)}{(104.167 + 200)} \approx \frac{123.8 \text{ lmg}/L}{104.167 + 200}$				
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So, it will be your solution is going to be like this. So, the sewage discharge if we try to convert in liter per second as we discussed this into 10 to the power 6 into 24 into 60 into 60. So, this gives 104.167; this is 350, this is already 200.

So, now these units are same and we can use this as 350 into this discharge; 6 BOD into the discharge in the river and then total discharge. So, this gives approximately 124 milligram per liter of the BOD ok. So, river purity initial BOD from 6; it can turns to 124 after this stream is; after this waste stream is discharged into the river. So, that is one simple problem of the dilution ok.

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Practice Problem: DO Deficit				
\succ A city discharges 1500 L/s of sewage into a stream whose minimum rate of flow is 6000 L/s. The				
temperature of sewage as well as water is 20°C. The DO content of sewage is zero, and that of stream is 90%				
of the saturation DO. Find the initial DO deficit. Assume instantaneous mixing of the sewage and stream.				
$DO Deficit = D = DO_{sal} - DO_{t} \qquad (R_{s}, DO_{s}^{20})^{20}$ $DO_{sal} = d 20^{\circ} c = 9.17 \qquad (R_{s}, DO_{s}^{20}, D)_{car}$				
$DO_{SM} = \frac{1}{20^{\circ}C} = 9.17$				
- 9.17 - (1) (1 × - 0.9 × 9.17				
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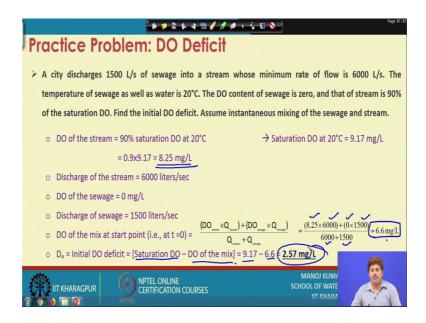
Now, let us take another problem on DO deficit ok. So, a city discharges say 1500 liters per second of sewage into a stream whose minimum rate of flow is 600 liters per second ok. So, this question you have again severe line coming in there is flow rate is given. So, of sewage flow rate of river is also given the temperature of the sewage as well as water is 20 degree. So, temperature of both is 20 degree; so if it is both 20 degree the mixture will also remain a 20 degree; the DO content of the sewage is 0.

So, the dissolved oxygen here is 0 and the stream is 90 percent of the saturation value; the DO here the Dissolved Oxygen of the river is 0.9 times of the DO saturation value ok. Assume the instantaneous mixing of the sewage in this stream and we need to find the initial DO deficit ok. So, what is the DO deficit? Now we know that DO deficit which typically we note by D is equal to DO saturation; saturation level DO minus the existing DO, DO at any given point of time T or whatsoever. So, remember we hired a table what are the different in the when we were discussing the DO as a parameter.

So, what are the different values of the dissolved oxygen at a different temperature the saturation values. So, DO saturation at 20 degree Celsius is 9.17 which is one of the pretty common temperature which is given. So, we know that DO at 20 degree DO saturation at 20 degree Celsius is 9.17 and in order to find the DO deficit we need 9.17 we know and then we need to find what is the dissolved oxygen of the mix and that way we can get the deficit.

And dissolved oxygen of the mix we can get from the dilution concept which we just earlier discussed. So, DO of river; existing DO of river is 0.9 times of 9.17 ok. So, that is the existing DO; DO R and DO of the sewage is 0. So, we have all these values and similar to the earlier problem; we can first do a dilution we can first do a mass balance for that and determine.

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So, this becomes 8.25 milligram per liter the DO of the stream. So, DO of the stream in to flow in the stream DO of the sewage in to flow in the sewage divided by flow in the sewage plus flow in the stream gives us at the DO of 6.6 milligram per liter and the initial DO deficit is saturation DO minus DO of the mix. So, that gives us 2.57 milligram per liter; so, that way simply we can actually determine what is the deficit of the dissolved oxygen ok; these are the simple problems.

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Practice Problem: BOD estimation			
\succ The ultimate BOD (L ₀) of a wastewater sample is estimated as 87% of COD. The COD of this wastewater is			
300 mg/L. Considering first order BOD reaction rate constant k (use natural log) = 0.23 per day and			
temperature coefficient θ = 1.047, what will be the BOD value (in mg/L, up to one decimal place) after three			
days of incubation at 27°C for this wastewater. (GATE Civil Engineering, 2018).			
$C_{0} = 300 \text{ mgl}$ $L_{0} = 0.8777300 =$			
Lo = 0.8770300 =			
101, 11 = 0.23 2 BODy = Lo(1-e-Kot)			
$L_{1}^{2} = \frac{1}{2} $			
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Now, let us take another practice problem on BOD estimation. So, BOD estimation is particularly from problem perspective is actually one of the topics which is often asked in several competitive exams as well. So, particularly those preparing for GATE or those kind of exams or years you are likely to see at least one question or so on this particular topic ok.

So, this is often like almost every year there will be some questions from this BOD part. Questions are simple they are not very complicated questions; so, let us see this question is from the GATE paper of the Civil Engineering of this year itself ok. The ultimate BOD of wastewater sample is estimated at 87 percent of COD, the COD of wastewater is 300 milligram per liter.

So, we have been given this COD which is 300 milligram per liter and the ultimate BOD or the L 0 is 87 percent of this. So, 0.87 into 300 will give us the L 0 value. So, straight forward the L 0 value is given; considering first order BOD reaction rate constant at natural log means at a base is 0.23. So, k D is given as 0.23 per day and a temperature coefficient is also given.

So temperature coefficient if you recall the rate of your de oxygenation or this thing are dependent on the temperature coefficient. So, if we want to determine the k D value at say a temperature T, we normally know at k D value at say any given temperature at 20

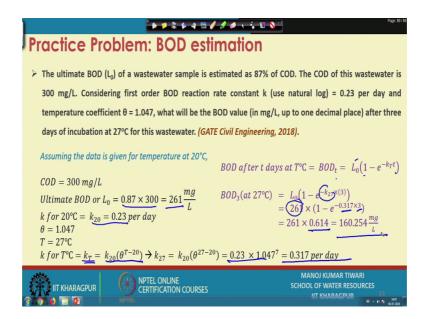
degree we know. So, it will be k D 20 times this value temperature coefficient theta to the power T minus 20 ok.

So, that way we can actually determine the k D value; add appropriate or given temperature. So, here a temperature coefficient is given and it asked what will be the BOD value in milligram per liter after 3 days of incubation at 27 degree Celsius for this wastewater. So, the time period is given to us as 3 days and incubation temperature is given as 27 degree Celsius. So, we know the L 0 value right and if the discussion that we had in the previous lecture, you know that the BOD exerted which is actually BOD at consumed or demand. So, like we said that BOD at any time t is equal to your L 0; 1 minus e to the power minus k D times t ok

So, this expression we can directly use; so for say if we want to determine after 3 days of incubation means how much; how much BOD has consumed? We are not talking about the left in the system; BOD left in the system BOD exerted that is what the 3 day BOD means. So, BOD is L 0 1 minus e to the power minus k D at 27 degree Celsius the temperature which is given to us into 3. L 0 value is can be taken from here; the k D value can be estimated at this temperature from this. So, the theta is given to us and temperature here is 27 degree Celsius ok.

So, we know the temperature, we know the value of theta and that way we can determine and T is also given to us.

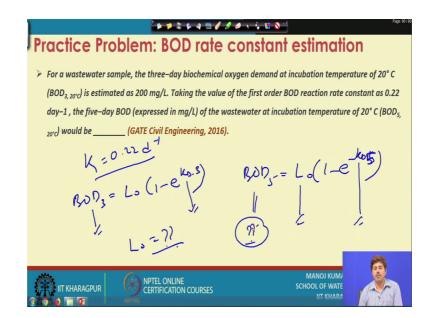
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So, we can actually get the value of the desired value of this thing. So, the ultimate BOD becomes 0.87 into 300 that is equal to 261 and k 20 is given to us, k at T degree temperature is k 20 minus theta to the power T minus 20. So, theta to the power 7 and this gives us 0.317 per day as the value of k.

So, as we said it will be BOD at time t will be L 0 1 minus e to the power minus k T into t. So, L 0 is 261 and k at 27 degree Celsius is this value which we have estimated time is 3. So, we get this number as 0.614 and the total 3 day BOD or 3 day exerted BOD is 160.254 milligram per liter. So, that way we can estimate the BOD or how the BODs they are present in the system.

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Let us take another problem on BOD rate constant estimation ok. So, how the; we can estimate the rate constant of BOD, this is a problem from GATE Civil Engineering of 2016 paper. So, it says that for a wastewater sample the 3 day BOD at incubation temperature of 20 degree Celsius is estimated at as 200 milligram per liter. Taking the value of the first order BOD reaction rate constant reaction rate constant is given to us. So, k value the k value here is given to us 0.22 per day ok. The 5 day BOD of the wastewater at incubation temperature 20 degree Celsius is to be estimated ok.

So, we have been given the value of k, we have been given the BOD rate we have been given the BOD value of 3. So, we know that BOD 3 is equal to $L \ 0 \ 1$ minus e to the power k D into 3 time 3 days right. So, for this we have been given this value and we

have been given this rate constant value. So, only unknown is $L \ 0$ and from this we can determine $L \ 0$ and then for BOD 5; we know that again it will be $L \ 0 \ 1$ minus e to the power minus k D times T and T here is 5 for BOD 5.

So, for this now we know the L 0 value from here. So, this is given to us; we know the k D value given to us, 5 is already there. So, we can determine what is going to be the BOD 5 simple problem.

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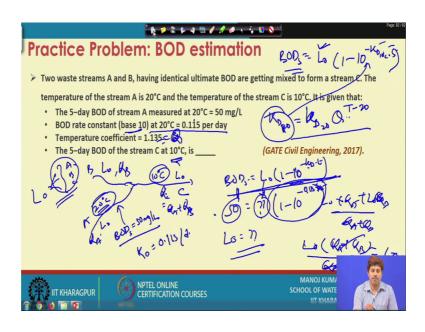
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Practice Problem: BOD estimation				
For a wastewater sample, the three-day biochemical oxygen demand at incubation temperature of 20° C (BOD _{3,20°}) is estimated as 200 mg/L. Taking the value of the first order BOD reaction rate constant as 0.22 day-1, the five-day BOD (expressed in mg/L) of the wastewater at incubation temperature of 20° C (GATE Civil Engineering, 2016). BOD after 3 days at 20°C = BOD ₃ = $200 \frac{mg}{L}$ BOD rate constant at 20°C = $BOD_3 = 200 \frac{mg}{L}$ Since, BOD after t days at T°C = $BOD_t = L_0(1 - e^{-k_T t})$ $L_0 = \frac{200}{0.483} = 414.078 \approx 414 \frac{mg}{L}$ BOD after 5 days at 20°C = $BOD_5 = L_0(1 - e^{-k_{20} \times 5}) = 414 \times (1 - e^{-0.22 \times 5}) = (276.2 mg/L)$				
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So, it will be the solution will be like this. So, at 3 days BOD is 200 milligram per liter, rate is given to us. So, BOD t is this we know; so, BOD at 3 days after 3 days is 200 and L 0 e to the power minus 3. So, this gives us the ultimate BOD as 414 milligram per liter and then BOD after 5 days at 20 degree Celsius, which is your BOD 5 is L 0 1 minus e to the power minus k 20 into 5 and this gives us the number as 276.2 milligram per liter; so that becomes the BOD 5 ok.

So, you see that BOD 3 was 200, BOD 5 was more and ultimate BOD is even higher. So, that is how the BOD estimation happens; BOD exerted if you see the oxygen demand exerted. So, initially as you see day 1, day 2, day 3, day 4, day 5 it may further increase actually that way around. So, it is lesser on day 3, higher on day 5 and ultimate BOD is eventually which is the maximum demand; maximum oxygen demand it can exert will be the highest.

So, eventually if you extend this curve for further longer and longer; it will get static around this value. So, that is what the ultimate biological oxygen demand is.

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This is another question on BOD estimation; we have 2 waste streams A and B having identical ultimate BOD and are getting mixed to form another stream C. So, there are 2 different streams; there is let us say this is stream A, this is stream B and they are mixing and forming another stream C ok. The ultimate BOD or L 0 for both the streams are same they are having identical ultimate BOD.

So, if this is same and they are getting mixed so ultimate BOD of stream C will also be same we can just apply simple mass balance also. So, let us say flow here is Q A, flow here is Q B and flow here is Q C which is equal to Q A plus Q B. So, because it is the same value L 0 into Q A plus L 0 into Q B divided by Q A plus Q B; L 0 is common from the top. So, this become Q A plus Q B divided by Q A plus Q B which gets canceled equal to L 0 ok; so, that is your the BOD over here.

The temperature of the stream A is 20 degree Celsius; temperature here is 20 degree Celsius and the temperature of steam C is 10 degree Celsius. So, here the temperature is different it gives that the 5 day BOD of stream A at 20 degree Celsius is 50 milligram per liter. So, BOD 5 here is 50 milligram per liter of stream A the BOD rate constant at base 10 is 20 at 20 degree Celsius is 0.115. So, k D at base 10 is 0.115 per day.

The temperature coefficient theta is this 0.135. So, we need to determine the 5 day BOD for stream C which is at 10 degree Celsius, this is a question from the last years GATE of the Civil Engineering ok. So, what we can do? Like if you see the total information provided in the system for stream A, we have been given the 5 day BOD and we have been given the BOD rate constant also ok.

So, we know that BOD 5 is equal to L 0 1 minus e to the power minus k D into T that is when it is exponential base; when k D is given at 10 base. So, this e becomes 10 ok; so, this application this equation will be applicable because this is given at base 10 and not at base e right; so, this becomes our BOD equation for this.

Now, here k D at the stream is at 20 degree Celsius and k D is given also at 20 degree Celsius; so, that is fine. So, we can basically use this. So, this value is 50 this value is not known to us; L 0 which we need to determine 1 minus 10 to the power this value is 0.115 per day and it is a 5 day BOD so, this value is 5 into 5. So, we know this number; we know this number only thing is L 0; so, we can determine L 0 from here and since the A and B are having identical L 0 the L 0 of the C is also same.

So, we know the L 0 value for steam C also. So, we know L 0 value for basically it is same for all 3 steam; so, steam A, B and C; we know L 0 values for all the streams now. The next thing is that steam C is at 20 degree Celsius. So, we need to determine the k D value at 20 degree Celsius; k D at 20 will be equal to sorry at 10 degree Celsius; so k D at 10 will be equal to k D at 20 degree Celsius and into theta to the power T minus 20, theta is given to us T is 10 here.

So, that way we can estimate the new value of k D and then for steam C; again we have the BOD 5 is equal to L 0, now we know the L 0 from here and 1 minus 10 to the power minus k D at 10 degree Celsius. So, we know now k D at 10 degree Celsius from here into 5 ok. So, we know all these things and we can estimate the BOD 5 value.

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Practice Problem: BOD estimation					
▹ Two waste streams A and B, having identical ultimate BOD are getting mixed to form a stream C. The					
temperature of the stream A is 20°C and the temperature of the stream C is 10°C. It is given that:					
 The 5-day BOD of stream A measured at 20°C = 50 mg/L BOD rate constant (base 10) at 20°C = 0.115 per day 					
Temperature coefficient = 1.135					
The 5-day BOD of the stream C at 10°C, is (GATE Civil Engineering, 2017).					
For stream A, BOD for 5 days at 20°C = BOD ₅ = 50 mg/L BOD rate constant = $k_{20} = 0.115$ per day (base 10) temperature coefficient(θ) = 1.135 BOD after t days at T°C = BOD ₁ = $L_0(1 - 10^{-k_T t})$ $\Rightarrow 50 = L_0(1 - 10^{-0.115 \times 5}) \Rightarrow L_0 = \frac{50}{0.734} = 68.12 \frac{mg}{L}$ (while considering base 10) Since both A and B streams have same ultimate BOD, the resulting mixture C, should also have the same ultimate BOD. For stream C, BOD rate constant for temp 10°C (k_{10}) $= k_{20} \times 1.135^{10-20} = 0.0324$ per day. 5 day BOD for stream C at 10°C $= L_0(1 - 10^{-k_{10} \times 5}) = 68.12 \times (1 - 10^{-0.0324 \times 5})$ = 21.222 mg/L					
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So, that is how we can actually solve this. So, if we see the solution for stream A; we have 5 day BOD as 50 milligram per liter given, rate constant given, theta value given. So, BOD this thing and we can get the L 0 value considering at base 10 and since the BOD are remains the same; so, C will also have the same ultimate BOD.

So for stream C; first the BOD rate constant at temperature 10 degree Celsius; so k 20 into theta which is given to us to the power 10 minus 20. So, this becomes our the 10 degree Celsius rate constant and then 5 day BOD for stream C at 10 degree Celsius will be L 0 1 minus e to the power k 10 which we have estimated here and this gives us 21.22 milligram per liter.

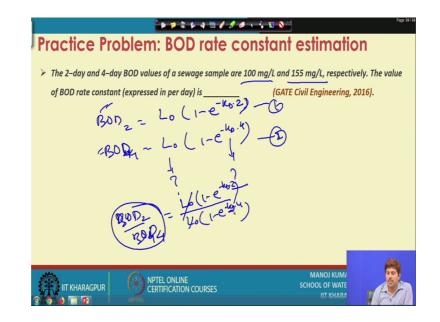
So, with the same ultimate BOD; this is interesting to see with the same ultimate BOD one of the stream which is at higher temperature is exerting an oxygen demand of 68.12. Same existing BOD means the; or the initial organic matter amount of initial ordinary matter is the same in both the streams. One is exerting a 5 day oxygen demand of 68.12 milligram per liter, while another is exerting a 5 day oxygen demand of just 21.22 milligram per liter.

The difference being that the stream C is 10 degree cooler as opposed to the stream B which as opposed to a stream A; which is at which is at actually 20 degree Celsius right. So, you can clearly see the effect of temperature over here with the lower temperature, the biological activities has reduced and the exerted demand of oxygen has reduced.

Organic matter is still the same, but exerted demand in the 5 day has actually decreased down because the rate of the decay has slowed down.

So, you see the rate of decay here is 0.115 per day while rate of decay here is 0.03. So, almost one order of magnitude lower ok; almost 5 times lower than this and that results in the reduction of the oxygen demand as well or biological oxygen demand as well ok. So, that is how it can actually be like solve. So, problems are simple if you see just one needs to understand the basic concept ok.

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And these are quite popular problems for exams; there is another problem from again GATE of 2016. There is 2 day and a 4 day BOD values are given of sample 100 milligram per liter and 155 milligram per liter and we need to determine the rate constant. So assuming that these values are at same temperature because we have to consider that the temperature is not changing; so, a 2 day BOD value means BOD exerted in 2 day BOD 2 is L 0 1 minus e to the power minus k D into 2.

And your 4 day BOD value; BOD 4 is L 0 1 minus e to the power minus k D into 4 right. So, we have been given these 2 values, but we have not been given neither have been given the ultimate BOD and nor have been given the rates ok. So, we have 2 equations equation 1 and equation 2 that way and 2 unknowns or one unknown in; one unknown is L 0 that we do not know and another unknown is the k D that we do not know and that is k D is the one that we need to determine. So, we can divide these 2 equations let us say; so if you divide these 2 equations. So, we get BOD 2 by BOD 4 or other way also we can divide BOD 4 by BOD 2 does not matter. L 0 1 minus e to the power minus k D into 2 divided by L 0 1 minus it the power minus k D into 4. L 0 gets cancelled ok; this number is known to us and now the only unknown that remains is the k D and we can determine the k D from here that way ok.

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Practice Problem: BOD rate constant estimation					
> The 2-day and 4-day BOD values of a sewage sample are 100 mg/L and 155 mg/L, respectively. The value					
of BOD rate constant (expressed in per day) is (GATE Civil I	Engineering, 2016).				
Assuming the temperature to be same and taking the BOD rate constant as k,					
BOD for 2 days = $BOD_2 = L_0(1 - e^{-2k})$					
BOD for 4 days = $BOD_4 = L_0(1 - e^{-4k})$ Dividing the above two equations,					
$\frac{BOD_2}{BOD_4} = \frac{L_0(1 - e^{-2k})}{L_0(1 - e^{-4k})} \xrightarrow{(100)} 100} 100 + \frac{1 - e^{-2k}}{100}$					
Solving above, $\underline{k} = 0.298 \approx 0.3 per \underline{day}$					
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So, it is a pretty simple problem that way one can see that. So, BOD for 2 day, BOD for 4 day and then if we take a ratio we get 100 by 155 as this number and we can easily solve this then and get a value of k which is coming out to be close to 0.3 per day.

So, that is how we can estimate the rate constant. So, these are some of the problems and we will conclude this discussion here and in the next class we will start so; so far what we have discussed actually and in this week was practically on to the natural systems.

So, when waste is introduced in the nature what happens how it happens, but now we will move towards the engineered systems. So, what happens when we try or what how we can conceptualize some engineering device for the treatment purpose. So, we will have those kind of discussions from the next class onwards.

Thank you.