Theory of Elasticity
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Lecture – 07
Concept of Stress and Strain

Hello everyone. This is the second week of the course. My name is Amit Shaw and it was mentioned in the introductory course, introductory class. I will be sharing this some of the lectures along with Professor Vishwanath Banerjee. So, this week, we will be discussing Concept of Stress and Strain. You see, some of you may be studying civil engineering, some of aero space engineering, some of from mechanical engineering.

So, you all are studying different engineering disciplines. As you go higher, you will see the gaps between different engineer discipline they reduce. And anything that comes out from industry they are essentially the outcome of handshaking of different engineering disciplines. It is not just for one engineering discipline, it is it is the concept of different engineering disciplines put together and then we get a product from an industry.

Now, therefore, there are few places where all these different discipline engineering disciplines they share a common trait and the subject of this course as it was mentioned, in the first lecture, subject of this course is one such thread. You see, there is a saying that nature is the best engineer, right. And we all we all try to understand nature, we all try to mimic nature.

Nature is governed by certain rules and we try to understand those rules. But, just understanding those rules is not enough. We need to translate that understanding into a form which can be used to develop a product; different kinds of product eventually for the development of mankind. Now, if I have to give you some examples of some of the products.

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These are some products; you see; these are some representative product. While selecting them, I try to keep in mind that different scales can be cover different the application domain can be covered. So, this product could be as large as say bridge building or as small as say, nano robot. This, this product could be different kinds of vehicles, then mother board of a computer, then the jackets, then a switchboard that you can see. You know in your house the optical fibers cd, the space pacemaker that we put in human body and they and even the sim cards.

So, all these product that you can see now on your screen, on the screen of your monitor, you can all relate to those products ok. In fact, there are many such things. Now, you see they are all designed to serve certain purposes and their purposes are different ok. And in order to in order to design these different products, we need different skills.

But, one of the major difference in different, all these products are, if you look at them, they all are made of different materials. For instance, the material that again I use for say, bridge, the same material cannot be used for nano robot or the materials used for bridge cannot be used for switch board or the material used for switch board cannot be used for bulletproof jackets or the same way the material used for bulletproof jackets cannot be used for the optical fiber.

So, every products has their own purpose to serve and depending on that purpose, these products are made of different materials, ok. Now, and these thing we all know that, now when we design anything, anything, when we design anything, one of the very crucial step in the design is selection of this material. And, that material selection has to be based on the purpose of the final design and the environment; that is, the final design the product is subjected to.

In some cases, the environment is extreme. Some cases, a environment is not extreme. For instance, if an airplane is flying, then the outside temperature could be minus 40 to minus 50 degrees Celsius. Whereas, if you take a same bridge, the outside temperature is not that extreme, it is just an one example. So, different products are subject to different kinds of environment. So, we select a material which can sustain that environment which can sustain that threat that may come onto the product.

Therefore, understanding of material the behavior of material when it is subjected to a certain kind of threat, it is a very important step.

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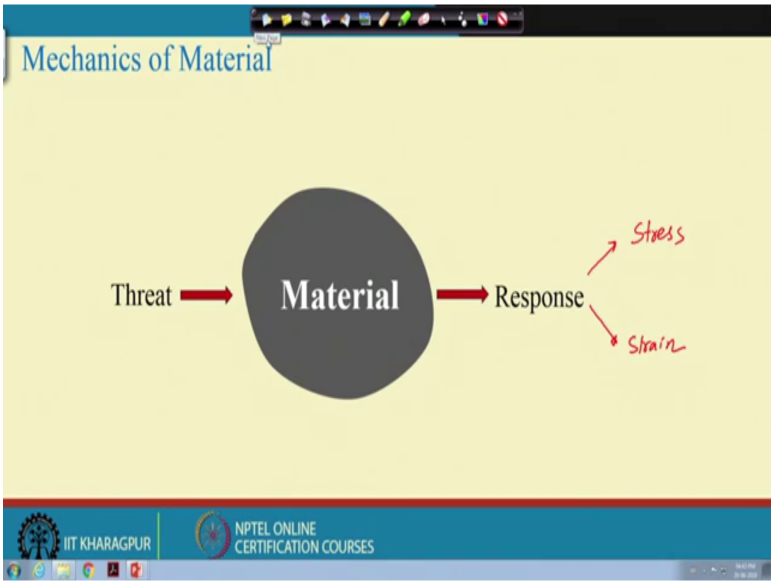

And, so, what is we have a material and which is subjected to some kind of threat and then what is the response of that material and that threat could be different. That depends, that is very subjective, that depends on the which is the purpose, for what purpose we are we are using this material.

Whether, and under what circumstances under what environment this final the product will be exposed to. So, depending on that, the threat has to be has to be has to be considered, ok. So, now, but overall the important thing is, the response of the understanding of the response of the material when it is subjected to certain kind of threat. And that is the crucial point, that understanding is the crucial point in any design, ok.

Now, this course is essentially a first step towards that understanding. Now, this response, this entire theme, there are 3 important part in that understanding.

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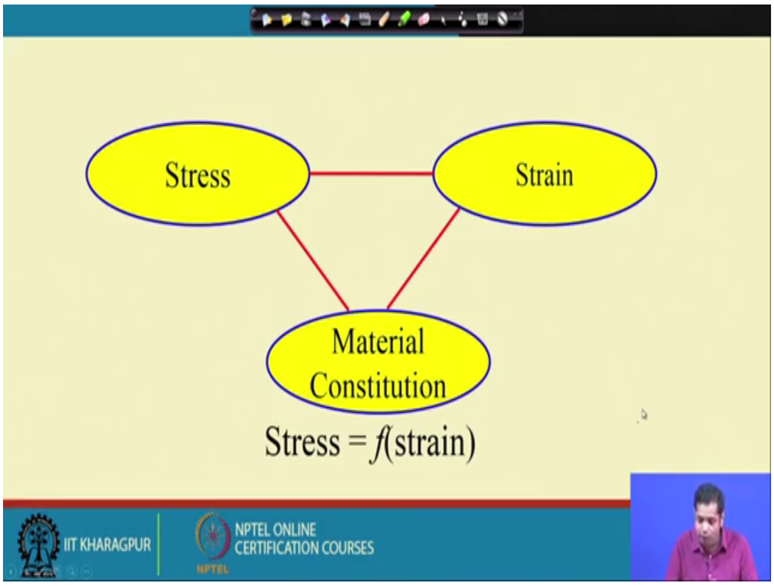

When we talk about to the understanding of material behavior, it has 3 important part, one is stress. Let us not bother about what is stress, what is strain. Now, we will come we will shortly come to that point one is stress, one is strain and then, how this stress and strain related to each other.

Stress is a function of strain which is called material constitution, ok. So, this is 3 important part in material characterization; material characterization is the understanding, how the material behaves when it is subjected to different loads or differing environment. Now, what we will be doing in this week is, we will try to understand what is stress and strain.

How they are defined, how they are measured, what are the physicality associated with stress and strain and the next 2 weeks, week 3 and week 4, we will see what is the relation between stress and strain. And that relation is not, if that relation is not same for difference for all material that depending on the material that relation may change.

So, in the next week, we will see, we will consider different kinds of material. We will idealize those material and then see what would be the relation between stress and strain for different idealized different idealization. So, let us start with stress today. Now, you see again stress is the term. It is not particularly associated with theory of elasticity.

It is a very general term and we all use this term very often in it in a different perspective. For instance, we often say that, do not take any stress, otherwise your blood pressure will increase, you see. So, stress is what? Suppose, you have a workload in office, you have you have exam around the corner, you have you were moving from one place to other, do not know the place, do not know the people, then you were stressed.

So, stress is essentially, it is response of human body. We are trying to define stress from a different context we will see what is their interpretation in the context of a material characterization in the context of elasticity. But, let us understand the word stress, what is the general use of stress and believe me, the general use of stress has the same meaning when we come to the come to the interpretation of that in the context of elasticity. It is exactly the similar thing.

Now, stress is what it is the body's way of responding to any kind of threat and that threat could be anything and we all have experienced stress in some point of some point of time.

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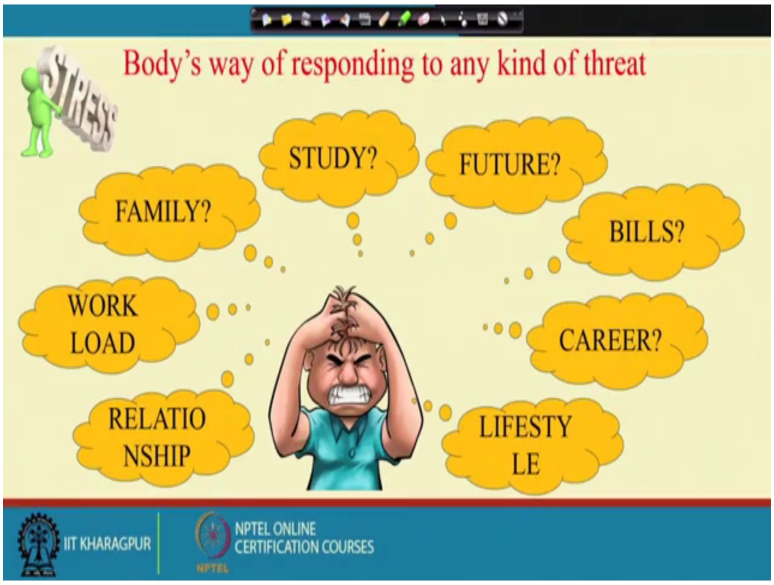

Now, let us entirely call all these different kinds of a, kinds of different causes, different reason for stress.

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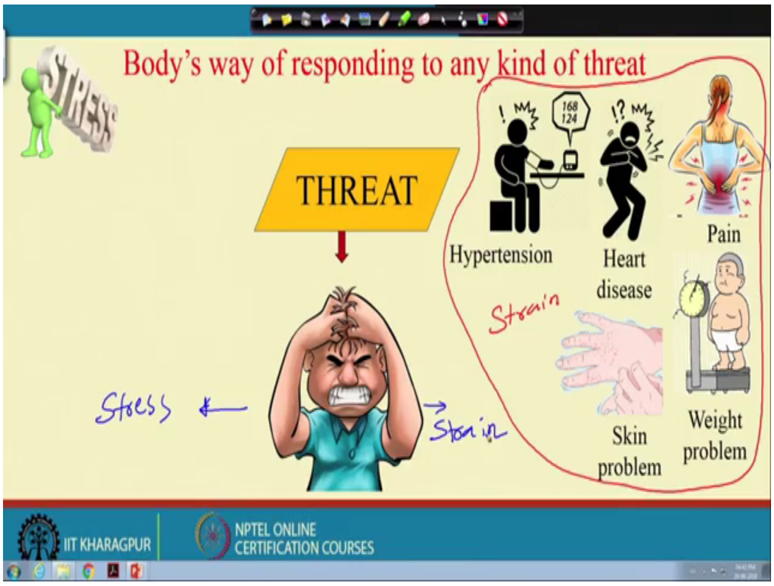

We can put together and let us call this threat, ok. So, when we say threat, it is very subjective. It depends on the what context we are talking.

Now, you see stress is something that, we cannot visualize. It is an experience, right. If I am stressed, I can experience that. But, someone, if someone looking at me cannot experience that stress that cannot visualize that stress. So, it is something which is which is experience, but it is not perceived by someone else, but there is. So, stress is, stress is a response of human body. But, it cannot be visualized by someone and this is an experience. But there is another response of the same human body which can be visualized.

For instance, if you are constantly, if someone is under constant threat, constant load and constant stress, then the health may get deteriorated the blood pressure may increase, sugar level may increase the weight, the person may experience weight problem. So, these are some consequences of that threat acting on the human body. But, that response can be measured. That can response can be visualized is not it.

So, these are the some response. This is just an example. These are the some responses which are which are visualized which can be which can be perceived unlike stress. So, let us call all together these entire thing, entire thing is strain, ok. So, when an human body is subjected to some kind of threat then the as a response, we have stress. And then, another response is strain right.

So, now and you, we all know, this we all know this. Now, now what we do is, not only just the human body, if a material is subjected to the some kind of threat, then the material is also experience stress. And, but against as I said, it is just an example. The threat is different for material, but again if the material is subjected to a different environment which then the material also experience stress.

But, again in the same way, that stress is something that material experience. It cannot be visualized, but there is another response which is strain, which can be visualized. For instance, take one object say it is a paper cup, ok. Now, if I apply some load like this, load like this, so, this paper come experience a stress, but we cannot see that. But, what we see is, as a consequence of this load, as a response to that load, this paper cup undergoes severe deformation and this deformation is related to stress, related to strength.

Now, so, what we have, what we have here is, of. So, again, we come to the mechanics of material which is essentially the understanding the response of material. Now, this response has 2 part; one is stress, one is strain, right. And then, we will now understand what is stress and what is strain and how they define, how they measure, ok.

Now, come to the, once we have understand, once we have understood that the stress and strain are 2 responses, 2 different kinds of responses of a material, now let us define what is stress. Now, before coming to the definition of stress, let us go back to the look, go back to your school days, this is the first time even in the context of mechanics. This is not the first time that you are you are encountering the term stress. In your school days also you use the term stress and remember, suppose if I have if I have a bar, longitudinal bar and then which is subjected to some kind of load P, like this P.

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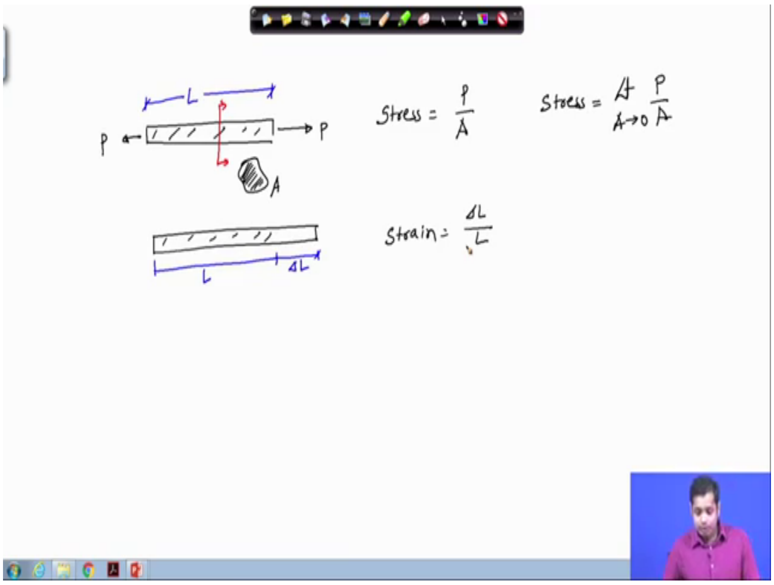

Suppose, this longitudinal bar as a cross section, this is the cross section of the bar. This cross section is suppose A. So, recall we define stress as stress is equal to P by A or more fine definition is, stress is equal to limit A tends to 0 P by A, ok; when force by area. That is how we define stress, ok. Now, how do we how did it? Now, whether it is the tensile, the load is tension or load is compression depending on that the stress can be coined as the tensile stress or compressive stress.

Now, these when you apply this load, this will cause change in length of this rod. Now, supposing after suppose that the initial length of the rod is initial length of the load is L and then after deformation, if the rod becomes like this, the law length may increase, length may decrease, depending on whether you whether it is compressive load or tensile load, it is delta L.

Then, recall. We define strain as strain as delta L by L right. So, this is the definition we used in our school days and we all are familiar with definition. Now, there is one what we have to do now is, we have to move beyond this definition. These there is nothing wrong in this definition. This definition absolutely fine definition and we will be using this similar definition.

But now, when we say the stress is equal to force by area, then the question naturally comes which area and which force or the strain is equal to change in length by original length. Then, the naturally the question comes is which length change, which change in which direction divided by length in which direction. So, depending on that, we have to move beyond this definition. For instance, let us see, let us take an example.

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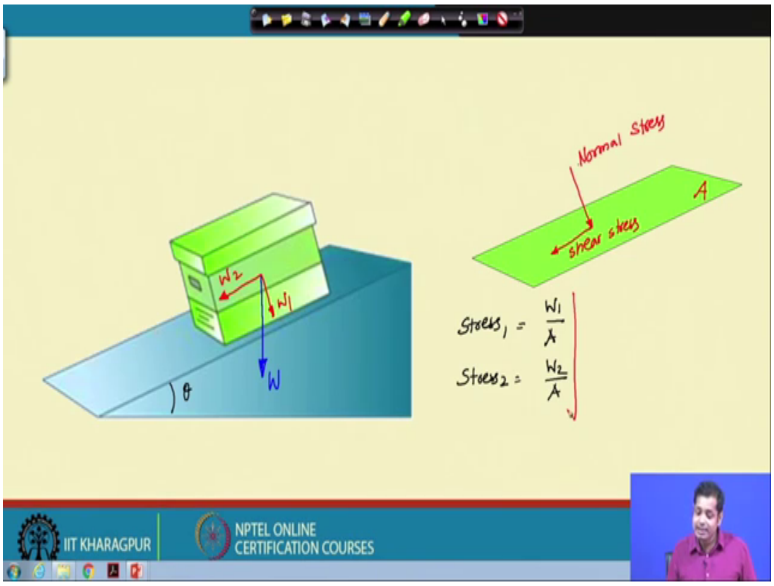

Suppose, we have an object suppose, this angle is theta ok. Now, suppose the load of this is acting downward like this and this is W. Now, so what exactly we are doing is, we will be discussing the stress and strain in detail. We are just trying to save the platform today being this, since this is the first lecture on stress and strain.

And then, from the next lecture, we will see the different stresses, how they are represented and how they are measured and everything. So, suppose the load is acting downward, so, naturally we have a 2 components of this load; one is the one is this tangential direction and one is normal direction, right. So, say it is W 1 and this is W 2 ok. Now, suppose this area is this, this area is suppose A, area is A.

Now, you see, on this area, we have to just now we discussed the force stress is equal to force by area. Now, we have on the same area, we have 2 forces; one force is normal to this area and another force is tangential to this area. So, if I stick to that definition that space is equal to force by area, then we have 2 forces here, 2 stresses here. Then, then stress 1 would be stress 1 is equal to W 1 by A and then stress 2 is equal to W 2 by A.

One thing, please note I am not yet writing any symbol for any notation for stress, stress 1 and stress 1 we are writing will formally introduce what are the notations are. Now, how it is to be written in subsequent classes. Now, we have 2 stresses. So, one stress is one stresses, this is one stress, this is one stress which is acting which is normal to this and another stresses which is acting to this.

Now, these stress is called normal stress, normal stress and this stress is called shear stress. So, the stress is now is just not 1 quantity it has in this context. At least, we can see it has 2 component one is acting normal to this plane and one is acting shear to this plane. Now, you see definition is again still same, the force per unit area. But, now since we have 2 forces of the same area, we have 2 stresses.

Similarly, we will not discussed, right. Now, when we when we come to the non-linear elasticity, then we will see that we may we may have different definition of area. Therefore, depending on which area we are talking about, we can have again different measure of stresses ok. Let us do it in 3 dimension; suppose, in the same problem, suppose this is first, let us fix the.

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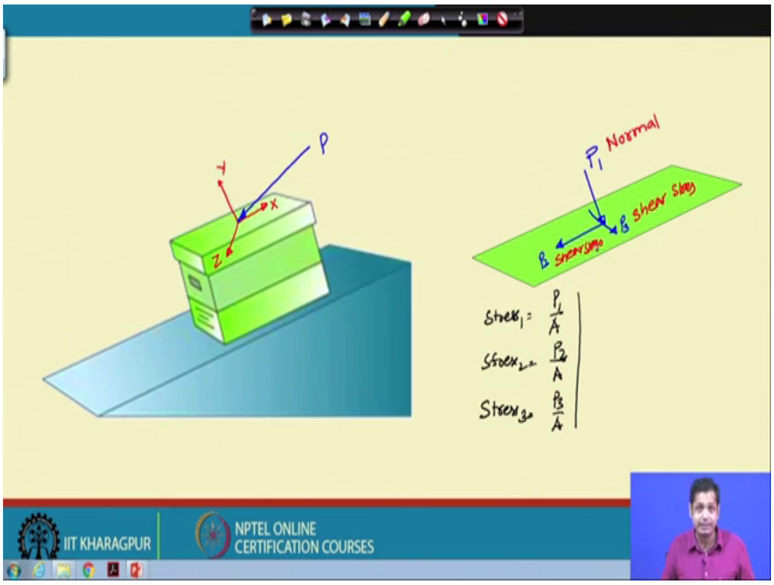

Suppose on this plane, suppose this is the let us fix the coordinate axis first this is this is and this is this is the coordinate axis say this is x y and z. Now, on this coordinate axis, suppose we have a load which is like this so P. Now, this P makes an angle with all this axis x is x y and z and if it is then, P will be having 3 components.

What are these 3 components? One component will be normal to this. Suppose, this is P 1, another component will be tangential to this in this direction which is say P 2 and another component will be tangential to this which is in this direction which is say P 3, then we have 3 stresses on the same plane, then stress 1 will be stress 1 will be P 1 by A.

And similarly, stress 2 will be P 2 by A and stress 3 will be P 3 by A P 3 by A. So, we have 3 stresses stress still force per unit area. But, now we have 3 component of stresses; 2 stresses are this is normal stress because, it is acting normal to the normal to the surface and this is shear stress this is and this is shear stress, right.

Now, once we have understood, once we have understood that we have to move beyond that definition because, now we may not be dealing with just one force or one area. We can have different kinds of forces acting on the same area. Or this or for the time being let us consider the same area different kinds of forces acting on the same. And therefore, we can have different measure of stresses, different components of stresses. Instead of measure, we say different component of stresses.

And these are the components of stresses. Normal stress is one component shear stress is another component. Now, the next is so, therefore, now next is when we see what is the stress in a body.

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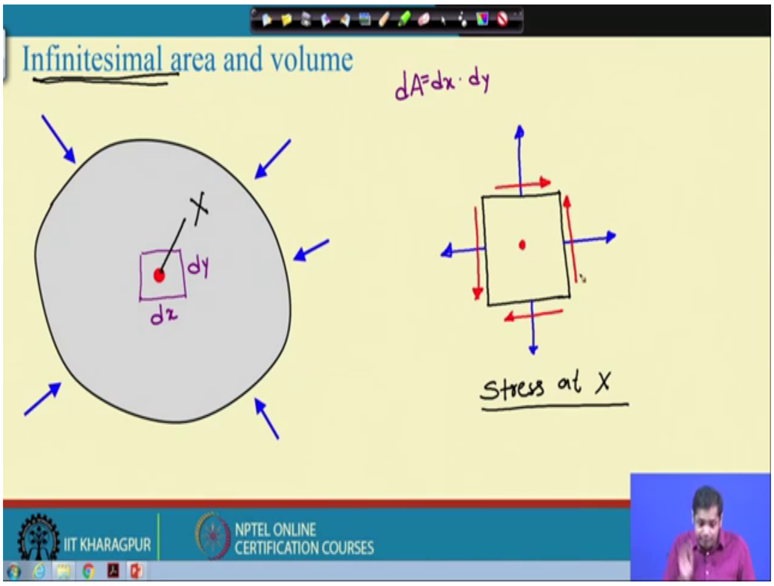

For instance, suppose, we have now we want to introduce a very important term called infinitesimal area and volume. Now, suppose take any object like this which is subjected to some external threat. In this case, the threat is some external loading.

Now, because of this threat, this object, this material will experience some stress ok. Now, when we say that, if I ask what is the stress in this material, what rather what is the stress in this body or in this object, this question is, wait this question is not a in this, question is not properly posed.

Because, stress is a point wise description; means if we take several points in the body, if we take several points like this in the body, in the same object, then the stress in at different points are not same. They may be different. So, if they are different, then we cannot ask. We cannot say what is the stress in the body, rather we have to ask what is the stress at a given point in the body.

So, stress is a point wise description. When we say what is the stress implicitly, it means what is the stress at a given point ok. Now, just now we discuss the stress is equal to force per unit area and then we are now we are now, we are now talking that stress is a point wise description and the point does not have any dimension. Then, the question comes on the other hand just now in the previous slides we see that, we took a plane and on this plane, we will show there are 2 components of 2 component of stresses or 3 component of stresses; one normal to shear stresses.

Now, if we if the stress is a point wise description, then how can we have the shear stress, normal stress at a point? Because, point has no dimension, right. How can we have a plane at a point where the point does not have any dimension, right. Now, then what we do? There we introduce a term infinitesimal area and volume. What it means is, the point is not exactly a point. The point is idealized as an area or as volume an infinitesimal volume. An infinitesimal area means, it is a very, very small area, very, very small volume which is the limit strengths actually a point.

For instance, now if we have a point here, take a point like this. Then, around this point, we take at a small area, small area if it is 2 dimension ok. Now, suppose this is dx [noise,] this is dx and this is dy and then this area is dA and da is equal to dx dy and dy is dy is very small. Now, once we have idealized this, we idealize this point as this infinitesimal area.

Then, let us let us draw this area as this represent this point as this as this. Now, remember this is just I am showing this is for the representation. This is as small as possible, this is infinitesimal these term is very important. Now, once we have this is the point, this is the point here, once we have this, then you see we have a plane here. We have a plane here, you have a plane here, you have a plane here, 2 dimension.

So, we can have stresses on this plane and what are the stresses on this plane? Now, on this plane, we can have a normal stress. Just now, we discuss a normal stress on this plane. We can have a normal stress on this plane, we can have a normal stress again on this plane, we can have a normal says all the 4 planes, we can have normal stress. Similarly, we can have shear stress on all this plane as well. We can have shear stress on this plane, shear stress on this plane, we can have shear stress on this plane, we can have shear stress on this plane and then we can have shear stress on this plane, ok.

You, if you are thinking that why these shear stresses are showing in these direction here, it is upward here, it is downward, then here this here this you bear with us. We will discuss this why they are how these the presentation has to be made and what are the sign convention and what are the coordinates we will fix that in the next class. But here, the point is, then we have to we have one shear stress and one normal stress on each plane.

How this normal stress and how this shear stress to be represented, what are the sign of this normal and shear stress, that we will discuss in the subsequent classes. Now, so, this is called the entire thing. When you take this point, you scoop out this thing here and represent this test. So, if this point is P, let us all any point if this point is say if this point is this point is X, then this is the stress at X, the presentation of stress at X.

So, how many components of stresses we have? We have on each plane we have one normal component and one shear component. Total how many components? We will see that ok, in the next class.

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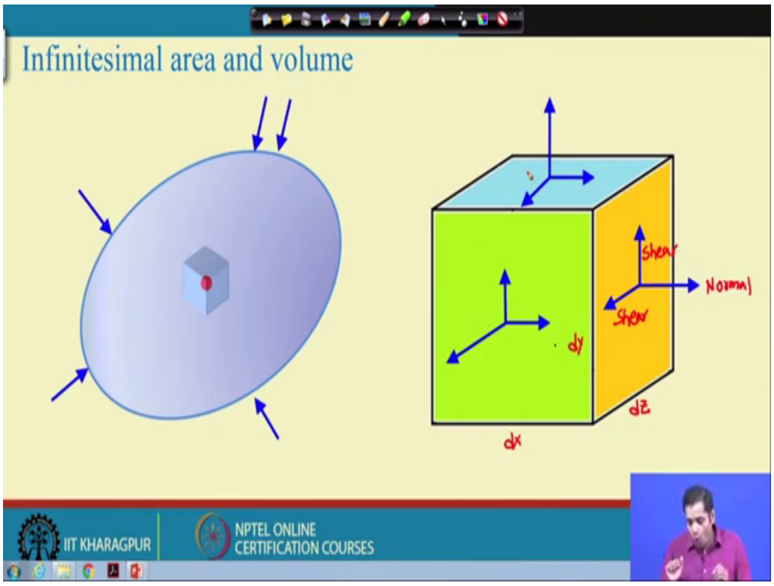

Now, similarly if we have if we take a 3 dimensional object, a 3 dimensional object which is subjected to this kind of load and then we take a point, now a around this point we take we consider a volume and then take the volume like this. And this is essentially then it will be dx. It will be dx, it will be dy and it will it will be dz ok. And this volume is very small, the infinitesimal volume and then we have normal stress. This is normal stress.

Since now, it is 3 d plane. So, we can have just you go back to the previous example when we have a load on 3 D direction, then we have 2 components, then we have a 1 normal stress and the 2 shear stress. These are all shear stress, this is shear stress. So, everywhere, all these planes, all these 6 planes we can have normal stresses and we can have shear stresses.

Now, so, what we have understood? What we have discussed today is that, stress material characterization or rather the understanding the material behavior when it is subjected to some kind of load or threat, it is an important because, it is a very crucial step required while selecting a material for a certain purpose. Then, we also understand we also discussed that, when we talk about material characterization, there are 3 important aspects.

One is stress, one strain and then how these 2 stress and strain are related to each other which is called material constitution. And then, we discussed stress and strain are essentially the response of the material, 2 different response of the material whether they are subjected to load.

Now, stress is something which cannot be visualized. Unlike strain, stress is an experience like the human body experiences stress it is the same way material experiences stress. But on the other hand, strain can be visualized, it can be perceived; it can be measured. Now, then we discussed the stress or strain is a point wise description and the since it is a point wise description and then also this stress is equal to force per unit area. To be consistent with these 2 definition we introduced infinitesimal area and infinitesimal volume concept.

And then, discuss how if we take an infinitesimal area and how the stress at a point can be represented, what are the different components of stresses in 2 dimension and in 3 dimension. We will stop here today. But, before that, next class what we do is, next class will start with fixing our sign convention of stresses, fixing our coordinate axis and then, see just here while representing these places in a graphical form, I showed shear stress in different directions normal stress in different directions.

Why they are shown like this? Will be discuss that as well in the next class, then. So, next class what we do is, next class we will discuss more on stress and of course, strain and then, we will see a different way how the different way the stress and strain can be representative; particularly in a form of tenser.

Thank you. See you in the next class.