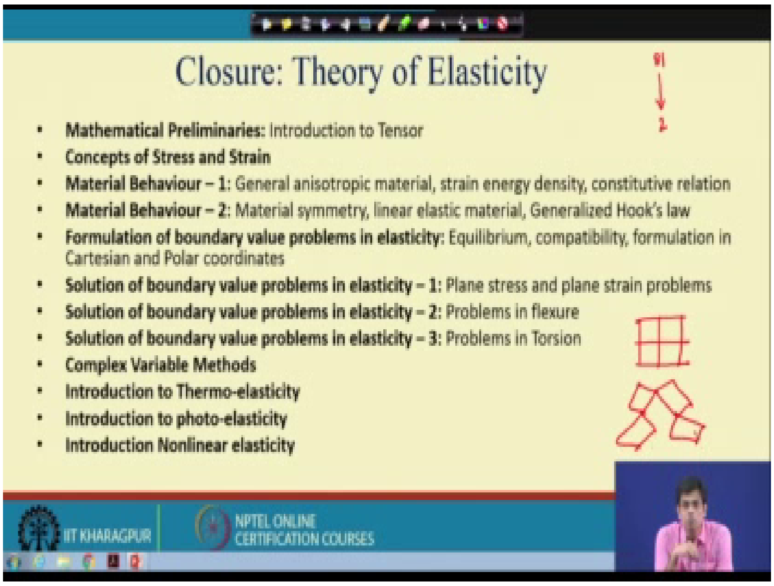
Theory of Elasticity
Prof. Biswanath Banerjee
Department of Civil Engineering
Indian Institute of Technology, Kharagpur

Lecture - 60
Closure

Welcome, so we have come to the last lecture of this course.

(Refer Slide Time: 00:26)



So, we basically what we have studied we started with a basic definition of what is elastic. So, we try to discuss that; what is the difference between plastic or the material behaviour which is permanently permanent deformations happens in the material or try to distinguish between the plasticity and elasticity. That means, there we have discussed one important thing that elasticity is not stress strain is linear essentially. The elasticity is that it the path dependence is also important.

So that means, the loading and unloading curve remains same it cannot be different in case of an elasticity even though it comes to the original position the loading and unloading. That means, in a other sense the energy dissipation is not occurring so the system energy remains constant so that is one of the important aspect of the elasticity. And when you go beyond that state then you really lose some energy or you add some energy into the system and the deformation goes to a different state which is plastic which we are not doing here.

Now, then we have introduced the concept of tensor; the tensor is actually we all know in terms of matrices. Now the we know for instance stresses, we know strains, we know displacements, displacement is a vector, stress and strain is a tensor. Why it is a second order tensor? Because it has two direction so third order tensor will have third three direction, fourth order tensor will have fourth four direction. So, in this same concept you have introduced scalar as zeroth orders tensor. So, then we have just dealt with some tensor operations; which is inner product and then some calculus of tensor calculus.

For instance there get out a riveting or freshly rivet you have introduced that or specifically the directional derivative and then divergence called and all those things we have introduced. So, this gives you the fundamental or the basic grammar of the course. And then we introduced first concept of stress and then strain in theory of elasticity; the stress and strain. Now, once we are equipped with stress and strain, then we go for the material behaviour.

Then a material behaviour we essentially defined from the very basic isotropic material to the general anisotropic material. And then to define that we need to define strain energy, which is an important aspects of the material behaviour by which we actually try to define the different anisotropic behaviour of the different anisotropic material, for instance; the strain energy will be rotationally invariant so these things we have defined.

Now in specifically in the material behaviour-2; we discuss the material symmetry and we defined we have learn what is orthotropic material, what is transverse isotropic material, what is cubic material, all those things we have learned. And how we can generalize these for instance from the 81 such material constant high, how we can come to a 2 material constant for the isotropic material. So, this was the basic discussion for the material behaviour-2.

Now then we start giving you the formulation for boundary value problems in elasticity. We discussed the governing equation, there we have also specifically discussed; what is strain stress based formulation and displacement based formulation. In the stress based formulation essentially we have derived the bi harmonic equation, which is with the help of compatibility equation. So, compatibility equation again we have given you a notion of what is physically what it means what it mathematically means.

So, when you have a stress when you have a strain given and if you want to find out displacement so that problem is an over determined problem. So, there is a constraint needed and that constant is essentially compatibility equation. That is a mathematical definition and what is the physical definition of compatibility. Physical definition of a compatibility; that means, there is no the deformation is essentially, if you remember this picture for instance the body, which is for instance this was here so the body cannot deform arbitrarily like this so this cannot happen for instance so this gives us the physical meaning of compatibility.

So, all those things we have discussed and we have also discussed in the stress based formulation that really; in case of solving the bi harmonic equation we do not need the constitutive equation to solve the by army equation. There we have applied the stress function approach, the stress function approach, we have seen that satisfies the compatibility also we constructed as a stress function in a different way. And then we try to use these stress functions for a different class of problem, for instance the bending problem, torsion problem, and then the plane stress plane strain problem, in the subsequent model.

And then we also discussed the displacement based formulations, where essentially we write the differential equation in terms of displacement and we try to find out the displacement u v or w in case of three d. But in case of a stress based formulation, we find out stresses which is sigma x x, sigma x y and sigma y y. Then finally, once you find out the stresses and then with the stress we need to substitute the constitutive equation which is relation between stress and strain.

And then that essentially gives you the strain, and then from that strain, we need to we go for the displacement calculations, so that is a basic stress based approach. Now in case of a displacement based approach the procedure is totally reverse, you write the differential equation in terms of displacement and from that displacement essentially you find out the strains and then stresses. So, that is the more general approach displacement formulation is more general compared to stress based approach because the construction of stress function is very tedious for a complicated problem.

So, now, then we have discussed the complex variable method. So, there are some problems which we can really solve in a exactly for a simple geometry and simple boundary condition; there we can use this type of method to solve the problem. Even the stress function approach is very much restricted to the very simple geometry and simple boundary condition. Then there are two topics that we have discussed very briefly; one of them was thermo elasticity and another one is photo elasticity.

So, photo elasticity is an experimental technique which is still valid and very relevant today; in terms of its use and the term elasticity is that the primary part of the thermo elasticity you have learned in your strength of material course, where essentially if you hit a bar then how its deformation will happen, so if there is a restrain in restrainment in the bar so, that we have discussed in the context of Damon Newman constitutive law.

Then we have also discussed the one dimensional heat equation, where and then from the energy consideration or with the use of second law of thermodynamics we have found out that how to, how this thermo elastic deformation happens. Then if this thermo elastic if there certain conditions are there, then these thermo elastic deformations can be uncoupled that means, the heat equation and the new Hefs equation of elasticity can be solved separately. Heat equation gives us the temperature distribution within the body and then we use the temperature this temperature distribution of the body it to solve the to find out the displacement or the deformation of the body with the help of Naviers equation of elasticity.

Now, this heat equation and the Naviers equation can be coupled and that is we have explicitly given, what term we need to we are neglecting and we why we are neglecting that. So, that was the basic issues in the thermo elasticity and then also we have solved some problems which is very much simply simple in nature. Because our approach in this course was analytical mainly, there is no numerical aspect of it.

And I have also discussed means that most of the problems that we encountered in real life cannot be solved analytically, because of the complication in the geometry, complication in the boundary condition, and complication in the load also. So, this most of the systems that we will encounter in the physical world is need to be solved by approximate method. And one of the such approx method is finite element method or mesh free method or any finite difference method or any other method approximate method. So, even semi analytical method for instance Fourier series approach and all those things, differential quadrature method those methods are also useful to solve some problems.

But most general way of approaching these problems for a very complex thing is the finite element and mesh free methods are the most popular. Now, the photo elasticity thing we have first we have discussed one important aspect is the birefringence, what is or the temporary a change in the material. When you load a material some photo elastic material it shows anisotropy behaviour in the refractive index; which was earlier before loading was isotropic.

That means, when you and the light passes through that material the refractive index remains same in different direction is same. That means, the material refractive index is isotropic essentially, when you load that material, then the refractive index in different direction becomes different. So, this is temporally change or temporarily doubly reflection property or these properties essentially the birefringence. So, this property actually helps us to find out the stress contours within the material given a loading.

So, this actually we calculate it and with the help of stress optics law we calculate the stresses and essentially we calculate the difference in stresses. And from this difference in stresses if we assume purely elastic material; then we can calculate the strains and then we can calculate the displacement from that. So, that is a very oldest approach of experimentally observing the stress pattern within the body. So, this approach actually cannot be generalized for all material because all materials are not photo elastic material.

So, what we do generally for a complicated structure? We make a model and then studying its stress strain behaviour. So it is a scaled version with a different material. Now this can be alleviated with the help of birefringent coating. So, these some aspects of it you coat for instance in the birefringent material by which we can get the surface strain or surface stresses of the body. So, that is a part of that is even though that is not in the full field we cannot get, we can get only the surface behaviour of the body.

So, we also discussed the some as very little aspects of digital image correlation there; where you compare two different image to get the stress to get the displacement and finally, the strengths so that was the whole course. Then in the last lecture we have also discussed some aspects of the non-linear elasticity or basically what is the difference between the linear and non-linear elasticity; so which can go non-linear which cannot go linear.

So, what we have seen is that the differential equation remains same and the basic difference is that deform and undeformed geometry cannot be remain same. Or it is the difference is distinguishable so we really cannot neglect the undeformed geometry here or we cannot say that undeformed and deformed geometry is almost same so; that means, the displacement radiant is small. So, that assumption when we violate that we get the geometric non-linearity. And when the stress strain relation for instance, the stress changes how a strain changes in this relation is becomes non-linear then we get the material non-linearity in the body.

So, so these are the basic difference of non-linear elasticity and non-linear elasticity itself is a separate course. So, we had no plan to discuss non-linear elasticity here in this course and naturally with the difference in the configurations of reference and difference undeformed or the deformed and undeformed configuration, we essentially get different measures of strain which is not there in the linear elasticity. For instance; Piola Kirchhoff stress and the Cauchy stress and the different other stress measures are also there for instance Weired stress.

So, these gives you different notion of the a differential equation we can write it in deformed configuration as well as on different configuration So, most of the quantities that we defined in the non-linear elasticity in the undeformed configuration does not have meaning, but mathematically we can define it and there are some had advantage on it. For instance the formulations can be based on the updated lagrangian, lagrangian, or (Refer Time: 16:04) in formulations where we for instance we when the difference geometry if we always referred or we find out the solution based on our reference geometry we go for lagrangian formulation.

And then if we go for the updated lagrangian formulation, once you get the one load step and then you update your geometry and that becomes your reference geometry then we go for the updated lagrangian formulation. Already in formulation is also very important for fluid mechanics, where the fluid the constant from volume formulation there we fix this domain and particle moves on so that is already in formulation.

There is another formulation known as co rotational formulation so which is body attached coordinate system so, these things actually are very different from what we have learned in the linear lessons. So, the next step for this after this course is a course on non-linear elasticity and continuum mechanics, which can be clubbed together. So, the and not only that the solution procedure, which is for the non-linear elasticity is entirely different compared to the linear elasticity. Because in a linear elasticity your governing equation incomes linear so what you after discretization you get a linear system of equation which can be solved in one shot.

So, which is just the matrix vector solution through the Gaussian elimination or any other solve solution methodology. But in case of a non-linear elasticity no differential equation becomes non-linear and so your discrete is finalized final discretized version of the governing equation becomes non-linear equations. So, there the solution strategy is necessary if for instance the Newton reaction method by which the successive linearization we can do and we can finally, solve the system for a incremental way. So, that is one of the major issue in the non-linear elasticity.

So, non-linear elasticity has two aspects; one is the theoretical aspects another is the solution aspect because the solution is nationally not same as the in case of linear elasticity. So, and after this we can go for plasticity solution of plasticity a plastic material, what will happen if the material does not become elastic. So, then we can go for the higher end mechanics or advanced mechanics for instance in the geometric mechanics which is more mathematical in nature.

So, the next course I would suggest is the continuum mechanics or the non-linear elasticity; where we basically depart from the linear regime to a non-linear in regime. So, with this I thank you all, and we stop here for theory of elasticity.

Thank you.