Theory of Elasticity

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Lecture - 58
Photo - Elasticity (Contd.)

Welcome. So, this is the last lecture of module 11, where we are discussing the Photo Elasticity. So, photo elasticity is essentially a experimental stress analysis technique.

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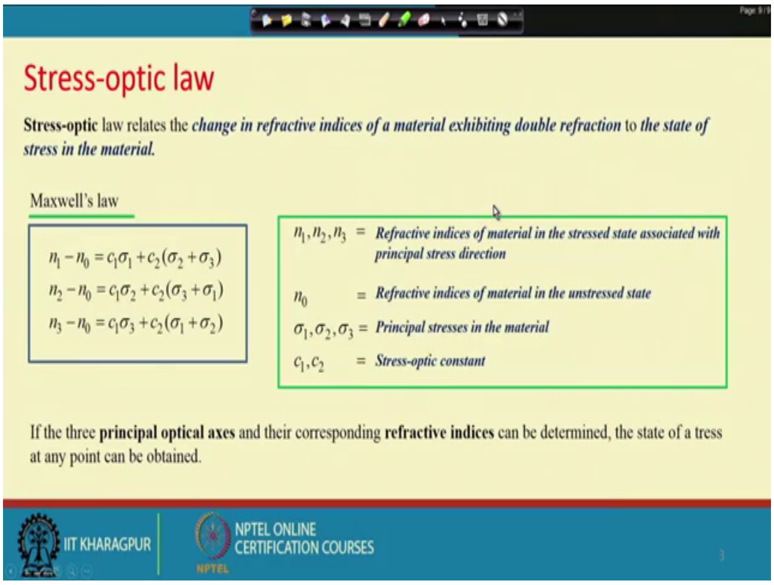

Now, what we have learned basically is the Maxwell's law of stress and which relates the stresses and the refractive indices. So, what it states that the change or relative refractive indices is essentially proportional with the relative principal stresses.

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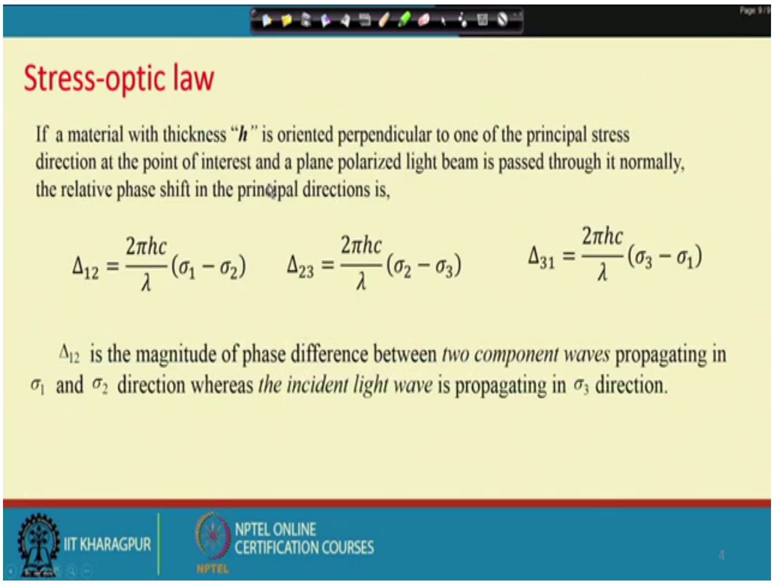

So now it is we also express in terms of retardations. So, what does this retardation means? The retardation means that the, if light is propagating in one direction, then to other direction what does the phase relative phase shift. So, this is expressed in terms of, for instance, delta 1 2 is expressing in terms of the thickness, and stress of the coefficient c, and lambda is a wavelength and sigma 1 minus sigma 2.

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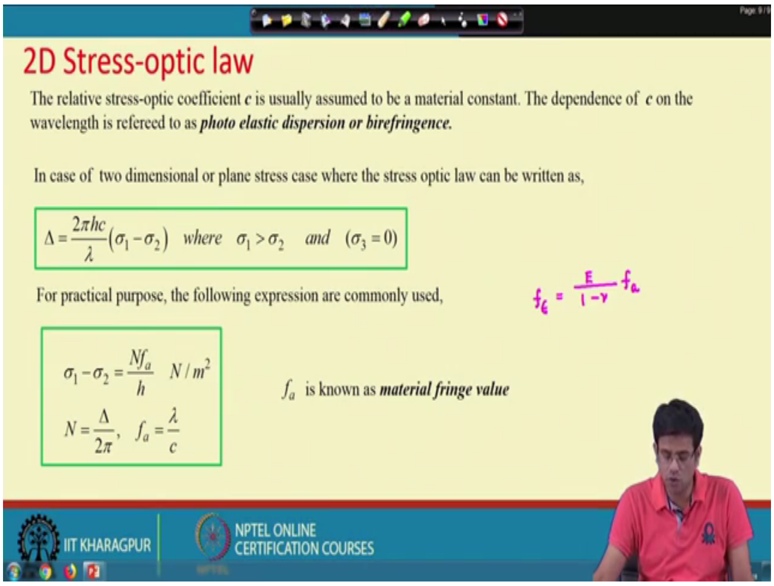

So, this we also further modify the in terms of plane stress condition, these plane stress condition it is sigma material fringe value.

So, this material fringe value if we know, and if we know the relative phase shift that is in here, that is delta by 2 pi, then we can find out the change in the principal strength, a principal stress. And if we know the change in the principal strengths, if we want to know the change in principal strengths, then we have to use the relation between the principal stress and principal strengths. So, which will modify the material fringe value naturally, because the material constant will be plugged into the active material fringe value.

And we have seen that in case of a plane strain, the material fringe value will be x epsilon is E E by 1 minus nu into f a. So, this we have seen and if we used essentially the relation between prints stress and strain for the 3D plane stress material. But I want to emphasize here that, here we assume that material is behaving purely elastic manner. So, there is novisco elasticity or other thing is present. So, at least the number of at least the strain regime is in the elastic range. So, it does not go beyond that.

So, that we have learned in the previous class, and this is due to the by fringes property or temporality d t a w refraction refractive property of the photo elastic material. So, that states that when you have the isotropic the when the material is unstressed, then the material behaves as isotropic in optical property; that means, the refractive indices are same in all direction. And when it is stressed the refractive indices are different in different direction. So, this property is exploited for finding out the difference between principal stresses in 3 different direction, when the photo elastic body is stressed and this is known as the Maxwell's law.

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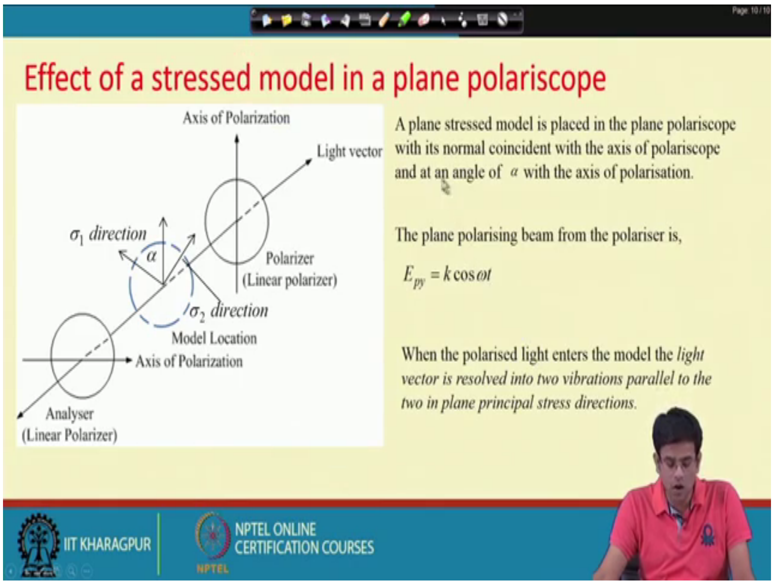

So now here will a understand if the effect of a stress model in a plane polariscope. So, this is the direction of the light vector. So, this is the analyzer and this is the polarizer. So, and here if we place a model. So, you have sigma one direction principle stress direction and principle sigma 2 directions to principle stress reaction.

Now, a plane model when it is placed in a plane polariscope with it is normal coincidence axis of the polariscope at an angle alpha, then the axis with the axis of polarization. So, alpha is the axis of polarization with the principal stress reaction. Now then the polarizing beam we know we are very much aware of that this all optical; that means, the electromagnetic wave essentially we can express in terms of simple harmonic motion.

So, this simple harmonic motion we express in terms of k cos of omega t k is amplitude, and when the polarized light enters into the model, light vector is resolved into 2 vibrations parallel to the 2 planes of principle relation. So, when the polarized light also we know what is polarized light. So, this when we put the model in under the polarized light, and then essentially it shows this equation.

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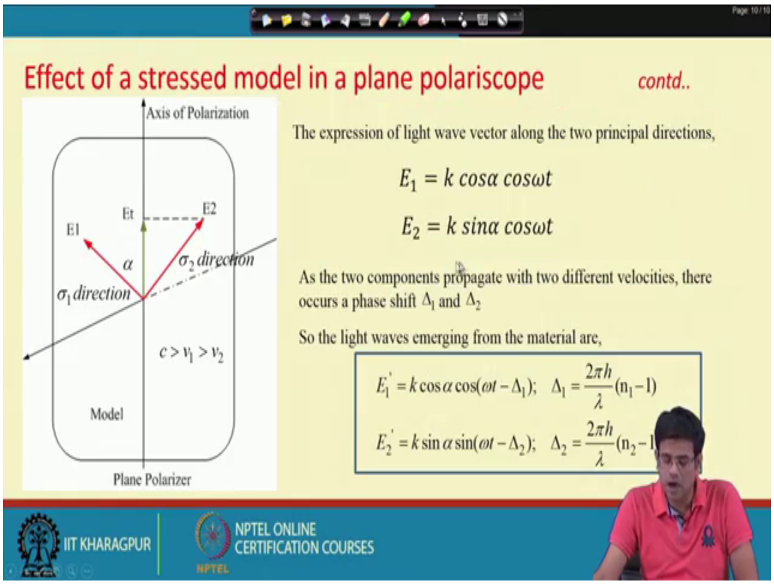

Now, when this light expression, light wave vector along 2 principal direction we can decompose which is k cos alpha cos omega t and k sin alpha cos omega t. So, this we know, and then as 2 component of propagation with 2 different velocity, because it shows since the refractive indices in 2 different direction will be different.

So, naturally the 2 different velocity will occur in 2 different direction. So, that will lead to phase shift. And we have found out this phase shift delta 1 and delta 2, which is essentially this 2 pi h by lambda n 1 minus 1 and 2 pi h by lambda is n 2 minus 1. Now this is the component of the light vector along the 2 different direction.

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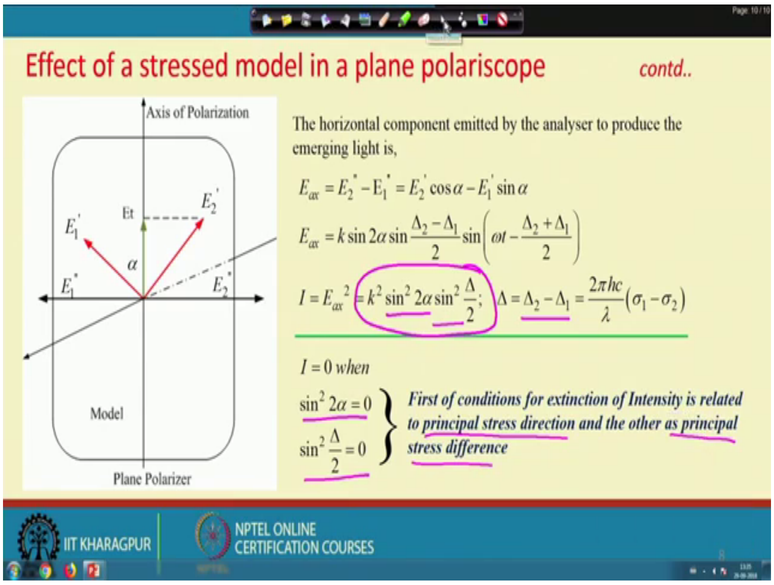

Now, these we can further little modify. So, we can actually a find out what if the horizontal component emitted by the analyzer. So, the horizontal component essentially we can find out by just taking the difference, and this difference is expressed in this form. So, with this the intensity of the light we have seen earlier that is what is intensity. So, E a x square which we can repress present in terms of this equation. So, this I is in terms of this and then delta 1, delta 2 by delta 1 also we can write it in this form.

Now, this is if you remember that this is the lambda by c is essentially material fridge constant. And so, when I equals to 0; that means, either this quantity will be 0 or this quantity will be 0. So, that means, intensity of the light will be 0 is either this quantity will be 0 or this quantity will be 0. Now the first condition of extinction of intensity is related to the principle stress direction another as the principal stress difference. So, we will understand what is this principle stress relation and what else principal stress difference. So, now if I consider that the principal stress, when the sine square alpha is 0, then what is the condition.

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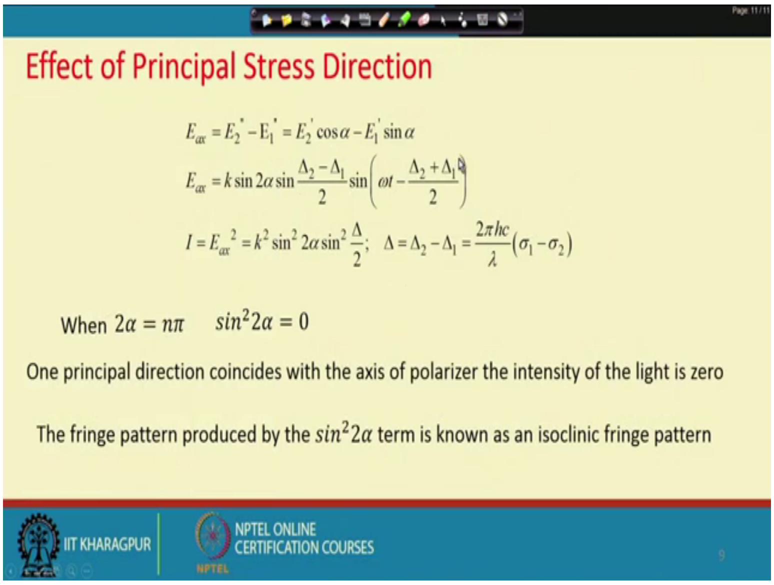

So, now if when sine squared alpha, when alpha is 2 alpha that is the angle between the polarizer and the light vector is or the axis of polarization is essentially in pi, then one principle then naturally sin squared to alpha will be 0. So, this intensity of the light will be 0. So, what this means the one principal direction coincides with the axis of polarization, which is since the alpha is 0, and the intensity of light is 0.

So, this gives us the extinction firstings extinction of the intensity of life. Now the fringe pattern produced by this sine square alpha term is known as the isoclinic fringe pattern. So, this fringe pattern is governed by the angle alpha. So, if you change the alpha the fringe pattern will be different here. So, another case where the intensity will be 0 is possibly a sin square delta by 2 is 0 that is, it is dependent on the relative phase shift.

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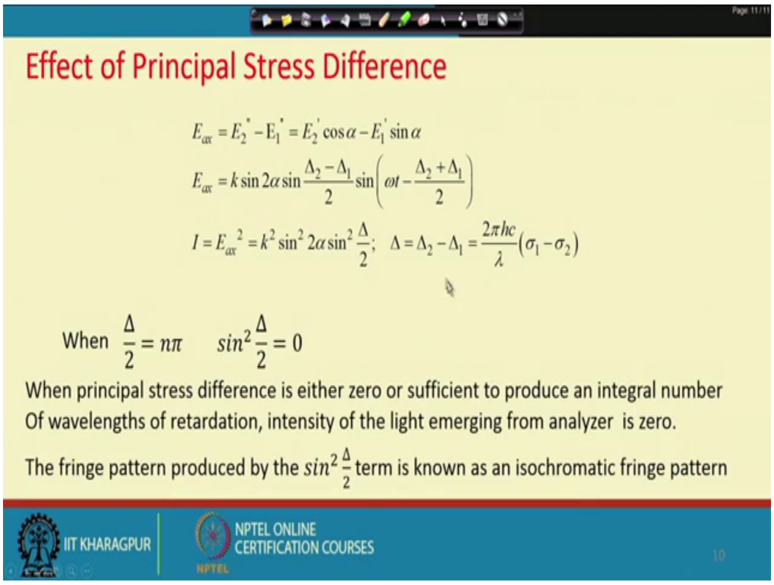

So, let us see if sin square delta by 2 has to be 0, then delta by 2 is n pi essentially. So, what does this means? That when the principal stress difference is either 0 or sufficient to produce an integral number of wavelengths of retardation then the intensity of light emerging from the analyzer is 0. So, essentially delta can be 0 or it is in can be 1 2 3 and so on.

So, if this is the condition, then this intensity of the light again is 0, and the fringe pattern produced by the sin square delta term is known as the isochromatic fringe pattern. Now this isochromatic fringe pattern essentially if you pass a monochromatic light; that is the light which is entering into the body, then we can actually get a very sharp fringes and very clear fringes. So, if we count the number of fringes then we can actually find out the principal stress.

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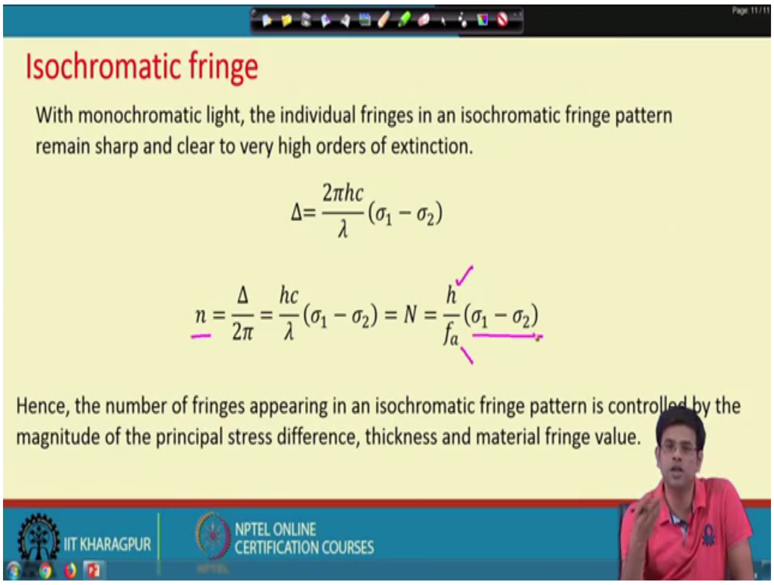

So, what this is essentially a with a monochromatic light the individual fringe in a nice tropic fringe power remains sharp and clear 2 very high orders of extinction. So, if this is the stress optics law, and then n equals to delta by 2 pi. So, that means, delta by 2 essentially n pi, and then only the isochromatic fringe occurs.

So now if we now substitute to this equation then n is h c by lambda into sigma 1 minus sigma 2 and which is delta by 2 pi also we denoted by capital N, and then that is equals to h by f a sigma 1 minus sigma 2. So, essentially n is the number of fringes and h is the thickness and f a is essentially the material fringe value and sigma 1 minus sigma 2 is the difference between principal stresses.

So, number of fringes appearing in a isochromatic fringe pattern is controlled by the magnitude of the principal stress difference thickness and the material fringe value, when mutual fringe value is essentially the photoelastic coefficient means photoelastic the property of the photo elastic material, thickness what are the thickness of the material you were considered. And so, the number of fringes will appear based on the different depends on the magnitude of the principal stress difference.

Now this is a special case and in most of the laboratory actually we use this property and in the photo elastic experiment, we actually count the number of fringes for an isochromatic fringe. And then we try to find out the difference in principal stress.

Now, once we know the difference in principle stress essentially we know the difference in principal strain. So, we can also calculate the strain, but remember that at that time the material fringe value has to be changed. So, it will be in terms of material parameter also. So, the final in case of a isotropic in case of a isochromatic fringe that is the number of fringe appearing in an isotropic fringe pattern is essentially dependent on thickness material fringe value and these restrain principal stress difference.

So, finally, we can express it in terms of strain and strains also. Now this completes our knowledge or our discussion about the transmission photoelasticity. So, where actually we can see the stress contour within the body and within the whole body essentially. So, that is why it is a full field technique or we can take the image of those thing and count the principal stress difference, and essentially we can find out the principal stresses.

So, this can be extended to do for 3D body, but as I have said earlier the in case of a 3D body counting of such isochromatic fringe is a very tedious job, and finding out those principal stress difference is also a difficult task. But nevertheless it can be done. Another case where essentially you do not have the photo elastic material or the material which exhibits the bio fringes property. There what we do? We use a bio fringe encoding essentially the coating material is a polymer essentially.

So, sometimes say suppose you want to have you want to be tested a still body under photo elastic experiments. So, steel is not a bio fringing material. So, it will not a non crystalline material. So, basically it will not show the perfringens property; that means, it will not show the temporary w refracted and refractive property. So, what we do in such cases? Essentially we coat with birefringent material or we quote a layer of birefringent material on that body. And then we do the photo elastic material, and when the body is stressed essentially the coating is also stressed. So, the stress we get from that birefringent coating is the surface strain.

So, this procedure is also used in photo elastic experiment where we really do not do with the non crystalline material; that means the material which proposes temporary w refractive property. So, this is another aspects of photo elasticity which is not transmission photo elasticity, a rather it is a, we use a polymer kind of material on the surface of the non crystalline and or the crystalline body essentially. And then try to find out the stress pattern along the surface of the material due to the application of load.

So, this procedure also helps how the surface stress pattern will be in case of a complicated body. So, here actually this completes our discussion on photo elasticity for this course. So, photo elastic experiments is generally performed in all our courses in laboratory courses. So, this theory is used there to find out actually the material behaviour or the rather the stress strain pattern of the material.

So, in this completes our this module. And in the next module we have another topic which is non-linearity. So, we will discuss the non-linearity in the next module where we will introduce what is non-linear so, in theory of elasticity. So, even though this is a non-linear mechanics or essentially the continuum mechanics is the next step of theory of elasticity, what we will introduce to you a basic concepts of what exactly non-linear, and why it is called non-linear and what is the difference with the linearized elasticity which we have studied till now. So, I stopped here so, will meet in the next lecture with the introduction of non-linearity.

Thank you.