Theory of Elasticity
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Lecture – 56
Photo – Elasticity (Contd.)

Welcome we are in a photoelasticity module. So, we are discussing the basics of optics which we know from our simple harmonic motion knowledge. So, in the last class we have discussed the polar the linear polarization or rather the superposition of 2 light waves in the same plane with same frequency, but different amplitude. So, and if we consider the we have also seen the equal amplitude case, what will be the intensity of light because, all polariscope and all those things polar the instrument by we which, we will use for the photo elastic experiment the intensity of light is the quantity that is required to be evaluated.

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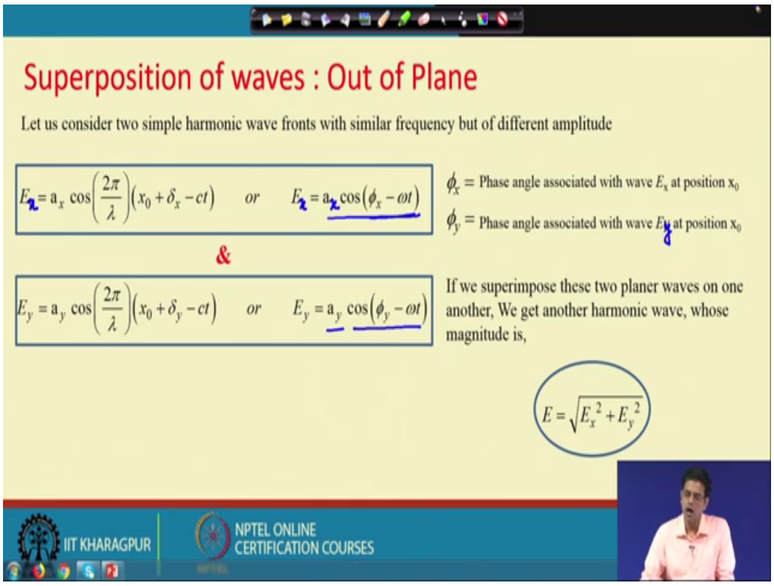

So, here in today we will consider the superposition of out of plane waves. So, here this let us consider 2 simple harmonic wave fronts with similar frequency, but different amplitude. So, this will be E x. So, let us consider that the one of the wave is E x is a x cos of 2 pi by lambda is 0 plus delta x minus c t and similarly, we can write it E x is a x cos phi x minus omega t and, but the frequency is same for 2 light waves. So, that is E y is a y cos of phi y and omega t.

So, now d t is, but the characteristics of this these are the out of plane waves. So, not in a plane. So, when we have out of plane waves. So, we can now write the resultant wave is root over E x square and E y square. So, this is phi x and phi y, E the phase angle associated with wave E x and E y.

So, now what will be the in this case? What will be the resultant wave? In the previous lecture, we have seen if it is a in plane then we can just add it just E 1 and E 2 we have seen, but in case of a out of plane super means, if the 2 waves are out of plane then it is E x square and E y square that we know right.

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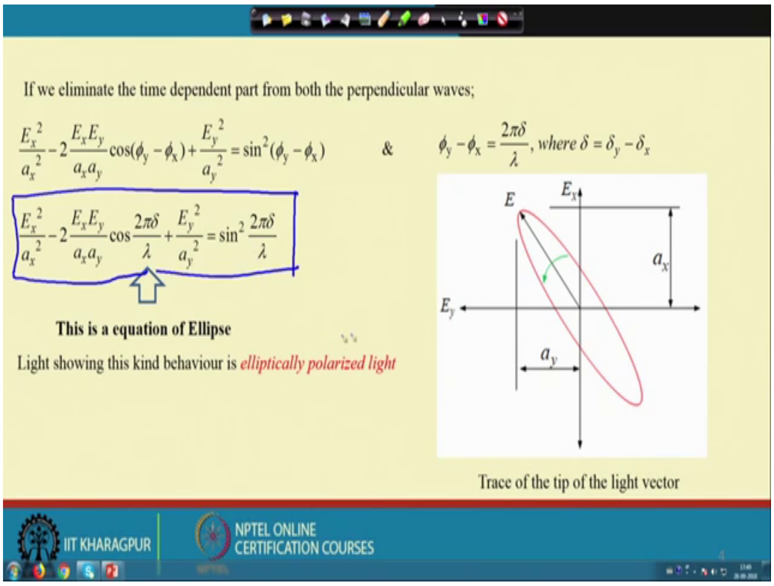

Now, if we now do some manipulation and try to eliminate the dependent part time dependent from the both the waves then, we arrive an equation of this form, this equation if you look carefully, this equation is nothing, but a elliptic equation. So, here E x square by a x square minus 2 E x E x y by a x by cos of 2 pi delta by lambda and then E y by this thing, now phi y minus phi x is 2 pi delta by lambda, where delta is delta y minus delta x. Now this equation is an equation of an ellipse. So, if I write it the in a graphically, if I show it graphically. So, this is a actually the E for the you are the root of E x square minus plus E y square and this is a y and this is a x. So, this is E x and this is E y. So, that is why this is a x and a y. So, the amplitude.

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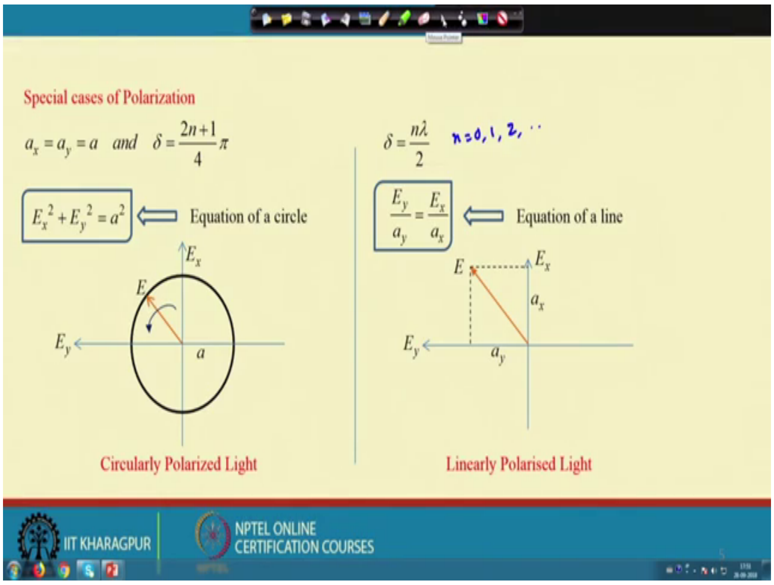

Now, this since this represents an equation of an ellipse. So, this is this we can say, it is elliptically polarized light. Now, if I now this if I consider a special case of polarization for instance, when a x and a y equals to a, and delta is to 2 n 2 n plus pi 2 n plus 1 by 4 pi then I can see that if, we substitute in the previous equation these conditions that delta is 2 n plus 1 4 by pi 2 n plus 1 by 4 into pi and a x and a y is a then, we obtain an equation of a circle the elliptical equation becomes a circular equation.

So, we say circularly polarized light. So, when essentially, two out of plane waves, if we superpose then we get a polarization the resultant wave is a polarized light and this polarized light is essentially depending on certain magnitude and phase difference, it either elliptically polarized or circularly polarized. Now if delta equals to n pi n lambda by 2, then it will represents an equation often line.

So, this is called linearly polarized light so; that means that if I just substitute n equals to 0, 1, 2 and so on. So, if I substitute then I see that this is this will represent a equation of a straight line and then this we called as a linearly polarized right. So in an essence, what it is actually that it gives you polarization, when you have 2 sets of wave, you superpose and then you we get the resultant wave, the result that it is trying to we are trying to understand, what will be the characteristics of the resulting wave.

So, essentially this is the principle of a polariscope. So, when you design a polariscope this essentially, you need to look how a unpolarized light enter into such polariscope and how it goes out from the polariscope? So, basically the polarization part of the light.

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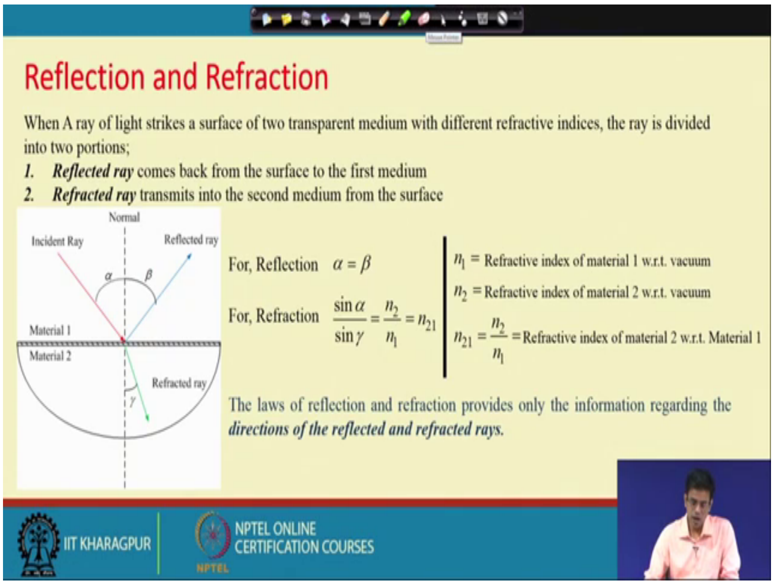

So, this polarization can be achieved in a different way also for instance, reflection, reflector or refract through that also it can be obtained. So now, reflection and refraction we know the snells laws and all those things from our basic plus 2 level knowledge. So, when a ray of light strike at the surface of 2 transparent medium with a different refractive index that is divided into 2 portion 1 is called reflection another is called refraction.

So, the reflected ray comes back in the same surface of the first medium, refracted ray transmitted into the second medium from the surface. So, these things we know, but just to review our or to study the essentially, the photo elasticity, we need to review this. So, for instance the reflection is when alpha and beta are same in this picture. So, this is a interface. So, this is material 1 and this is material 2.

So, when an incident ray comes and so it is reflected, when alpha and beta is same then, it is called pure reflection and then when some of the rays goes in the second medium. So, if this is a gamma this with a normal, if this makes a gamma then sin alpha by sin gamma is n 2 by n 1 or that is the refractive index of material 1 with respect to vacuum and N 2 is refractive index of material 2 with respect to that you. So, n 2 1 is the ratio n 2 by n 1 or refractive index of the material with material 2 with respect to material 1.

So, the laws of refraction reflection and refraction provides, only information regarding the direction of the reflected and refracted rays. So, basically this n 1 and n 2 is a medium property, which affect how the light reflects and refract in a medium.

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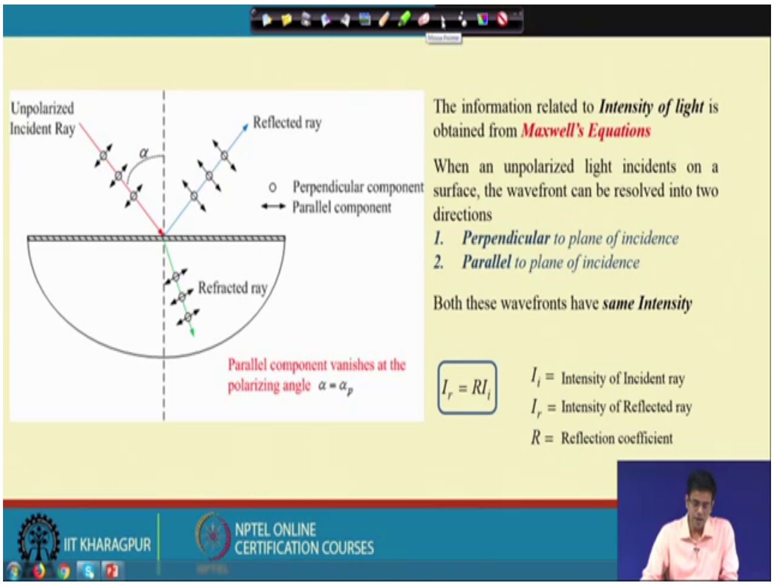

So, this thing we know it from our previous knowledge. Now, the information related to the intensity of light is obtained from the Maxwell equation. So, Maxwell equation, even though we will not discuss it here. So, it is better to know that is coming from the Maxwell equation, when an unpolarized light incidence on a surface the wave form can be resolved into a 2 direction, one is this is unpolarized incident ray and then it one of them can goes out.

So, parallel component term and the there are a perpendicular component also perpendicular to this plane and then this refracted ray in the polarized part the parallel component actually vanish. So, both of this wave form have same intensity. So, and this is how the intensity of the intense intensity of the incident ray and intensity of the reflected ray, how it is related with the refraction coefficient R? So, this also we have seen in our previous knowledge.

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So, now, the basic idea of this to construct a polariscope. So, it is a known optical instrument that, use properties of the polarized light in it is operation for experimental straight analysis in stress analysis, we generally use a circular polariscope, but plane polariscope is also used. So, it used plane polarized light which is produced by plane or linear polarizer. So, we have we know now, what is linear polarization of the light means. So, when you have 2 superimposed or the unpolarized light, when it gives certain properties, which we have studied in the previous lecture.

So now, circular polariscope, it uses polarized light which requires linear polarizer along with a wave plate essentially. So essentially circularly polarized light so, the equation of the light vector becomes circular or so equation of a circle that, we have seen in this previous slide. So, this is known as the circular polarizer.

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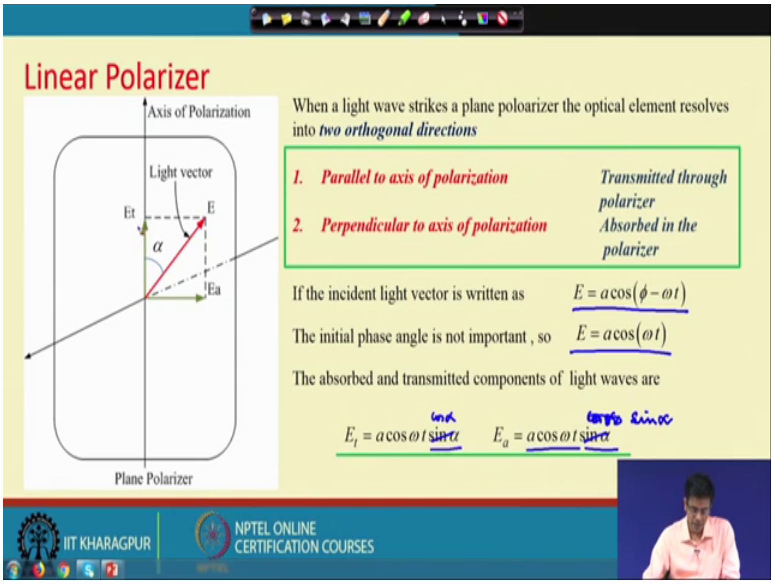

So now, this finally, if we look the linear polarizer. So, when a light wave strikes in a plane polarizer, the optical element results into 2 orthogonal direction. One is parallel to the axis of the polarization. So, if I consider this is the axis of polarization. So, if this is my red one is my light vector and if it makes an axis of the polarization with alpha t. So, it is alpha and then I can do it in a component wise. So, in this component wise, one component will be parallel to the axis of the polarization, this one and perpendicular to the axis of the polarization is this one. So, if I consider this is a axis of polarization.

So, if the incident light vector can be written as this. So, E is a cos, just a simple harmonic wave and then initial phase angle is not important. So, I can write it as E cos E cos omega t. So, the absorbed and transmitted component of the light, I can write E t is a cos omega t into sin alpha and E a the absorbed part of the light is E a cos omega t sin alpha. So, a cos omega t sin alpha now.

So, I think this will be cos alpha. So, this will be cos alpha. So, E a will be a cos omega t cos alpha. So, actually this is the axis of polarization. So, this will be cos alpha I think and this will be sin alpha I think. So, this will be cos alpha I think, this will be sin alpha.

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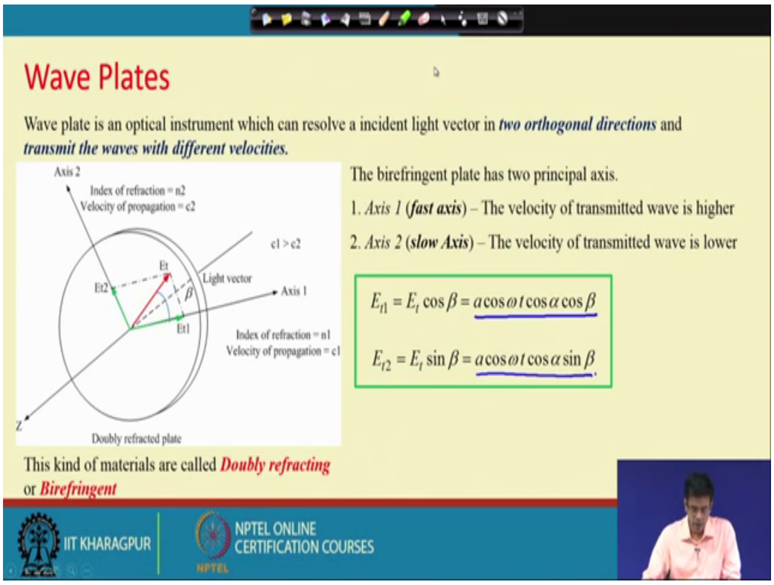

So, axis of polarization along that is E t and perpendicular to that is E a. Now, this wants you in case of a circular polariscope again, this is doubly refracted doubly refracted plate and this wave plate is an optical instrument, which can resolve incident light into 2 orthogonal direction this type of polariscope actually we use. So, if index of refraction is n 2 and velocity of propagation is c 2 then c 1 and c 2, if it is greater than then there are 2 medium that we are considered, which is having refractive index n 1 and N 2 and velocity of propagation is c 1 and c 2, then how we can resolve any incident light wave. So, one axis will be faster, one axis will be slower.

So, now this can be resolved into 2 E t cos beta and E t 2 is sin beta, which with the previous expression, we can write this into this form. Now this kind of material is called doubly refracting or the temporary doubly refractive minty material, we are talking about in the previous class. So, this property is essentially known as the buyer fringes.

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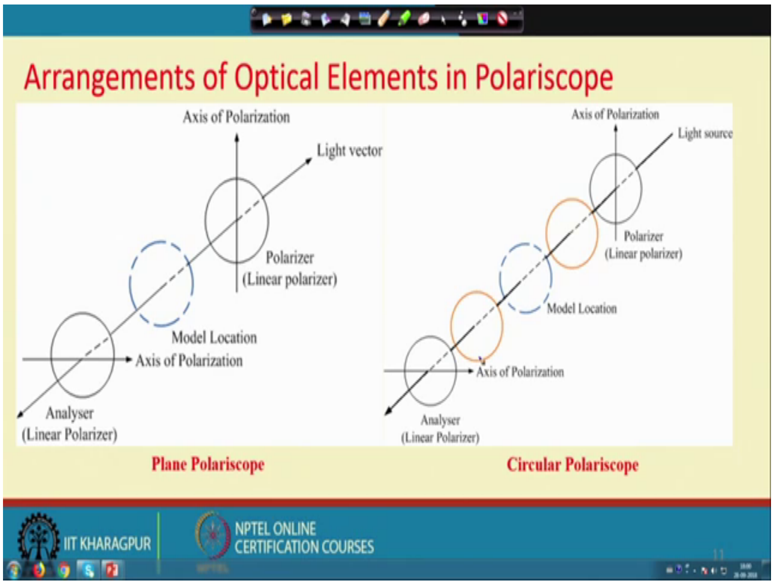

So now, these are the basic principles of designing the polariscope. So, which naturally is very important to us because we will be using this polariscope in our photo elastic measurement.

So now, arrangement of optical elements in a polariscope. So, this is essentially a linear polarizer where the if this is a model location and if this is a light vector going through the. So, there and the axis of polarization for the polarizer will be perpendicular and this is one of them will be perpendicular and another will be the orthogonal to the other direction.

So, this is a setup for plane polariscope and this is a setup for circular polariscope, we are essentially use a 2 circular 2 different refractive index material and then you have one axis of polarization and there is in the other direction axis of polarization. So, this is the basic principle, how circular polariscope works in the photo elastic measurement.

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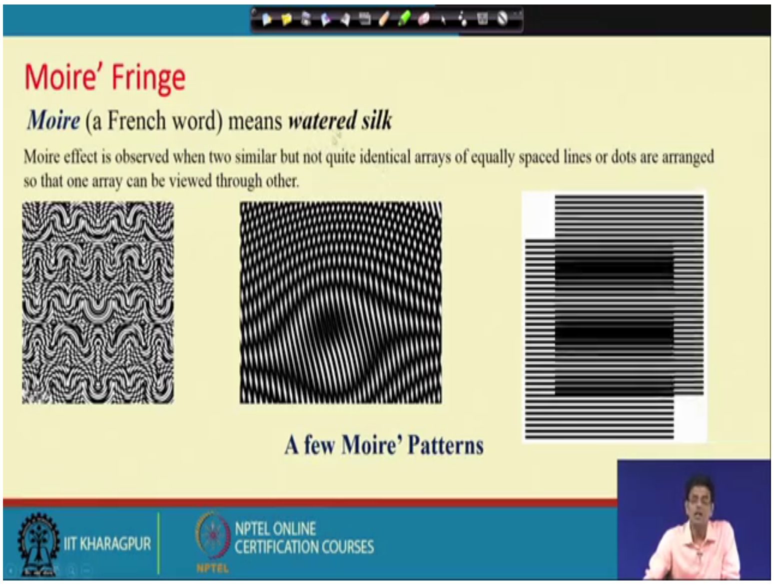

So, now another thing that is important, we will not study it in detail here, rather we will just understand or we will discuss it those who really want to understand it, they can go through the experimental stress analysis by Dally and Riley. So, this is Moire Fringes, the Moiré Fringes is it is a French word actually this is a special type of material is known as the watered silk.

So, if you look at the most right hand side picture then, you will understand that if you place this 2 grated body and then you see the inner layers are very much visible in this common zone. So, this is Moire effect is observed, when 2 similar, but not quite identical arrays of equally spaced lines or doors are an inch so, that the one array can be viewed through the other. So, that is identical or may not be identical. So, this generates some patterns in a body.

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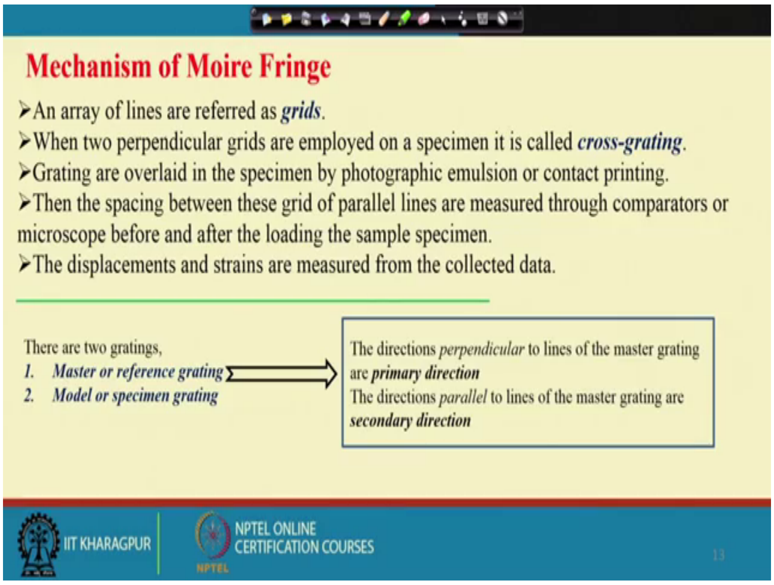

So, this is also helpful for our case. Now, in the essential mechanism of the Moire fringes is an array of lines are referred, which you have seen earlier this is grids when 2 perpendicular grids are employed on a specimen it is called cross grating. So, gratings are overlaid in the specimen by photographic emulsion or contact renting in or any other means then, the spacing between these grids are of parallel lines are measured through the comparators or microscopes before and after the loading of the specimen and the displacement and strains are measured.

So, one is reference grating essentially it is a correlation. So, the direction perpendicular to the line of the master grating our primary direction and the direction parallel to the lines of the master grating are secondary direction.

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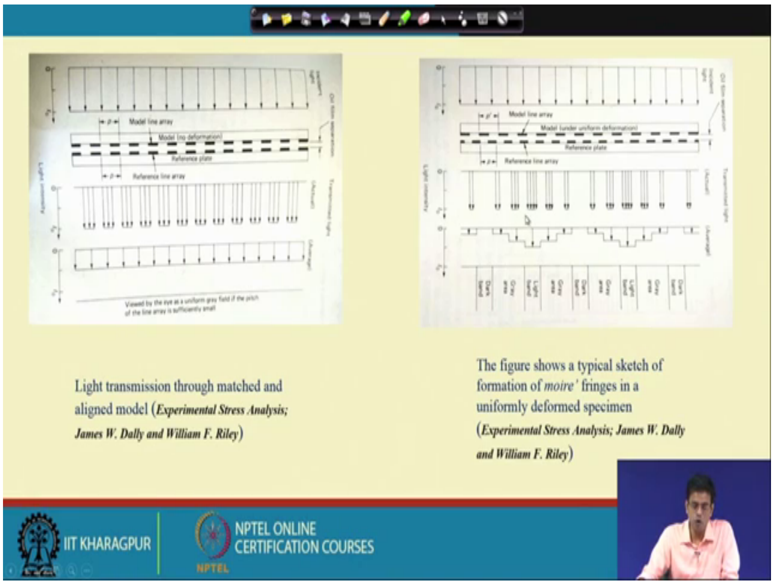

So, master or there are 2 gratings, one is reference grating and another is specimen grating. So, when you compare these two gratings essentially you get the strains in the body.

So, how if you see. So, this is a model information. So, this is a reference plate and then this light transmission through a match and aligned model. So, this is essentially from Dally and Riley, I just pictured it for you for just to understand what is it is and so this is a reference grating if you see this is the pitch here and so we can calculate from this p and p dash the center the center distance, these are center to center this distance so, we can calculate the strains in the body.

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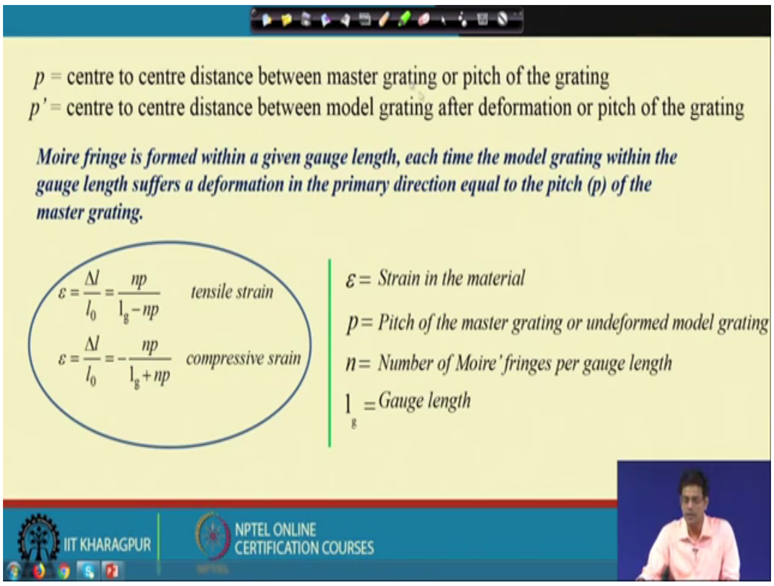

So, for instance if p is the center to center distance between master grating or the pitch of the grating and then, p dash is a center to center between model grating in or pitch of the model grating then Moire fringes can be written as this. So, delta l by l 0, which is essentially the tensile strain and so, in p is the number of Moire fringes per gauge length. So, gauge length also we know that number of Moire fringes in that gauge length. So, that is n and then if i g is the gauge length then 1 minus n p is my l 0 and 1 plus n g 1 plus n 0 is my for compressive strength. So, this is the formula for the Moire fringe.

So now, this is just to introduce the topic to you we are really not will be doing this thing. So, this require some experimental setup also to understand these things. So, in a overall sense what was our objective here? Just to introduce the, what is Moire fringes and how the objective of it we can calculate it is strain patterns essentially, it is comparing 2 gratings, one is master grating one second grating.

So, one is associated with the reference point and one is as a reference body and one is associated with the model for which, we which we will be evaluating which for which we will be evaluating the grating. So, by comparing these 2 grating, we can essentially find out the strains in the body.

So,. So, what we have discussed here? We discussed there are 2 types of polariscope, one is linear polariscope, another in circular polariscope. The circular polariscope, which we will be mostly using in our experimental setup and this circular polariscope is worked on the how it worked?

Because, if 2 out of plane lights, if we superimpose essentially, we get an equation of ellipse and under certain condition that is if the amplitudes are for instance the amplitude of the light is the same and then the phase difference, if we arrange the phase difference of these incident waves in such that that, delta is 2 n plus 1 by 4 into pi, then we get equation of that ellipse is transformed to a equation of a circle. So, then we get the resultant wave is circularly a polarized light and using this principle actually, we conduct the photo elastic experiment.

So, this is overall theory of the polariscope and how we have also seen, how the circular and linear polariscope can be? What are the components of the circular and in linear polariscope? So, the objective is to give you an idea, how this photoelastic experiments happens. So, in the next class actually, we will discuss what is the stress ellipsoid? And how it is essentially related to the refractive indices? And how we can really conduct, if we have a photoelastic body your photo elastic model?

And if we load it and then try to find out the fringes if you observe the fringes pattern, how really we can find out the stresses in the body? And how we can really calculate the stresses in the body? So based on the number of fringe patterns and other quantities. So, in the next class, we will start the theory of photo elasticity by which most of us will do the experiments. So, I stop here.

Thank you.