Theory of Elasticity
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Lecture – 01
Introduction

Welcome this is a NPTEL online certification course on Theory of Elasticity. So, before starting the course formally, we will have today the introductory lecture of this course. So, before going into the lecture, I just wanted to mention that I am Professor Vishwanath Banerjee, I am faculty member of department of civil engineering IIT Kharagpur and one of my colleague Professor Amit Shah, who will also be taking part of the course with me and I welcome all of your questions and suggestions ah, but you can contact me over email or in the NPTEL blog wherever you are comfortable, but before sending me email ah where the email id's are given here ah.

Before sending me the email I request all of you to give a subject ah of the email as NOC -TE, TE is stands for theory of elasticity. So please mention this ah subject ah, you know CTE ah, so that I will understand or I and Professor Shaw will understand that this is related to this course. So, ah in this this is the first lecture of module 1 where we will discuss what is actually the course content and some primary objective of the course.

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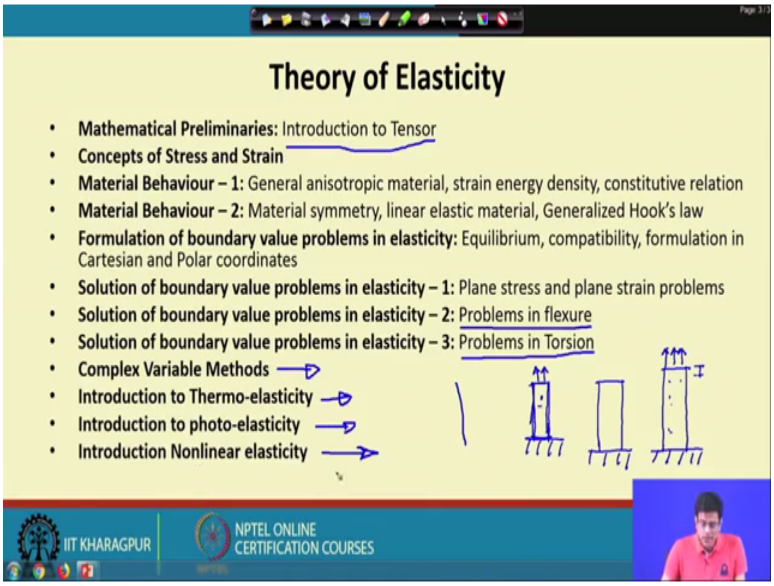

So, first let me go through the content once again ah, because theory of elasticity is undergraduate as well as postgraduate level course. So, in this course what we plan to cover, is mathematical preliminaries initially; we where you will learn one of the important thing of higher ah for understanding advanced mechanics is tensor. So, we will learn tensor ; what is tensor here or we will learn here and then concept of stress, strain. Then we will emphasize on material behavior.

Material behavior will actually take 2 weeks ah lecture ah, so where we will discuss general and isotropic material ah, linear elastic material and then different constitutive relations, and its implication in solving the mechanics related problem. Then we will go for the boundary value problem which is the main ah, in contrast with the strength of material where we do not really bother about the boundary value problem.

Here we will go for the boundary value problem; not only boundary value problem in the Cartesian coordinate system, we will also try to give you glimpses of general curvilinear coordinate system; for instance the polar coordinate system and the spherical, cylindrical coordinate system.

Then we will talk about the different problems related to plane strain, plane stress which probably you have heard those who have done solid mechanics ah. They have done these planes change, the word plane stress what does this mean, and plane strain problems. Then I think all of you must have done the problems in flexure; that is bending. So, the bending problems ah, we have already solved in, strength of material concept strength of ah, from the understanding of strength of material.

Then we will see what is the difference between these flexure problems, if we approach where the theory of elasticity approach ah. So, this is a one distinctive feature, then we will talk about the torsion. Torsion also probably you have gone through torsion of a circular section where this is probably in mechanics or solid mechanics or strength of materials.

So, here we will go for the torsion of a rectangular section, where warping is a important ah part. So, this, this is a distinctive feature of this course where we will learn in depth what is a problems. Now then we will go for the some new thing like complex variable method which ah those who have not taken this course earlier ah, they will find it new and this is a interesting topic where we can solve the solids ah, problems in solids to the complex variable methods.

Then another important aspect, probably you have heard this is a theory of thermo-elasticity. So, we will be mostly restrictive on the linear thermo elasticity or which is popularly known as uncoupling thermo elasticity where the we do not solve the heat equation, we only take the temperature changes and we really do not bother about the propagation of the heat. So, this is an important aspect in the elasticity, and then we will go for the photo elasticity.

Photo elasticity is very interesting and very important topic in today's world, because earlier days what we used to do is that we take any measurement through the strain gauges which is point-wise we fix ah; suppose there is a sample ah, sample you want to test and then ah now this sample what you do basically, you take the you load it and then you put some strain gauges here, some strain gauges here to measure the strain at these points.

So, but as the technology developed and the development in the sensors and say development of the technology ah, now what we do is basically is take the full image of the this specimen. For instance if I give you the example ah, if I take an image of the un deformed sample ah, and then once it is deformed, if I take another image and correlate these two image, then we can actually calculate the displacement at any point of the sample.

So, for instance this is the displacement ah, so this is the displacement in this direction. So at any point or every point of the sample, we can calculate the displacement. So, this is the popularly known as the digital image correlation. Basically what we do? We compare two image and find out the relative ah changes in the displacement and then from there actually we can compute the strengths ah. So, ah instead of a sensor or the strain gauge approach, this gives us full field displacement.

Another thing is we will be mostly talking about the linear elasticity, but it is essential to know ah the concepts of non-linear elasticity, those who want to pursue higher studies and very involved simulation jobs. So they need to know non-linear elasticity as well. So, this is the overall plan for this course ah. Now, before we get into the first topic which will be which will discuss from the next lecture. Let us study some of the basic things.

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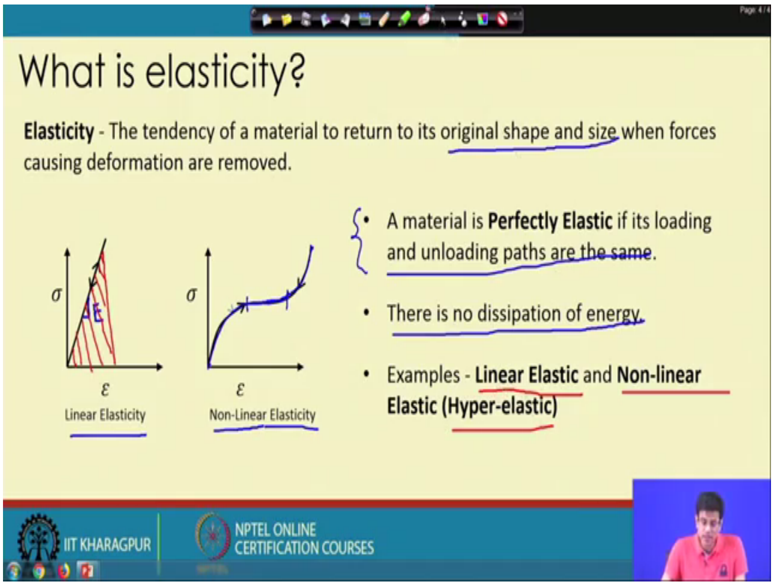

So, to start with what is the elasticity? So let us clear our concept about the elasticity. Ah I found some of us having concepts; like elasticity is where the stress is proportional to strain and all those things ah; for instance when stress ah, when I reload ah, when I load a material, when I load a sample and unload it, it comes to original position then it is called elasticity.

So, these definitions we will relook it here. So, first of all let us put it in this way that the tendency of a material to return to its original shape and size when forces causing deformation are removed. So, there are important thing; for instance the original shape and size, what do we mean by that? For instance here this is a stress strain curve of a material where I when I give stress it deforms and when I unload it after certain point, then it comes back to a original position; but this curve is a straight line right. So, we know the equation of a straight line which is a Y equals to MX plus C right.

So, M is the slope. So, we also know that this slope is essentially the Young's modulus. So this we know from your our basic knowledge of strength of material. But in case where this these curve is essentially not linear, it can be any function; for instance the quadratic function cubic function, or any other kind of function. So, this need not be a straight line, this could be a parabola, this could be a anything. For instance ah so, these again then should you call it elasticity or should you call it plasticity.

So, let us investigate this kind of thing. So, for instance this curve this curve is actually when I load from here and it reaches to this point and then when I unload it, it follows the same path, same path here and reaches here. So, can I say its elasticity? Yes, it is elasticity; it is non-linear elasticity. So, first we learn what is linear elasticity and then what is non-linear elasticity.

So, elasticity can be linear as well as non-linear. So this is an important distinction. Most of us we will confused with that, when the stress is proportional to strain, then it is called the elasticity. See in this case the second case stress is not proportional to strain. If you look for this part of the stress strain curve ah, the amount of strain increase and strain as amount of strain is increased very high, but corresponding stress increment is not very high ah, but after certain time it is again increasing.

So, this kind of thing is known as non non-linear elasticity. So, a material is perfectly elastic, if its loading and unloading paths are same. This is very important we must understand this, what does this mean? Ok we will see and example. So, a material is perfectly elastic if its loading and unloading path are same, So unloading path cannot be different right and why so? We learn, see it through an example.

So, there is no dissipation of energy right; so this is a reason. For instance if you have studied from your strength of material concept the ah, when stress strain ah, stress strain curve if I draw, what is the stress strain in the curve representing or the area under the stress strain curve; if you have seen this is known as the strain energy right. So, this part and this part is known as the strain energy.

Now you see the consequence of this is there is no dissipation of energy or the loading and unloading path are same; consequence to this if the loading and unloading part different, the energy will be different right, because for instance if ah if the loading path for this is different, the area under the unloading curve and area under the loading curve will be different. So strain energy will be different for the two loading case and unloading case, which is not possible for the elasticity, because ; that means, there are some changes in the energy ; either it is gone from the system or some energy has been added to the system, which cannot happen in case of a elasticity.

So, as we have stated that it is a linear elastic or non-linear elastic, sometimes we also talk about hypervelocity, a hyper elasticity where the strain energy density there is a ah form of strain energy density. And then we ah in case of a elasticity you also distinguish it that it is not ah time independent.

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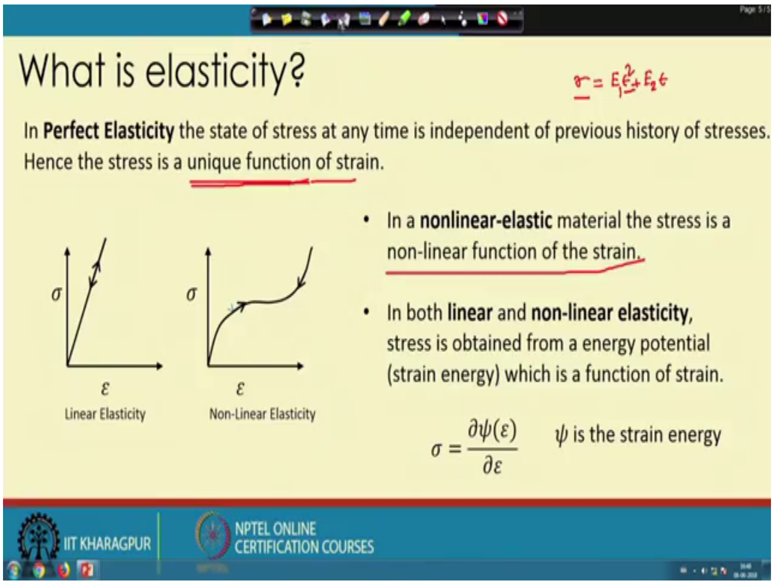

So for instance a perfect elasticity the step of stress at any time is independent of the previous history of stress. Hence stress is a unique function of strain. So, this is very important, because there is no the material can be defined in two ways, it is time dependent material or time independent material.

So, ah; that means, if how I load the specimen and how it shows the deformation, if it is dependent on the previous history how I load then it is time dependent material. So, the in case of a perfect elasticity this is not possible; so it is independent. So, now, if that mean if that condition is preserved, then stress has to be the unique function of strain; that means, the derivative of the strain energy function with respect to stress or with respect to strain, you should be given the stress.

So, in case of a non-linear elastic material, the stress is a non-linear function of strain. So, naturally, because this stress and strain relation for a linear material what we know from our basic knowledge of strength of material is the Hooke's Law. Stress is proportional to strain. So, the proportionality constant which is the slope of the stress strain curve is a Young's modulus right, but if this relation, this relation is a equation or this equation is a equation of a straight line; for instance if I write this sigma is essentially E epsilon. Now this is an a equation of a straight line where sigma and y are the hm variable and e is the slope of the sigma epsilon curve.

Now, if ah, in case of a non-linear elasticity, this curve is not a straight line, so naturally the stress has to be the function of strain. So; that means, suppose if I take say epsilon squared or some other term some E 1 E 2 something, some epsilon or something.

So, this is a non-linear function, naturally the curve will not be a straight line. So this is an important distinction between linear and non-linear elasticity. So, now, in both linear and non-linear elasticity stress is obtained from a energy potential which is known as the strain energy and which is a function of strain.

So, basically what it says, is that the if I take the partial derivative of the strain energy function with respect to the strain, it will give me the stress. So, this is true for the both the case; in case of a non-linear, in case of a linear elastic also. I think for a linear elastic case you have seen it earlier.

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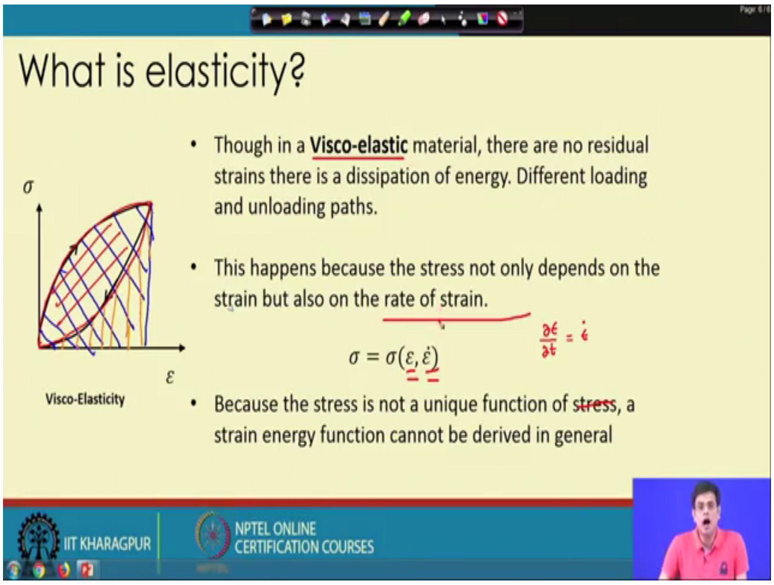

Now another instance ah for instance let us see in a see an example. This is the a material which starts if I load it its follows this path, it follows this path and when I, it follows this path and it reaches here. Now from here I remove the load and then it comes back through this path, not from the original path. What will happen?

So, as I have said earlier if you look carefully what will happen that when I was when I up to at the loading point, if I draw this area under this curve, this was my the this portion was my the strain energy right.

Now, when I unload it, when I unload it, when I unload it my unloading curve is different, so this part is my now the strain energy. So, this portion some portion of the energy is actually lost in the process. So, this portion of the energy is actually lost.So, this portion is out of the system due to this loading unloading process. So this cannot be elastic, because there is a loss of energy. So, this is known as, even though in means contrary to the definition that it reaches to the point where it started right. So, there is no permanent deformation, there is no residual strain, there is nothing. So it reaches to the point from where it is started loading or we started loading. Even though this is true, but this material is not elastic, why? Because it loses some energy and this type of material is known as the Visco-elastic material.

So, it though ah in a Visco-elastic material there are no residual strain, there is a dissipation of energy different loading and unloading path, because this happened, because stress is not only depends on the strain, but also rate of strain right. So, this we will not cover in this course, but it is better to know from this course that stress cannot be only function of strain, it can be function of strain rate also.

So, in case of a vict ah this is time derivative; that is del epsilon del t right. So how I load the material, this my stress will also depends on that. So, because stress is not a unique function of strain and a strain energy function cannot be derived in general. So this is k, this is for a case of a Visco-elastic material. So, the in a nutshell it is very important how I load and how I unload the loading and unloading path remain same, it cannot be changed. This is the basic thing for elastic material.

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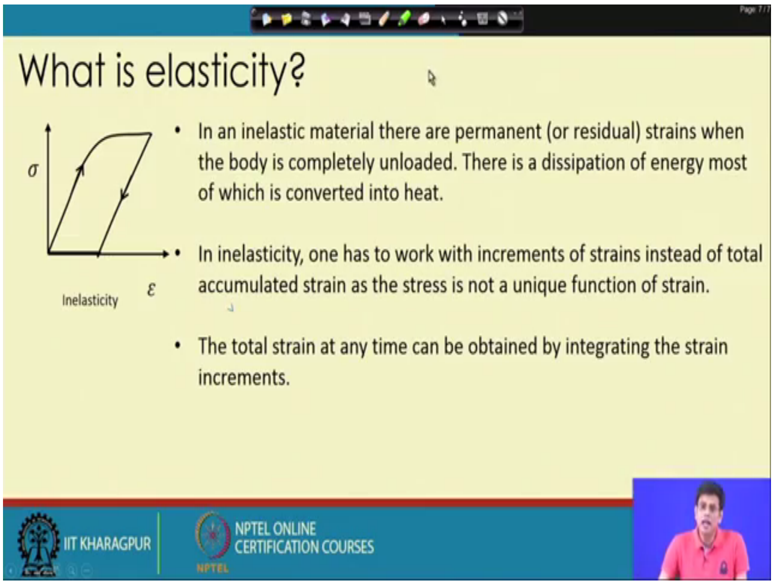

Now, in case of a inelastic material, for instance if you see any material or anything which after loading, it does not come to original position; it comes to a different position, then we say that there is a permanent strain induced in the material or the sample. This kind of material is called inelastic material. So, inelastic materials there are permanent or residual strain when the body is completely unloaded. So this is a dissipation of energy and most of which is converted to heat.

So, this is ah this, this is also a not part of this course because when will talk about the ela inelastic material, then we will discuss about it, but it is better to know this distinction from the elasticity.

So, this kind of material sometimes we call it plastic material. So, in elasticity one has to work with the increment of strain instead of total accumulated strain, as stress is not a unique function of strain. So, the total strain at any time can be obtained by integrating the strain increments.

So, this is a general plasticity ah formulation, but in this course we will not discuss about these things, but what we will discuss here and why we will discuss here. So, all of you probably have gone through strength of material.

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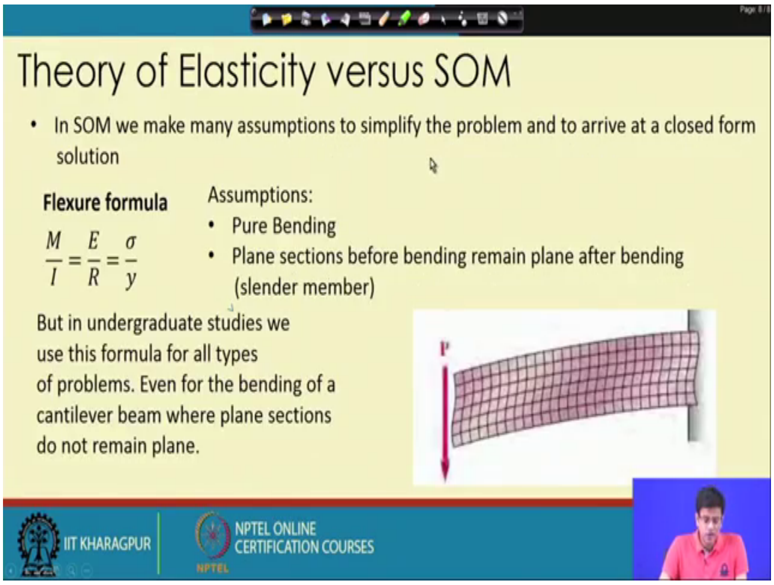

So, why to learn another course theory of elasticity? So, this theory of elasticity for instance in strength of material we make assumptions to simplify problems and to arrive close form solutions. If you remember the flexure formula that M by I E by R sigma by y, what was our assumption? If you remember carefully that plane section remains plane and all those before bending and after bending, and we assume that it is a pure bending case.

So, this kind of assumptions this is actually valid for the slender member, but if it is not a slender member, we can ah we can we cannot apply this kind of equations. So, but we have not distinguished whether the member is a slender member or whether is a member is a shorter member or start or short beam, whatever you call.

So, we apply M by I E by R and sigma by y, but it is not true because in reality every member is a 3 dimensional member. So it has an effect of cross section. So, it is not necessarily true that only the longitudinal sec, longitudinal section or the longitudinal direction stress or strain will be important. So, cross sectional stress strain will all ah will also be important. So, this is a clear distinction that our knowledge of strength of material is not adequate to solve this kind of problem.

For instance another problem probably I have discussed little earlier is the kind of torsion problem. So, in a torsion problem what we have seen that we always go for the circular section, why circular section? Or the hollow section with the constant thickness ah because we will not observe any warping in it.

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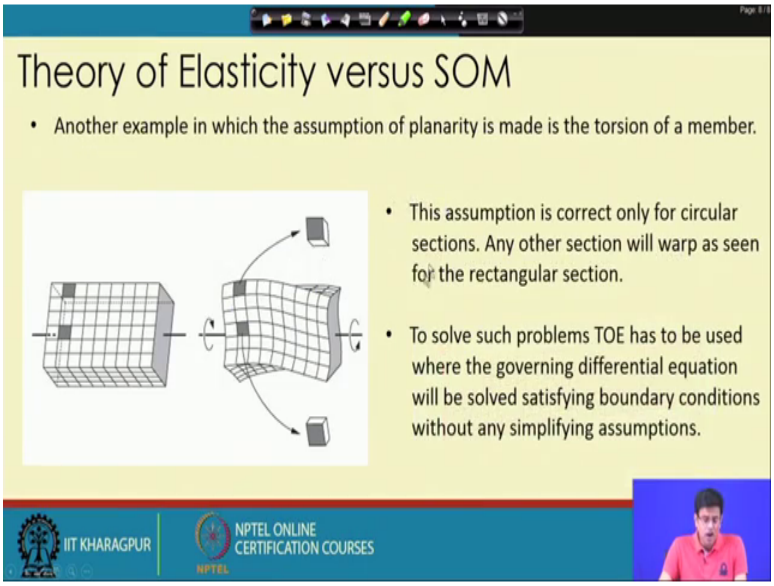

So, what is warping? So for instance if I ah, in a rectangular section ah, if I ah give a torque ; if I give a torque then there is a ah there is a change in the section ah. In the longitudinal case, if you see this carefully they will this change or different stress strain level appears. So, this kind of ah, this kind of phenomena is known as the warping. We will discuss it in detail.

So the ah actually the assumption is correct only for there is no warping, this assumption is correct only for the circular member. So, to solve such problem theory of elasticity has to be used where the basic approach will be forming a governing differential equation and then applying boundary condition to solve the problem finally ah. So, without making any simplification or any simplified assumptions.

So, this is important thing and that I feel will motivate you to learn theory of elasticity, because the assumptions are true for certain class of problem. So it is not a in general true for all classes of problem. For instance you may always encounter to I have this problem that a rectangular member or a ah non rectangular or non circular member experiencing torque or torsion. So this kind of problem we cannot solve through the knowledge of strength of material; so, this is important.

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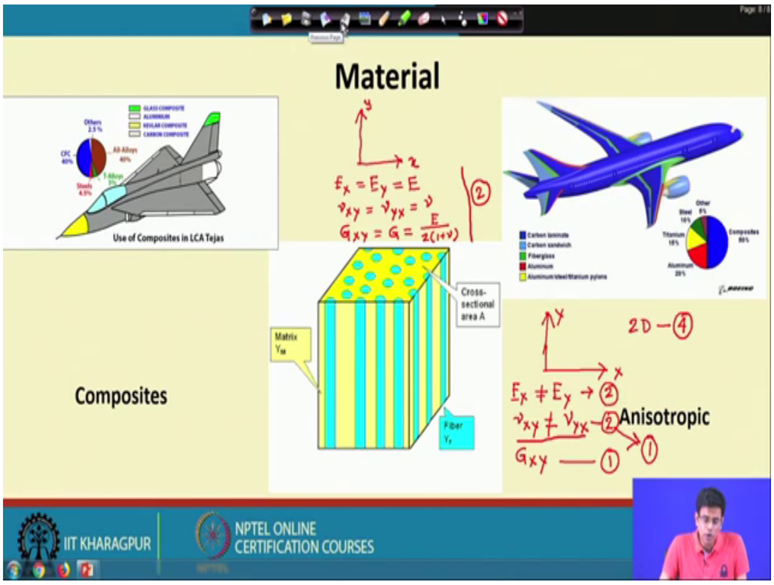

Now, another aspect which we must understand is that material. In today's world if you see the left hand side ah, the right hand side picture is actually the Boeing dream liner ah which is a huge portion of this plane is made of composites because why composite? So it is a new kind of material where the main objective is that if we make it lightweight or the weight of the structure is less, but strength is moved. So this kind of material, if we can use it in your structure ah, the cost of will is will be naturally less for this kind of problem; so this kind of structure. So, that is why we try to use composite kind of material and another important, it is not only the passenger flight, it is also the ah Tejas. For instance, a Tejas is also used a huge amount of composites to build.

So, now ah there is an interesting distinction. Most of us I have learned that material is isotropic right, what do you mean by isotropic? The isotropic means that the ah, any direction material property is same. For instance if I take a steel or any other isotropic material; so if there is a direction x ah, y direction and x direction, the material is remain same. So, if I now say that E x equals to E y equals to E and Poisson's ratio, Poisson's ratio nu x y equals to nu y a y x equals to nu.

So, and shear modulus G x y is actually G and we say it is E by 2 into 1 plus nu right. So, ultimately the in case of an isotropic material you are familiar with this relation. So, ultimately there is a 2 actually, if we know the two independent constant E and nu we can characterize that material, but this kind of composite material, it is not possible for characterizing with the two only independent constant. For instance this is a matrix and there is a fiber. You have seen this kind of structure or this kind of material in day to day of daily life. So it can be glass fiber, it can be carbon fiber, it can be Kevlar or anything. So ah, but this kind of material cannot be characterized as isotropic material; for instance it could be an orthotropic material or in general it could be an isotropic material.

So, in case of a anisotropic material, these relations are not again valid. So, if I write it for an anisotropic material, so E X should not be equals to E Y right.So; that means, Young's modulus in x direction and y direction will be not same and Poisson's ratio nu XY should not be equals to nu YX right and G XY cannot be represented in terms of E X and E Y.

So, finally, if there is ah, for a 2 dimensional if I take 2 D orthotropic material, from here we will have two material constant or two Young's modulus, from here there will be 2 and there will be 1; so total 5, but this nu XY and nu YX are not just independent. So, from here there is a restriction. If I fix nu XY, then nu YX has to have some value. So, from here after this restriction which we will discuss in detail in this course, so there will be 1 constant. So, in case of a 2 D orthotropic material this becomes 4 independent constants or 4 material constants. So, 4 independent constants is required to characterize this kind of material. So this is one aspect of the course.

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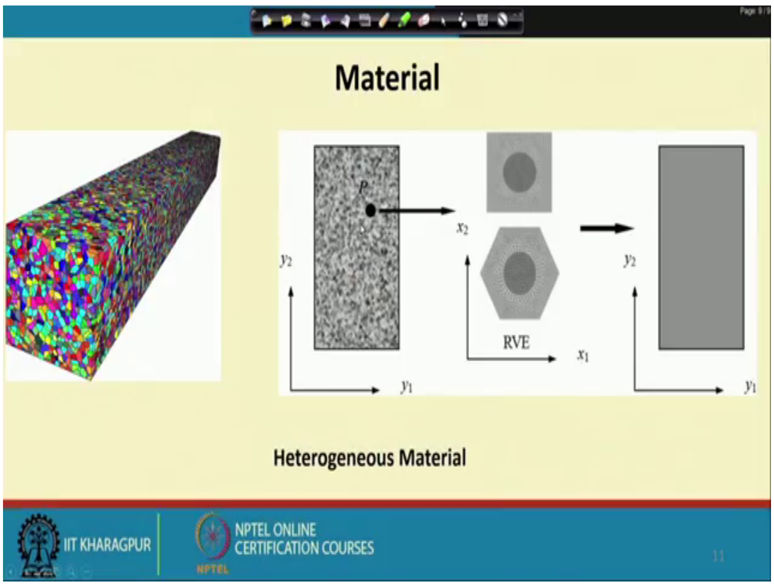

Another aspect probably all of you have seen it like the material heterogeneity. For instance the material hetero; we always take isotropic material, so we may encounter; for instance if I talk, all of us know what is concrete. So, concrete is a heterogeneous material, but we take, always sometimes isotropic. So, this is required when you need to go for the advanced studies where isotropic, material isotropy is not necessarily true. Especially if you consider the micro mechanics or the advanced composite material where the microstructure of the problem is very important, then actually you need to know what is the heterogeneous material property or hetero or the material profile.

So, for instance those who do not know which is heterogeneous property; that means, a material can be isotropic as well as a heterogeneous, but ah the heterogeneity means that at different location, material property will be different, but not necessary that at different direction it will be different right. So this is one aspect of the course. So, that is all for today’s lecture. So we will ah, from the next class we will start the tensor. So, from the lecture 2, we will go for the tensor analysis.

Thank you.