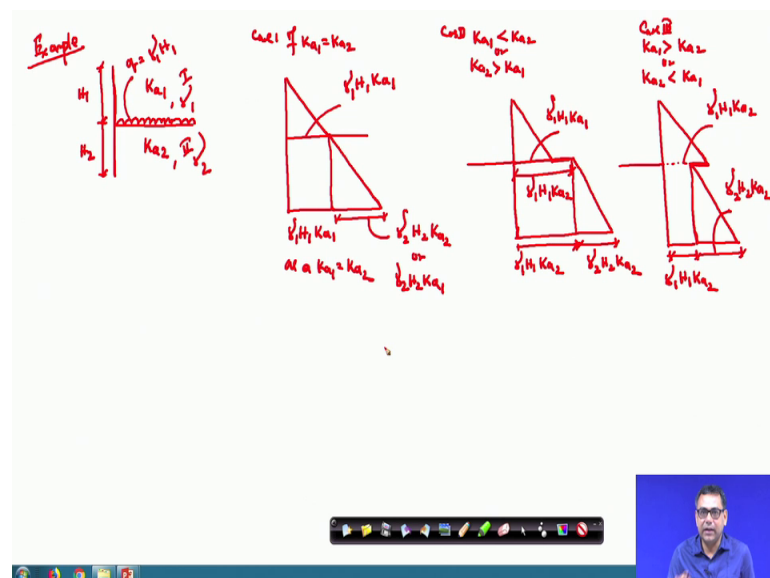


**Foundation Engineering**  
**Prof. Kousik Deb**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Kharagpur**

**Lecture - 45**  
**Earth Pressure - V**

So, this class I will solve a few example problems related to Earth Pressure Theory, and then I will show how we can calculate the forces, that is acting on a wall or the retaining structure due to this, due this is the backfill ok.

(Refer Slide Time: 00:39)



So, first problem that I have chosen that I have 2 soil layer; so, 2 soil layer, one is the thickness  $H_1$ , another thickness is  $H_2$ . So, one soil layer this is  $K_{a1}$  one soil layer this is  $K_{a2}$ . So, this is the  $K_{a1}$  is the coefficient of active earth pressure for layer 1, and  $K_{a2}$  is the coefficient of active earth pressure for layer 2.

Now, I have chosen 3 cases. The case one that if your  $K_{a1}$  is equal to  $K_{a2}$ , then what will happen? Case 2 if  $K_{a1}$  is less than  $K_{a2}$ , or  $K_{a2}$  is greater than  $K_{a1}$  ok. And case 3, where your  $K_{a1}$  is greater than  $K_{a2}$  or  $K_{a2}$  is less than  $K_{a1}$  ok. So, these 3 cases and I will draw only the earth pressure. So, draw the active earth pressure.

So, for the first case, this is the first case. So, I will draw the first case earth pressure, this is for the first layer, and this value is  $\gamma$ . So, here you can say the both the cases

gamma are same. And I have not considered the water table effect also. So, I can write this is gamma, or you can write this is gamma 1 this is gamma 2 also. So, if this is gamma 1 H 1 into K a 1. So, this is this value

Now, this first layer soil overburden pressure or will a stress will act as a surcharge on the second layer ok. So, first layer overburden pressure will act as a surcharge on the second layer, ok. And surcharge with q is equal to gamma 1 into H 1. So, that q will acts for those for the second layer ok. So, that and as I mentioned for the surcharge this will be q into k. Now here the K a is equal to K 2. So, this value will again remain same, the surcharge value will remain same because, this is your gamma 1 H 1 into K a 1, because your K as K a 1 is equal to K a 2. That is why I can write that.

Now, this is the surcharge for the first layer. Now the second layer earth pressure will be again this one ok. So, this part is second layer earth pressure and that will be gamma 2 H 2 K a 2. Or gamma 2 H 2 K a 1 also you can write, because or you can write gamma 2 H 2 K a 1 ok. Because, you have the K a 1 is equal to K a 2. But in the second case, this is for the layer 1 ok. So, layer 1 as usual the same thing, I will draw the same thing that is gamma 1 H 1 K a 1.

Now, your K a 1 K a 2 is greater than K a 1. Now again q value is gamma 1 H 2, now this will not remain same so, that is very important. When you calculate the earth pressure in a layer soil, if you have different K a value then you have to check for the intersection of every layer 1 for the first top layer 1 for the bottom layer, the same point ok. So, that mean gamma 1 H 1 K a 1 is what the earth pressure at this point corresponding to first layer.

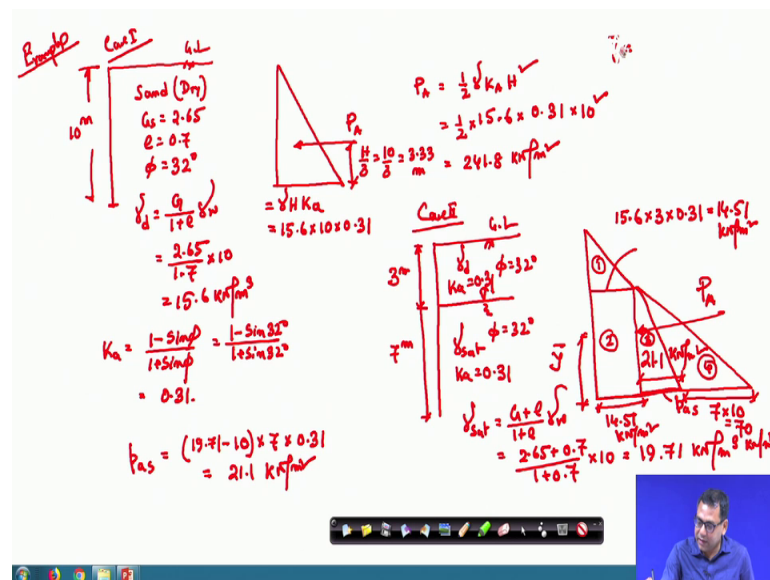
Now, corresponding to second layer, because that will act as a surcharge gamma 1 H 1 now it is into K a 2, and K a 2 is greater than K a 1. So, your earth pressure will increase, you will go this direction, this is very important ok. So now, this value is nothing but gamma 1 H 1 into K a 2, ok. That is why I have chosen this problem. So, that I can show you that what would be the difference.

So now this is for the UDL part. Now for the second layer earth pressure will act. So, this is for the second layer earth pressure that will be gamma 2 H 2 K a 2 ok. And this one is gamma 1 H 1 K a 2, this is very important. And another one is case 3, that we are taking

the same problem and up to this first layer will not be an issue ok. And this value as usual, these value will be  $\gamma H K a 1$ .

Now, your  $K a 1$  is greater than  $K a 2$  or  $K a 2$  is less than  $K a 1$ . So, this value will decrease, and then you have this UDL part. And this value is  $\gamma H K a 2$ . Now your  $K a 2$  is less than  $K a 1$  so, that is why it will decrease. Then from here this force or the stress we act and that is  $\gamma H K a 2$ . So, these 3 figures are very important. Remember these 3 cases, that if in the changing of  $K a$  value. So, you cannot always draw this stress diagram ok. If it can be this one also, it can be this one also, it can be this one also. And, I will show later on how we will calculate the forces and the point of application, but this is the earth pressure diagram for these 3 cases.

(Refer Slide Time: 08:12)



So, next example problem that I will take that this is again the example problem; that we have a earth pressure draw the earth pressure for this type of soil this is a sand and it is in dry condition. So now, here  $G_s$  is given 2.65  $e$  is given 0.7 and  $\phi$  is given 32 degree. So, what is the difference? In previous problems also, in the previous problems I have given directly the unit weight of the soil. But sometimes you will find that directly unit weight of the soil may not be given. It is given in terms of this is the specific gravity  $G_s$  is the specific gravity  $e$  is the void ratio. In terms of void ratio if you have to determine the unit weight either dry or saturated or bulk depending upon the values given ok

So now here the dry soil; so, you know the unit weight of dry soil is  $G \frac{1 + e}{1 + e \gamma_w}$ , ok. This is the dry unit weight expression. So, these 2.65, this is 1.7  $\gamma_w$  if I take one. So, this will be 15.6 kilo Newton per meter cube. And we have a  $K_a$  value is equal to  $\frac{1 - \sin \phi}{1 + \sin \phi}$ . So, this will be  $\frac{1 - \sin 32^\circ}{1 + \sin 32^\circ}$  and this will be 0.31. So,  $K_a$  value is 0.31.

So, my distribution of the earth pressure will be because this is the homogeneous soil. So, distribution of the earth pressure will be like this, and these value is equal to this is  $\gamma H$  value is 10 meter, this is the  $H$  is 10 meter. So,  $\gamma H$  into  $K_a$  ok. And so, your this value is  $\gamma$  is 15.6 into  $H$  is 10 into  $K_a$  is 0.31. And if I draw the final force  $P_A$ ; so, this  $P_A$  is equal to  $\frac{1}{2} \gamma H^2 K_a$ . So, this is  $\frac{1}{2} \gamma H^2 K_a$  is 15.6 into  $K_a$  is 0.31 into 10 square. So, this will be 241.8 kilo Newton per meter square. This is the case one.

Now, in the case 2, I have taken the same soil layer here, but the water table is at 3 meter below the ground level, ok. So, this is the ground level  $G L$  this is the wall here also this is the ground level this is 3 meter and this one is 7 meter. The same problem I have taken. So, this is the  $\gamma_{dry}$ , but this is  $\gamma_{sat}$ , the same  $G$  value  $e$  values are given. So, I have to calculated the  $\gamma_{sat}$ ; which is equal to  $G \frac{1 + e}{1 + e \gamma_w}$ ; so,  $G$  value is 2.65  $e$  value is 0.7, and  $\frac{1 + 0.7}{1 + 0.7 \times 10}$ . So, this is 19.71 kilo Newton per meter cube. So, this is the  $\gamma_{sat}$  and  $\gamma_{dry}$  we know that 15.6 kilo Newton per meter cube, ok.

And here one thing that your  $P_A$  is acting  $H$  by 3. So, that is 10 by 3, that is 3.33 meter from the base of the wall. Now in this case if the water table is considered, now is this case up to the water table, we can draw these earth pressure and these value is nothing but 15.6, this is  $\gamma_{dry}$  into the  $H$  is 3 into the  $K_a$  is 0.31, ok. Because your  $\phi$  value will remain the same, whether it is dry soil and it is saturated soil, here given in this way  $\phi$  value  $G$   $s$  value  $e$  value will remain same. So, this is the 30.31. So, this value is 14.51 kilo Newton per meter square.

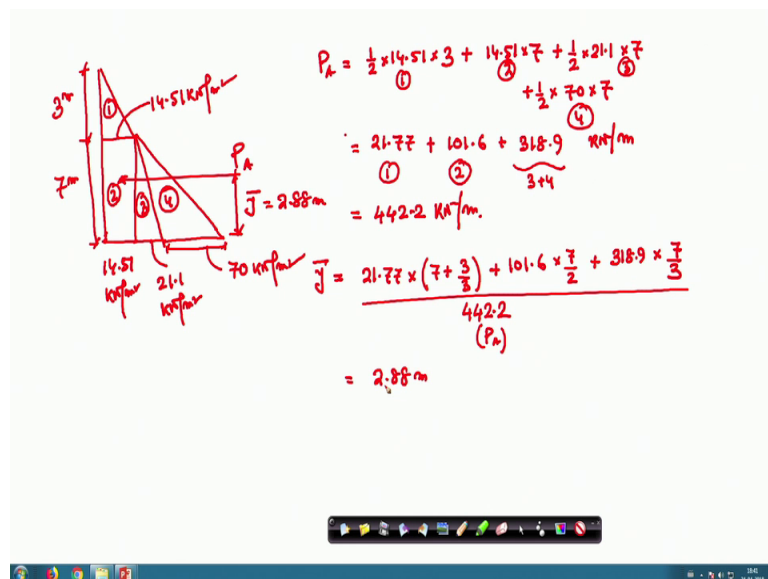
Now, as the  $\phi$  value is not changing so, your  $K_a$  here and  $K_a$  here we will also remain same. So, this is also remain same, it will not change ok. So, that means, here the your this is the 15 into 6 into 3 is the surcharge and  $K_a$  will remain same. So, it will continue here. So, this value will also be 14.51, ok. This is kilo Newton per meter square.

Now, I will calculate the stress for the soil. And that stress is here this one is, this is this gamma is in that case, this stress or p a I can write that here p as for the soil, I can write here that gamma is here saturated is 19.71. So, this will be the sub ok, into 10 into H is 7 meter into K a is 0.31. So, this will be 21.1 kilo Newton per meter square. So, this value is 21.1 kilo Newton per meter square. So, this stresses is for the soil.

Now, for the water will also give you the stress. So, this is the water stress, and this value is nothing but 7 into 10 ok, because water we will not multiply any earth pressure coefficient factor. Because for the water it is factor value coefficient is will not multiply, will multiply it only for the soil. So, this third part is will be 70 kilo Newton per meter square ok. So, finally, we have these expression where this is 14.51 this is 21.1 and this is 70; 70 is due to the water, 21.1 and 14.51 is due to the soil ok.

So, now this is the earth pressure distribution if we have water ok. Now here the P A is acting at H by 3, but here where the P A is acting. So, that we have to calculate. We have to calculate that if your P A is acting here ok, and say that is a height of y bar ok, height of y bar. So now, how I will calculate y bar? Y bar is so; it has 4 parts ok. This is the part 1, this is part 2, this is part 3 and this is part 4. So, I am taking all the forces. So, a first I will calculate the force P A, then I will calculate the y bar ok. So, first I will calculate the force and then I will calculate the y bar. So, I am using on another table.

(Refer Slide Time: 17:10)



So, quickly I am drawing that another sheet. And quickly I am drawing these values. So, this is the values is 14.51 kilo Newton per meter square, and this is 3 meter, then that will continue and this is also 14.51 kilo Newton per meter square, then there will be a triangular zone; whose this is 21.1 kilo Newton per meter square, then this water part that value is 70 kilo Newton per meter square and this height is 7 meter.

So, I am calculating the P A which is acting at the height of  $\bar{y}$  from the base of the one. So now, the P A is so, I have 4 parts 1, 2, 3 and 4. So, first part I am talking it is a triangle ok. So, it is triangle this will be half into 14.51 into the H 3. This is for the first part. For the second part it is the rectangle. So, this will be 14.51 into 7 ok. Then it is a triangle third part is a triangle. So, this will be the half into 21.1 into 7. And then we have another triangle. So, then it will be again plus half into 70 into 7. So, these are the all forces that we have considered ok.

So now if I take these 2, this is for the first case for the second one, this is for the third one and this is for the 4th one. So, 1, 2, 3 is due to the soil fourth one is due to the water ok. So, and finally, if I calculate this one this first one is given 21.7. Second one is given 101.6. And the third one and fourth one is given 318.9. This is the second one, first one second one, this is third plus fourth. Why I have taken third plus fourth because third plus fourth you can considered as a whole triangle, base is 21 points 1 plus 70, 21 plus 1 plus 70, and then the both cases it is 7 and a half. So, that is why I have taken as a same 3 and 4. So, that is the finally, the P A value is 442.2 kilo Newton per meter. This is kilo Newton per meter, remember that this is kilo Newton per meter, because in the plain strain condition.

So, again now  $\bar{y}$  I will calculate now for these 4 cases ok. So, first the  $\bar{y}$  for the first case, your force is 21.7. The lever arm I am calculating from the base. So, lever arm will be 7 plus, it will act one third from this base. Because it is a triangle first one, one third from the base so, this is 3 divided by 3. This is what the first one the second one, the force is 101.6 and it is a rectangle it will act at the middle. So, it will be 7 divided by 2 ok.

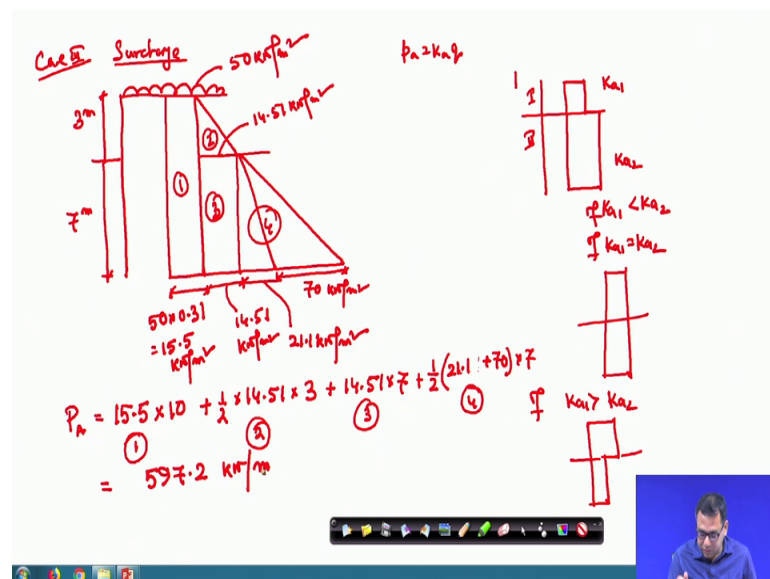
And then I am taking third and fourth one as one triangle, and the force is 318.9, and it will act at one third from the base ok. So, this will act one third from the base. So, it will be 7 divided by 3, because it is a one triangle I am considering. Third and fourth I am

considering as a one triangle. So, it will act one third from the base. So, 7 by 3 and divided by the total force 44.2. This is the total force P A ok, this one is nothing but as a P A.

So, finally, the y bar will be 2.88 meter. So, y bar will be 2.88 meter. So, if I see this problem here y y y bar was 3.33 meter now due to and the force was sorry this is P A. So, this P A is kilo Newton per meter, correct that, this is kilo Newton per meter because this is force. And small p is the stress kilo Newton per meter square, but the capital P is the force it will be kilo Newton per meter ok. Just correct that. And here it was 241.8. Now it is 442.2 and this is the pointer application of this force is 2.88 from the base.

So, that is that change. So, that is why when you design the retaining value always avoid these water ok. We will make such that make such arrangement; that means, we can avoid this water, because water will increase the force in significant amount, you can see that. So now the next problem, that I will I will solve, that is case 3.

(Refer Slide Time: 23:20)



So, in the case 3, I will consider one surcharge, for the surcharge, I am talking the same problem with water, the same problem I am taking the same problem 7 meter and this is 3 meter, but with a surcharge 50 kilo Newton meter square. And the rest of the things are same. So now for this total problem so, you know the surcharge P A is K a into q. So, and here K a is not changing. So, suppose if we have a soil where K a value is different for 2 layer. This is first layer this is second layer.

Then for the surcharge your  $K_a$  value will be may be something like that depending upon the type of  $K_a$ . If this is  $K_{a1}$  this is  $K_{a2}$ , now if  $K_{a1}$  is less than  $K_{a2}$ , then this will be the distribution, ok. But here if the  $K_{a1}$  is equal to  $K_{a2}$ , then this will be the distribution. And if  $K_{a1}$  is greater than  $K_{a2}$ , then this will be the distribution, ok. So, but our case this  $K_{a1}$  is greater than  $K_{a2}$  by  $K_{a1}$  is equal to  $K_{a2}$  because  $K_{a1}$   $K_{a2}$  is not changing. So, I can write a uniform surcharge that will act throughout this depth.

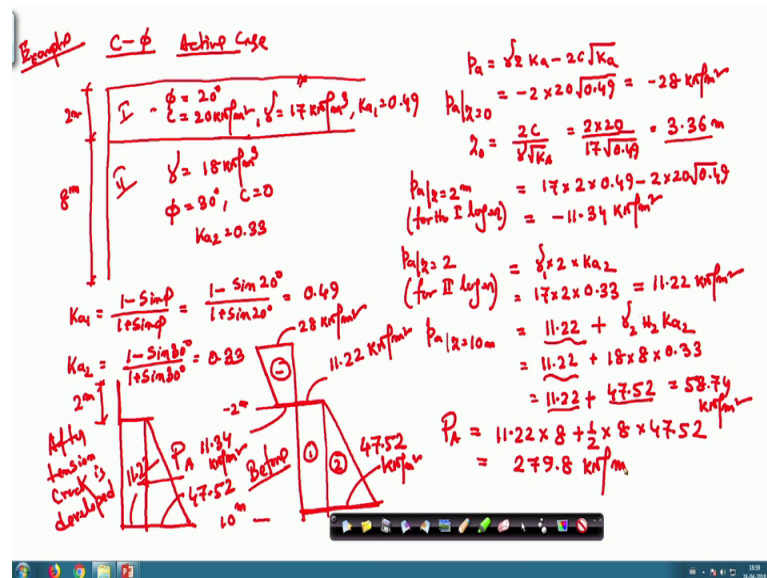
So, and that value will be equal to 50 into 0.31, ok; so, 50 into 0.31 that is 15.5 kilo Newton per meter square. Now, for the next one, this is for the first layer, that I have already calculated. This is 14.51 kilo Newton per meter square, that 14.51 will continue in this layer. So, this is also 14.51 kilo Newton per meter square, then for this soil and this then for the water. So, for the soil and for the water; so, for the water this will be 70 kilo Newton per meter square, and for the soil this will be again 21.1 kilo Newton per meter square. So, this except this 15.5 all the other stresses I have already calculated, this problem this is 14.51, this is 21.2 this is 70; except this additional force or this due to surcharge is added additional stress.

So, finally, the  $P_A$  value also we can calculate from here. So now, instead of 4 there are 5 components. So,  $P_A$  value will be here and if I take this is one triangle so, then this will be 4 components. So now,  $P_A$  value will be this is for the rectangle; this will be 15.5 into 10, ok. Plus, the first circle first triangle half into 14.51 into 3, then the next one this rectangle. This is so, this is first, second, third and together this one it is 4. So, this is for the first case first one this is for the second one, and for the third one it is 14.51 into 7, this is the rectangle for the third one and for the fourth one again half into 14.51 plus 70 into 7. This is for the fourth one half into sorry this one 21.

This is for the 14 into 3 half, this is 14 into 7 ok, and this one we have added. This is 21.1, 21.1 plus 70 into 7, 21 and the final value is 597.2 kilo Newton per meter ok. So, this is the case 3 when we have consider a surcharge, and then we can calculate what would be the stress or what would be the force  $P_A$  acting on the wall, ok.



(Refer Slide Time: 28:56)



So, next problem that I will consider, next example problem where I am considering for the c phi soil ok; till now I have solve one particular homogeneous layer with a phi soil. Now I will consider c phi soil, and we have to consider or we have to calculate what is the active earth pressure or P A. So now, all these things I am considered active ok. This is also active, active case.

So, the problem is that, we have a wall with 2 different layer, this is a wall 2 meter layer, and this is 8-meter thickness ok. Here it is 5 value is 20 degree, c value is 20 kilo Newton per meter square. And we have a unit weight is 17 kilo Newton per meter cube for the first layer. And second layer unit weight is 18 kilo Newton per meter cube phi, phi value is 30 degree and c value is 0. Water table effect is not considered, this is the ground level which is far below, ok.

So, first one we have 2 different layer, this is layer 1 this is layer 2 and 2 different phi. So, we will calculate K a 1 is 1 minus sine phi divided by 1 plus sine phi. So, this is equal to 1 minus sine phi is 20 degree, and 1 plus sine 20 degree. So, that is equal to 0.49. And K a 2 is also 1 minus sine 30 degree 1 plus sine 30 degree so, this is equal to 0.33. This is K a 2 is 0.33, this is K a 1 is 0.49. So now we have the we have the P A expression for the c phi soil is gamma z K a minus 2 c root K a for the active case. Now this is the expression. So, first layer the P A at z equal to 0. So, that will be this is 0

minus  $2c$  value is 20 and root 0.49. So, that value is minus 28 kilo Newton per meter square; so, minus 28 kilo Newton per meter square.

Now, we can calculate: what is a  $z_0$  value;  $z_0$  value is this expression  $z_0 = \frac{2c}{\gamma \sqrt{K_a}}$  this is the  $z_0$  expression. So,  $2$  into  $20$  divided by  $\gamma$  is  $17$  and root  $0.49$ , ok. So, that  $z_0$  value is coming to it  $3.36$  meter. So, I am, but the first layer is  $2$  meter ok. So,  $z_0$  will not cross this layer so,  $z_0$  will not cross this layer. So now, let me draw the calculate the other stresses also.

Now, the  $P_A$  at  $z$  equal to  $2$  meter for the first layer, for the first layer; as I mention, that if we have a  $2$  different layers we have to calculate the stress for a intersection of or inter-phase of these  $2$  layer for the first layer as well as the second layer. So,  $z$  equal to  $2$  for the first layer, it will be  $\gamma$  is  $17$   $z$  is  $2$   $K_a$  is  $0.49$  minus  $2$  into  $20$  minus  $0.49$  ok. So, this is equal to minus  $11.34$  kilo Newton per meter square. Because it is why it is also still minus, because your  $z_0$  value is  $3.36$  and your thickness of this layer is  $2$  meter. So, this layer totally will be the under the minus negative stress. If this layer thickness is greater than the  $z_0$  value, then there will be a positive stress, otherwise it will be negative stress.

Now, the  $P_A$   $z$  equal to  $2$  meter for second layer will be; now second layer is only  $\phi$  soil ok. So, the first layer stress will act as a surcharge and first layer stress is  $\gamma$  into  $2$  meter,  $\gamma_1$  and that will be  $K_a$   $2$ . Into  $K_a$   $2$  and this is a surcharge coming from the first layer. So, the  $\gamma$  value is  $17$  into  $2$  into  $K_a$   $2$  is  $0.33$ . So now if I draw this diagram that this is the  $2$  meter minus  $2$  meter this is minus  $10$  meter and  $0$ . So, here this is the diagram for the first layer. So, this value is  $28$  kilo Newton per meter square, and this one is  $11.34$  kilo Newton per meter square. This is for the first layer  $2$  meter and this is total negative.

Now, for the second layer it is the sandy soil, and for the second layer this is  $K_a$   $2$  into the surcharge coming from the first layer. And surcharge coming from the first layer is  $\gamma$  into  $2$  meter this is  $\gamma$  into  $2$  meter. So, this value is  $11.22$  kilo Newton per meter square ah. Remember that that will be positive stress, because it is only  $\phi$  soil. So, that will be the positive stress and it will go this direction. And that will continue up to this point ok. So, this one is  $11.22$  kilo Newton per meter square.

Now, P A at z equal to 10 meter, that mean at the end of this wall or the base of the wall. So, that a 10 meter the this 11.22 will remain there, then the additional one, that because of this second layer the additional one I am calculating. So, this is the same as this rectangle. So, additional one will be this  $\gamma H^2$  or the  $H^2$  into  $K a^2$ , because it is phi soil; so, this will be 11.22 plus  $\gamma$  is 18  $H^2$  is 8 and  $K a^2$  is 0.33, ok. So, this is 11.22 plus 47.52. So, the total one is 58.74 kilo Newton per meter square.

So, this one is 47.52 kilo Newton per meter square. So, because this is for the rectangle, this is for the triangle. So, this is 11.22 is already there. And so, the total one this one is 58.74. So, you can calculate in this way. So, finally, the P A value finally, P A value I am calculating here. So, as I mention when you calculate the P A value we will not consider the negative one. So, I will consider only the positive one, this is one and the second one.

So, now as I mention that here also if it is mention that you draw the earth pressure before the tension crack is developed, then this will be earth pressure ok. And then if you consider you have to calculate the force then you have to consider the negative one. And if you have to this is the before the tension crack, now the after the tension crack is developed. So, this will be your earth pressure diagram, ok. This is the earth pressure diagram, this is after ok, this is after and this is before.

So, after this will be the 2 meter and this value is 11.22 and this one is 47.52. This is after this is before, and we are calculating the force after the tension crack is developed. So, I am calculating for this one. So, I am calculating this is the P A, ok. Generally, we do not consider this one, but if it is specifically mention that you determine the force before the tension crack is developed, then you have to consider the negative one also. But here we will not consider because we consider only the positive one.

So, positive one P A will be this is the 11.22 into the 8-meter height for the rectangle, then the triangle half into 8, 8 meter into 47.52. So, this will be 279.8 kilo Newton per meter, ok. And similarly you can calculate what will be the P A acting; that means,  $y$  bar also, you can calculate as I have already shown in the previous examples. So, you can calculate the point of application also from the base. One is considering rectangle; another is considering triangle. In triangular it is  $H$  by 3 for the rectangle it is  $H$  by 2, then you can calculate the  $y$  bar.

So, in the next class I will also I will solve few more problems, considering more number of layers  $\phi$  as well as  $c \phi$  soil. And then I will show you how to calculate the lateral earth pressure in those cases.

Thank you.