Foundation Engineering
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Lecture – 42
Earth Pressure – II

In the last class, I have discussed about the earth pressure theories and I was discussing about the active earth pressure. And, I have also discussed that what is the failure plane a direction and it is making an angle 45 degree plus phi by 2 with the horizontal. So, if I continue the same thing that as I have discussed that, if this is sigma plane sigma axis and this is the tau axis. Sigma is the normal stress, this is normal stress and this is shear stress and this is the Mohr coulomb failure envelop.

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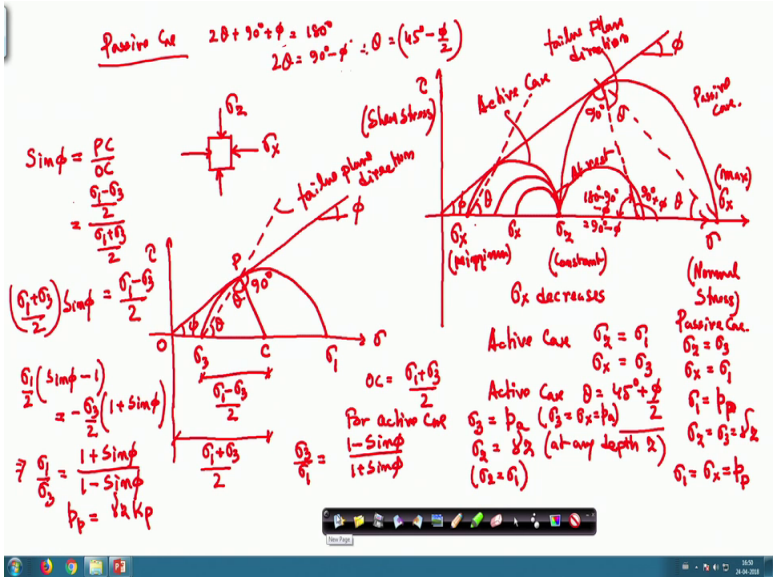

And we are discussing these things for cohesion less soil. So, that is why Mohr coulomb failure envelope will pass through the origin and it is making an angle phi with the horizontal. So, phi is the friction angle. So, as I am discussed as I have discussed that for a at rest condition if this is the Mohr circle; that means, this is sigma z and this is sigma x ok. And that is I have discuss that if we are taking an element which is making this a sigma z is the vertical normal stress and this is sigma x which is the lateral stress.

Now, as I mention this is at rest condition. Now, as wall moves away from the backfill then the stresses in the horizontal direction decreases, but this sigma z value will remain

constant. So, sigma z is constant and sigma x decreases ok. So, the sigma z a sigma x decreases and sigma z constant; so, sigma x decreases means that this sigma x value will shift towards the origin ok. So that means, next Mohr circle will be this one and this will be the Mohr circle and where this is the sigma x minimum ok. This is the minimum sigma z x, we cannot get less than that because this Mohr circle as already touch this Mohr coulomb failure envelope. So that means, as I have already mentioned that any Mohr circle will not cross this line. So, this is the lowest sigma x value.

And so, I have mentioned that for a active condition for a active case that your sigma z is equal to sigma 1 and sigma x equal to sigma 3. So, if you represent them in terms of effective stress then it will be bar or dash ok. So, sigma a z is sigma 1 because, in this plane there is no shear stress. So, the all the stresses are principal stresses and this is as I mention the principal stresses are sigma 1 and sigma 3. And so, I can write that this is the final Mohr circle where this is sigma 1, this is sigma 3 ok. And this is I am talking about this is the active case ok. And, here as I have already mentioned that if I draw join these failure this point where the Mohr circle touches the Mohr coulomb envelope or failure envelope and if I join this sigma x and this line. So, this will make an angle theta and I have drawn for a active case this theta value is 45 degree plus phi by 2.

So, I have already proved that that this is 45 degree plus phi by 2. So, this is the same line failure envelop and I am taking the, this last Mohr circle so, this is the sigma 3. Now, we have to prove what is the value of or we have to determine what is the value of active earth pressure. So, this is the center C and this is the origin O and center C if I join this C point with this point where this a line touches this Mohr circle or Mohr circle touches this line. So, actually on this point P this line or failure envelop is a tangent acting along this point P on this circle.

So, this angle is 90 degree and similarly you can also if we join these two points or this point so, this is the theta and this angle is also theta. So, this theta value is 45 degree plus phi by 2, that I have already proved and now sigma and tau. Now, if I take this triangle so, before you try this is the triangle so, if this is the Mohr circle. So, you know the radius we can determine by sigma 1 minus sigma 3 divided by 2. This is the radius of the circle and the center of the circle C from the origin is sigma 1 plus sigma 3 divided by 2.

So, I can write that OC is sigma 1 plus sigma 3 divided by 2 and the radius r of the circle is sigma 1 minus sigma 3 divided by 2. So, if we have a Mohr circle with sigma 1 and the sigma 3 are the two principal stresses. Now, from this circle we can so, this is the phi value we can write that that sin phi is equal to so, we can write that sin phi value which is equal to PC divided by OC ok. So, this is the PC and this is the OC because, this total angle is 90 degree, this is the perpendicular line on this failure envelop. So, we can write that sin phi is PC by OC.

And as I mentioned this is the failure plane and the direction ok. So, in the active case the failure plane will make an angle 45 degree plus phi by 2 with the horizontal that I have discussed. So, this is sin phi is PC by OC and PC is the, this PC is the radius of this circle. So, PC is the radius of the circle means this is sigma 1 minus sigma 3 divided by 2 and OC is the distance of this center from the origin O and OC is sigma 1 plus sigma 3 divided by 2 ok. So, I can write that sigma 1 plus sigma 3 divided by 2 into sin phi is equal to sigma 1 minus sigma 3 divided by 2. And, if I take all the sigma 1 this side so, sigma 1 divided by 2 will be sin phi minus 1 equal to if I take a minus sigma 3 divided by 2. So, this will be 1 plus sin phi ok.

So, I am taking I am just separating sigma 1 plus divided by 2 and sigma 3 divided by 2 from this equation. So, I can write sigma 1 divided by sigma 3 will be equal to because, here it is minus. So, I can write 1 plus sin phi divided by 1 minus sin phi because this is minus so, I am taking minus also this side 1 minus sin phi. So, the sigma 1 divided by sigma 3 will be 1 plus sin phi 1 minus sin phi. So, as it is mentioned that your for active case your sigma x is equal to sigma 3. So, I can write sigma 3 is equal to the active earth pressure because, that sigma x is acting in the lateral direction which is the lateral a earth pressure. So, I can write that sigma 3 here is equal to p a; because, the sigma x is acting in the lateral direction and for the active case sigma is equal to sigma 3. So, sigma 3 is equal to p a and sigma z will be gamma into z at any depth at any depth z. So, finally I can write that sigma 3 divided by sigma 1 is equal to 1 minus sin phi divided by 1 plus sin phi ok.

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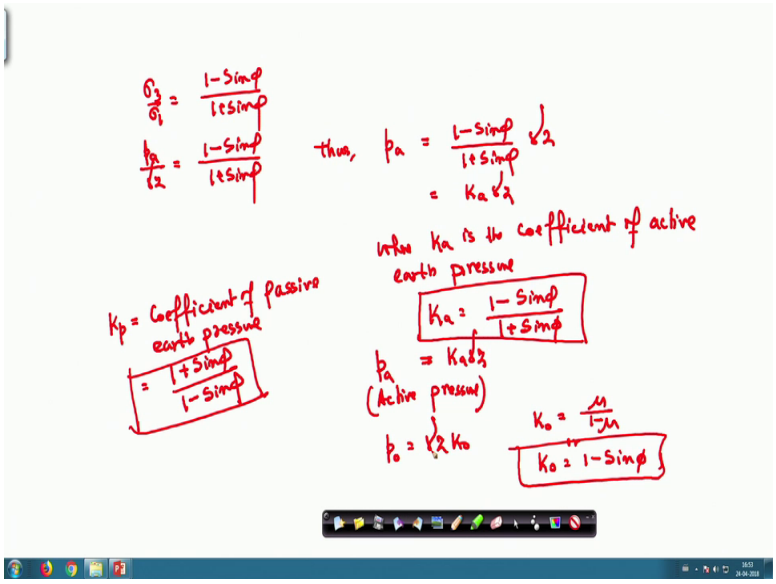

And, again further I can write that sigma 3 divided by sigma 1 is equal to 1 minus sin phi divided by 1 plus sin phi and sigma 3 is equal to p a and sigma 1 is gamma z. So, this will be 1 minus sin phi 1 plus sin phi thus, p a is equal to 1 minus sin phi 1 plus sin phi into gamma z ok. And, this we can write this is equal to K a gamma z where, K a is the coefficient of active earth pressure and K a is equal to 1 minus sin phi divided by 1 plus sin phi.

So, finally I can write that p a is the active pressure which is equal to K a into gamma into z, gamma is the unit weight of the soil and z is the depth at any depth from the ground surface within the soil ok. So, this way we can calculate the active earth pressure or active pressure which is K a into gamma z. And, as per this Rankine's theory this K a is 1 minus sin phi divided by 1 plus sin phi ok. Now, if I go back to the previous slide so, this is our active case.

Now, in the passive case, what will happen? The wall will move towards the backfill. Now, if wall moves towards the backfill then the lateral stress on the soil element will increase ok. So, if its moves away from the backfill, then there will be an extension in the element and the stresses lateral stresses in the element that will decrease. But, if wall moves towards the backfill then there will be a compression in the element so; that means, the lateral stress in the element will increase.

But, your sigma z value will remain same again, here also whether it is active a condition or passive condition your sigma z value will remains same. So, this is the sigma z value, it is it is remain same for the active case now, passive case also it will remains same. So, now, at suppose this is our, at rest Mohr circle. So, when it is moves away from the backfill then a sigma z remains same. So, this Mohr circle will shift towards the origin because your sigma x value decreases.

Now, if it moves towards the backfill now, sigma x value will increase. So, what will happen from the, at rest condition this sigma x value will start moving away from the origin. So, there will be it will it will move this side and there will be point where this sigma x and sigma a z will remain say I will be same. Then it will moves in this direction and there will be one particular Mohr circle for which you will get the maximum sigma x value, this sigma x value this will be the maximum ok. So, this is also sigma x value maximum as a, if you look at this active case where sigma x value is minimum and here it is maximum.

So, in case of passive it is maximum this is an active case, this is the passive case ok. So, remember that because here also these Mohr circle touches the failure envelop or this line. So, it we will we will not get sigma x value more than this, but this is the maximum sigma x. And, again if I join these two lines these two points and if this is also theta so, again this is the active passive case. So, in the passive case this theta value will be 45 degree minus phi by 2, you can prove that all this one also. Because, this is if this is the center if you draw a line if you draw a line so, this is the 90 degree this is the 90 degree. So, this angle is how much? This is the 90 degree, this angle is this is 90 degree, this is phi.

So, this angle is 180 degree minus 90 degree minus phi. So, this angle is 90 minus phi ok. So, this angle will be 90 plus phi fine because, this is 90 minus phi so, this will be 90 plus phi. So, now if I consider this triangle because this is also theta, if this is theta this will be theta because this is the radius of the circle. So, this is a one radius and this is the radius so, these two are same. So, now we can write for the passive case or for the passive case your 2 theta plus 90 degree plus phi by 2 is equal to 180 degree ok. So, your theta value is 90 minus phi by 2 a 90, 2 theta is equal to 90 by this is phi sorry and this will also be phi. So, this is 2 theta plus 90 plus phi that will be equal to 180. So, 2 theta will be 90 divide minus phi so, theta will be 45 degree minus phi by 2.

So, if you look at this thing that for the active case this failure plane is making an angle with horizontal is 45 degree plus phi by 2, but for the passive case it is 45 degree minus phi by 2. So, this is also failure plane direction ok. So that means, here remember that for the active case failure plane is making an angle 45 degree plus phi by 2 with horizontal and for the passive case it is making an angle 45 minus phi by 2 with the horizontal. So, now as I mentioned that for the active case your sigma z equal to sigma 1 and sigma x is equal to sigma 3. But, for the passive case for the passive case it is a opposite ok. For the passive case your sigma z is equal to sigma 3 and sigma x is equal to sigma 1 ok.

Because now, in the active case you see that your sigma z is greater than sigma x, but for the passive case your sigma x is greater than sigma z ok. You can see that for the passive active case sigma x is a less sigma z is greater than sigma x, for the active case and for the passive case sigma x is greater than sigma z. So, for the passive case sigma z is equal to sigma 3 and sigma x equal to sigma 1, sigma 1 and we can say that that for the passive case your sigma 1 is equal to p passive ok.

And, again sigma z is equal to sigma 3 which is gamma into z ok. So, here I have seen that sigma 3 is equal to sigma x equal to p a, similarly here it is sigma 1 is equal to sigma x is equal to p ok. So, remember that for active case sigma 3 is p a lateral earth pressure and in that case your sigma z is equal to sigma 1 and that is sigma z is equal to sigma 1 which is gamma into sigma z equal to sigma 1 which is gamma into z. But, for the passive case your sigma 1 is equal to p p and sigma z is equal to gamma into z which is sigma 3 because, here your active case sigma z is greater than sigma x, but for the passive case sigma x is greater than sigma z. So, that is why sigma z is here sigma 3 for the passive case and sigma x is sigma 1. So, now we have this expression sigma 3 or sigma 1 divided by sigma 3 is 1 plus sin phi 1 minus sin phi. So, passive case I can write now its sigma 1 is equal to p p.

So, I can write p p is equal to sigma 3 is nothing, but gamma into z gamma into z into K p ok. So, what is K p? Your K p is the coefficient of passive earth pressure or passive pressure and this K p is 1 plus sin phi divided by 1 minus sin phi, you can see that this is 1 plus sin phi 1 minus sin phi. So, this is the K a expression and this is the K p expression. So, you can write from this correlation that K p is 1 by K a or K a is 1 by K p ok. So, but these are the two coefficient as per the Rankine's earth pressure theories and I

have already mentioned that K 0 you can write by mu 1 minus mu or K 0 you can write 1 minus sin phi by Jockey's expression.

So, these are the three expressions for three different conditions of earth pressure. For the at rest K 0 is 1 minus sin phi, for active 1 minus sin phi divided by 1 plus sin phi. And, for the passive coefficient of earth pressure 1 plus sin phi divided by 1 minus sin phi ok. So, these are the as for the Rankine's earth pressure theory and before I go to the next part, just remember that in case of a active your sigma z is equal to sigma 1 and sigma x is equal to sigma 3. But, in case of passive sigma z is equal to sigma 3 and sigma x equal to sigma 1 ok.

So, you can see from this Mohr circles these two Mohr circles ok. So, this is very important and in case of active a failure plane is making an angle 45 degree plus phi by 2 in the horizontal. And, for the passive failure plane is making an angle 45 degree minus phi by 2 with the horizontal ok. So, these are the all expressions for active and passive condition. So, again the passive is gamma z into K p and active is also gamma z into K a and at rest p 0 is equal to gamma z into K 0. So, all the cases it is gamma z multiply with the coefficient of active or passive or at rest earth pressure ok.

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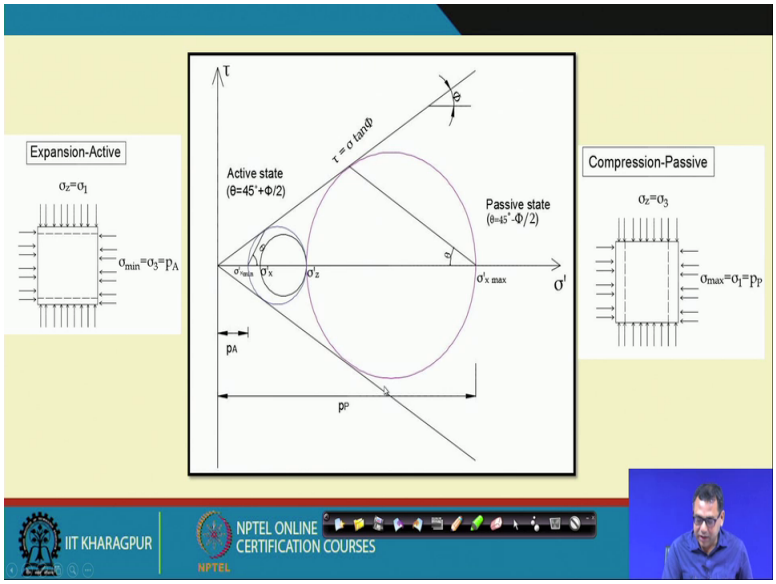

So, next part that I will discuss so; this is the figure that I have already discussed. So, in case of active it is expansion, in case of passive it is the compression. So, here sigma x sigma x value increases, here sigma x value decreases and this is the two a active and

passive cases. And, this active cases theta value is 45 plus phi by 2, for the passive case this theta value 45 degree minus phi by 2. So, these things I have already discussed.

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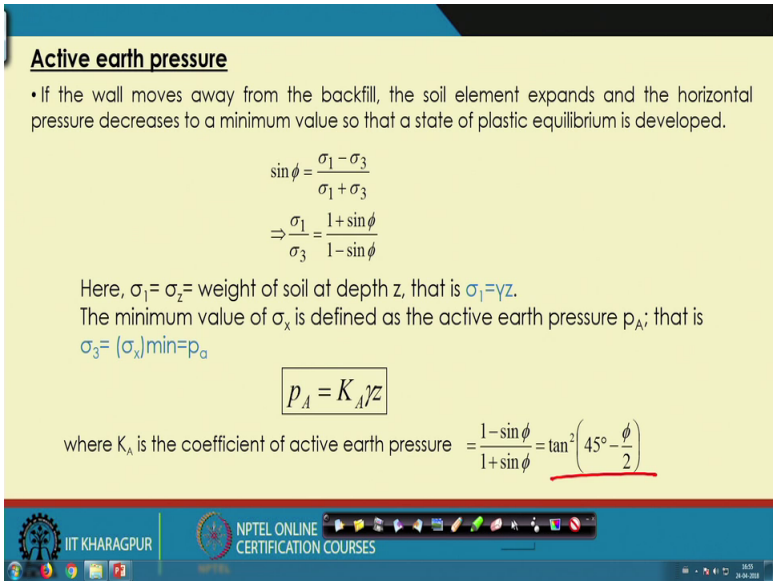

So, and this is the expression I have already discussed, this is your p A is K into gamma z, K is equal to 1 minus sin phi divided by 1 plus sin phi or you can write in this form tan square 45 degree minus phi by 2. You can write with this form also tan square 45, which is the same thing in different form tan square 45 degree minus phi by 2 ok.

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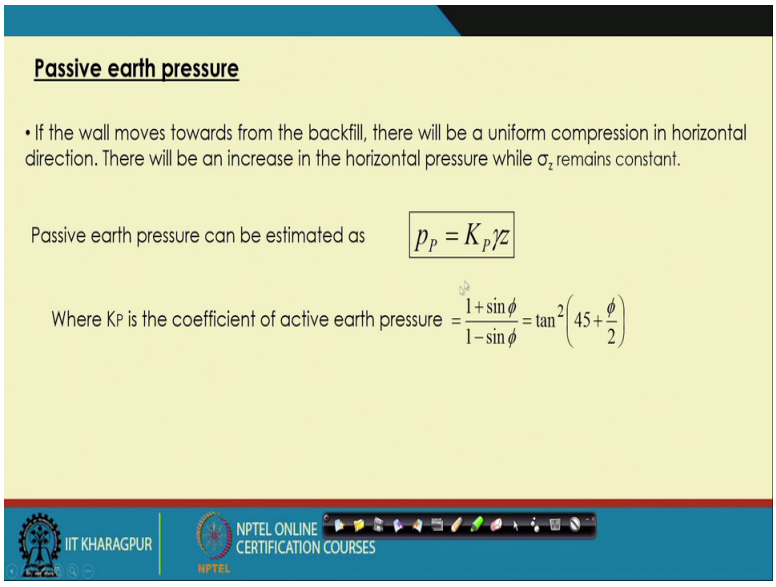

And, for the passive K p into gamma into z and K p is 1 plus sin phi divided by 1 minus sin phi or tan square 45 degree plus phi by 2 ok.

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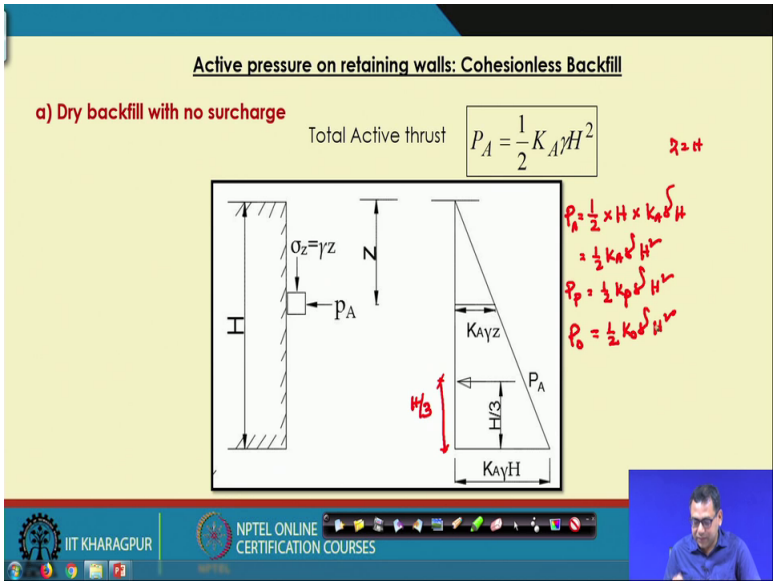

Then the next one that and then this how we will get this is the pressure. How we will get the force? The same thing that it is making an triangle so, area of this triangle. So that means, the half height is H is the height of the wall is H and into this is K A into gamma into H, here z is equal to H ok. So, this is the P A is equal to so, I can write half K A gamma H square and it will act with an height of H by 3 from the base of the wall ok.

If it is passive then P p will be half into K p gamma H square, if it is P 0 at rest this will be half K 0 gamma H square. But, all the three cases this force will act from the a act at H by 3 from the base of the wall where, H is the height of the wall ok. So, next part that here all the derivations, all the things I have derived without any surcharge ok, that mean I have consider only the backfill soil weight ok.

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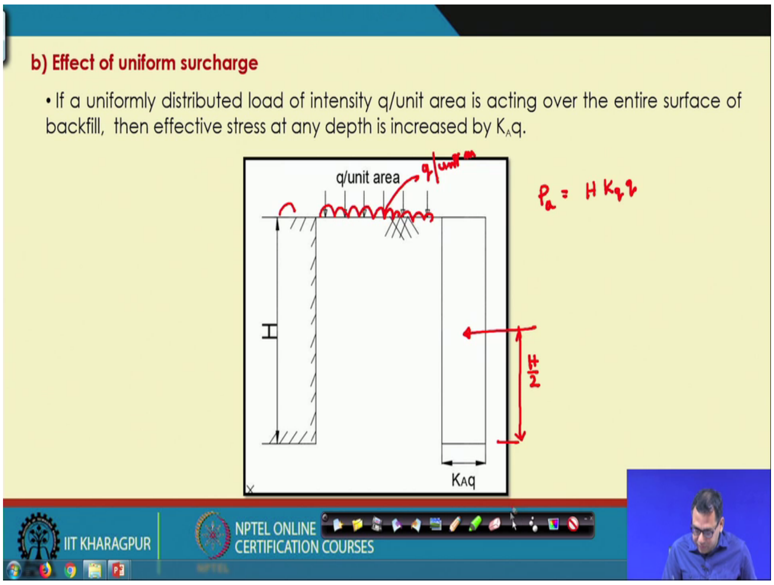

Now, if we have a surcharge here so, this is the surcharge here is surcharge is applied and that surcharge is q per unit area or unit meter; so, because it is in the plane strain condition. So, in that case your distribution of the earth pressure will be K A into q ok, K A is the coefficient of active earth pressure; if it is passive then K p into q ok. So that means, here K into q and the force will be definitely area of this rectangle. So, in that case your P a active will be area is H into K a into q ok. So, that is the force and again this will act because it is a rectangle. So, it will act at H by 2 from the base or from the top so, this is this will act at H by 2.

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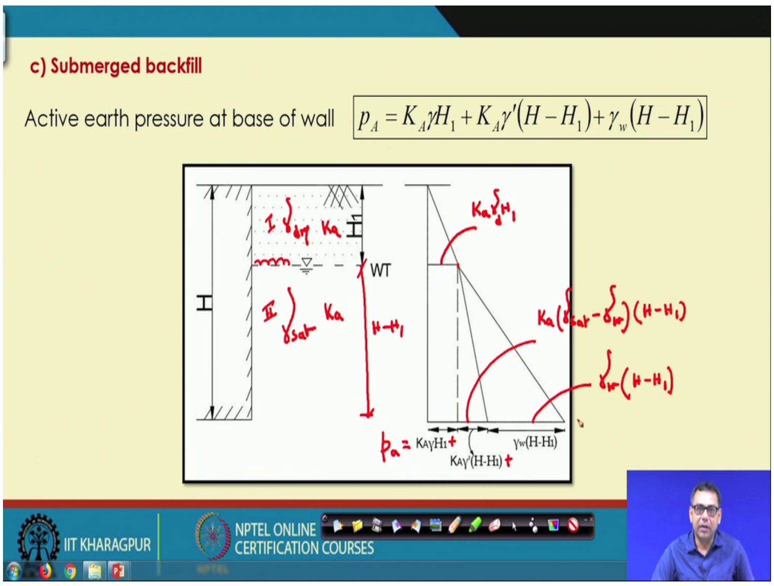

And so remember so, next one that is that if there is a water table is there ok, all these two cases I have discussed without considering the water table effect. Now, if there is water table, then what will be the changes ok? So, now for the first case suppose we have a water table at a height of H 1 from the ground level. So, up to H 1 this value is simple, if I am talking about K A active case which is the submerge backfill that water table is at a height of H 1 from the from the top or ground level. So, this value is K a small a or you can write capital A both are same; K a into gamma into H 1 that I have discussed and here this will be the gamma dry or gamma bulk and here it is gamma sat ok.

So, I can write this is K gamma dry or gamma bulk and then this weight of the soil will act as a surcharge because, if I know now if I take two different layers say this is one layer, this is another layer. So, for the first suppose this is a one layer and this is the second layer. So, for the first layer this is the earth pressure. Now, on the second layer this first layers load will act an surcharge and that surcharge is gamma d into H 1. And, if this surcharge is acting over here so, that will be K a into that surcharge because, the soil is same so, that is why here also it is K a, here also it is K a.

If K a changes later on I will discuss that then there will be a different story, but now both it is the same a soil. But, to assume it is two different layer; one is the above water table another is the below water table and the both the cases your K a is same because, the phi value will remains same about water table and below water table. So, your K a value will also remain same. So, I have consider this is K, now for the first layer it will be K a gamma d into H 1 the soil above the water table and that soil will act as a surcharge this soil below the a water table.

So, in that surcharge will be gamma d into H 1 into the K a so, that is the case. So, it will act as surcharge rectangular, then it will additional part for the earth pressure below this a soil ok, for this soil below the water table. So, this one will be this value will be K a into gamma sat minus gamma w because, this is below the water table into this height H minus H 1 because, this is H minus H 1. So, now up to this the earth pressure due to the soil. Now, this third-one or the second triangle this is the earth pressure due to water ok. So, these earth pressure due to water will be w into H minus H 1 because, the water table height is also from the base of the wall is H minus H 1.

So, now this is the total earth pressure diagram. This is above the water table where you have to consider the gamma bulk or gamma dry. Then this is the below water table where first one is the surcharge of the upper layer, second-one is the additional stress due to the a lower layer and third-one is the stress due to the water. So, the final stress you have to add this one plus this one and this one. This is the p a acting at the base of the wall ok. So, this is the effect of water table.

Now, in the next class I will discuss if we have a inclined backfill. Now, the backfill in the soil we have assumed that it is perfectly horizontal, even the initial Rankine's theory also assumes that backfill is perfectly horizontal. But, later on this theory is been extended for inclined backfill also. So, that thing we will discuss in the next class.

Thank you.