Foundation Engineering
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Lecture – 41
Earth Pressure – I

So, this class I am going to start a new chapter that is Earth Pressure or the lateral earth pressure. Now, what is a lateral earth pressure? When we are constructing any type of geotechnical structure, where that we have to resist some soil or earth then that earth will give you pressure ok.

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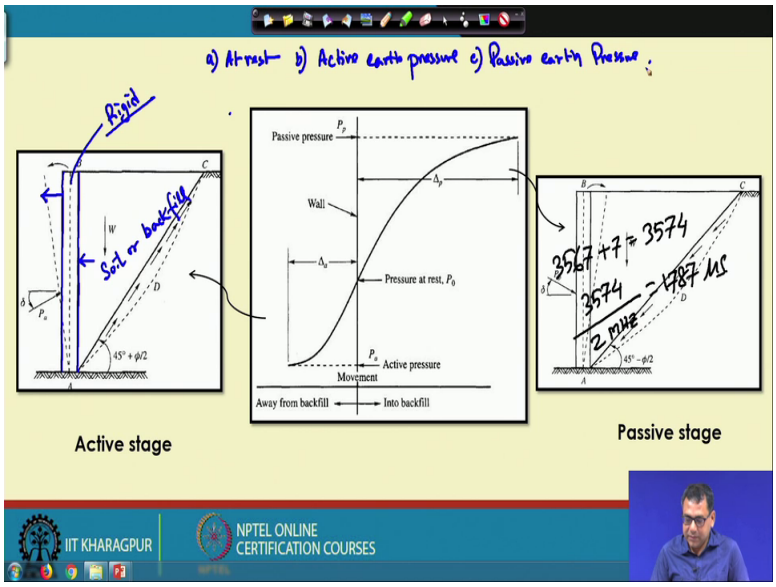

So, that pressure we are talking about that the lateral earth pressure. If we look at this figure here this here, this is the wall or a structure which is supporting this soil ok. So, this side is soil or it is called backfill and this side there is nothing. So, this soil will give you a pressure on the wall. So, that pressure is called the earth pressure. Now, this earth pressure can be three types ok. So, this earth pressure one can be at rest. So, earth pressure can be three types that is at rest. What is at rest? At rest means this structure or the wall. So, this is a if I say this is a rigid wall. So, this wall is not moving what is the; that means, when you are this soil is applying stress on this wall so, wall is so, rigid that it is not moving. So, there is no movement in the wall. So, it is at rest condition.

So, in that condition the earth pressure that is applying on the wall is called the earth pressure at rest ok. So that means there is no movement of the wall and the second one is active earth pressure. What is active earth pressure? Now, when this soil is giving a stress or the pressure on the wall so, this if the wall moves in this direction ok. So, the direction means direction to the direction of the force because, you are force direction is this way; wall is also moving in that way.

So, in that that portion of earth pressure is called the active earth pressure and there is a third type of earth pressure which is called there is the passive earth pressure ok. Now what is passive earth pressure? Now, if you that here direction of force is here. This direction as usual because this is the backfill or soil and the soil is giving pressure on the wall. But, your wall is moving this direction, this is the wall movement direction and this is the pressure direction ok.

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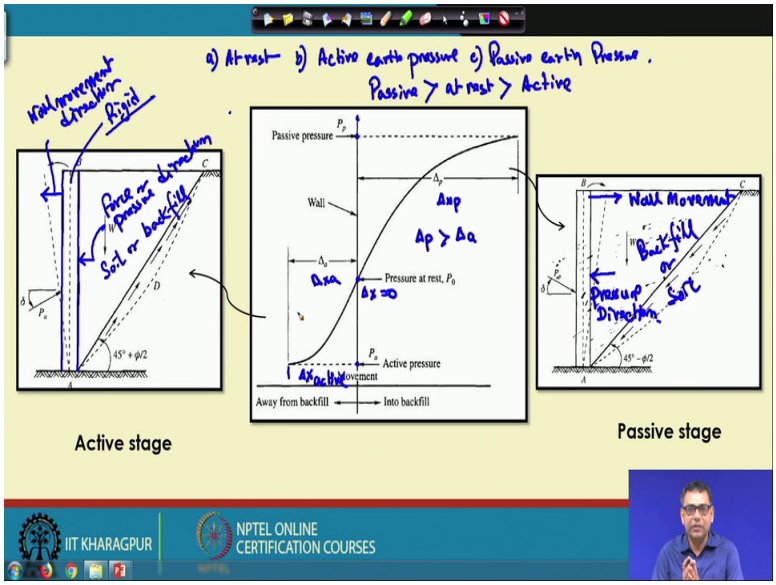

So that means, if your wall moves opposite to the direction of the force then it is called the passive earth pressure. So, there is three types of earth pressure: one is earth pressure at rest; that means, wall is so rigid it is not moving any direction. So, that is at rest, then if the wall moves towards the direction of the force; that means, your this is the wall movement direction and this is the force or pressure direction.

So, it is moving towards the direction of the force then it is called active earth pressure; if wall moves towards the opposite to the direction of the force; that is called passive

earth pressure. In other way active and passive force can be defined as that if your wall moves away from the backfill. So, this is the backfill wall is moving away from the backfill then it is called the active earth pressure. If your wall moves towards the backfill, this is the backfill it is wall is moving towards the backfill, this is then this is called the passive earth pressure.

Now, you can see that earth at rest the pressure when this wall is subjected to from the soil and the pressure where wall is subjected to during the active will not be same. Because, at rest condition wall has to be very rigid because, it is this force you have to resist such that there will be no movement. But, in the active condition wall is allowed to move so; that means, some stress is released ok.

So, your pressure at rest condition is more than the pressure at active condition because, at the active condition you are allowing wall to move away from the backfill or towards the direction of the force. So, your pressure is released because you are applying some movement ok. At rest condition you are not applying any movement. So, you have to resist the full pressure which is acting, which is coming from the soil. So, at rest condition your more pressure is acting as compared to the active condition.

But, if it is a passive condition then the wall is moving towards the backfill so; that means, it is opposite to the direction of the force. So, the pressure that is applied on the wall is greater than even the at rest condition because, it is towards the direction at opposite to the direction of the force. So that means, your active earth pressure is the lowest earth pressure, the wall is subjected to then at rest, then passive. So, I can write that your passive earth pressure is greater than at rest, is greater than active ok. So, active will give you the lowest earth pressure, then at rest, then passive. So, if you look at this figure this middle figure here, also you can see that this is the x axis is the wall movement and y axis is the pressure which is acting on the wall.

So, you can see this is the wall movement for the active and this is the pressure which is lowest. So, and there is the amount of wall movement, this is the amount of wall movement del x for active ok. This amount of wall is moved due to the active pressure or to develop the full active pressure you need this much of the wall movement. And, in the at rest you can see there is no wall movement, your wall movement del x is 0. But, this pressure at rest condition is greater than the active and then the passive you can see this

passive force is maximum and you need this much of the wall movement x delta x p, this is delta x or delta x a.

So, you can see from this figure also that your delta p or delta x p is greater than delta a, which means that to develop this full passive force you need more deformation compared to develop the full active force ok. So that means, the active force deformation is or the active stage deformation is lower than the passive stage deformation. So, that is the or wall movement ok. So that means, here this is the away from the backfill. This is towards the backfill which is passive; away from the backfill which is active. So, now these are the three pressures active, at rest and passive. So, we will discuss one by one all these three pressures, how we will calculate these three pressures on a retaining wall or the earth retaining structure.

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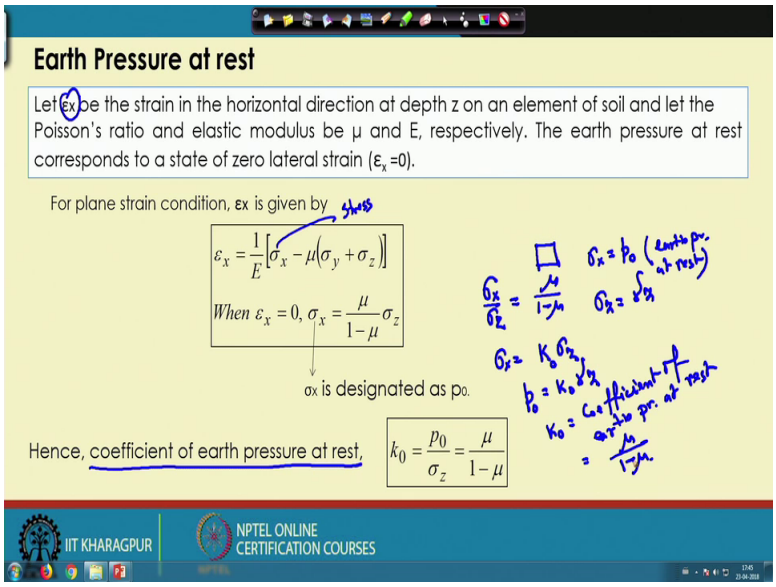

The first is the active first is the earth pressure at rest. So, now if this is the strain ok; this is the strain epsilon x is the strain of a horizontal direction at depth z of an elements. Suppose this if this is the element so, we have a strain at the horizontal direction of an element and then let the Poisson ratio and the elastic modulus is mu and E. Then the earth pressure at rest corresponding to that strain zero lateral earth strain will be will be the earth pressure at rest.

So that means, here if I write this expression this is the strain expression this epsilon x if the strain expression and this sigma are the stress. So, this is xy because this is the

retaining wall problem or earth pressure theory which is plane strain problem ok. So, plane strain problem so, that so we are taking x xyz. So, these are the three stress component that we are considering that sigma x, sigma y and sigma z so, this is the expression. So, I am not deriving this expression you will get this expression from any standard book that what is the expression of x so, in terms of E and stress. So, strain expression in terms of E and stress.

So, now when this strain is 0 so, your sigma x will be this ok. So, this will be the mu 1 minus mu sigma z ok, if your strain is equal to 0. Now, this the way we can determine that earth pressure coefficient. Now, if I write that sigma x by sigma z is equal to mu 1 minus mu and sigma z will be the earth pressure at sigma x, will be the earth pressure at rest. So, I can write that sigma z is the at any point gamma into z ok, that is the vertical stress which is acting. So, I can write that the K sigma x is equal to K into or K 0 into sigma z.

Now, this is the earth pressure at rest which is K 0 into gamma into z, this K 0 is called coefficient of earth pressure at rest ok. So, this is the coefficient of earth pressure at rest. So, which we can calculate by mu 1 minus mu, if you know the mu you can determine what would be the coefficient of earth pressure at rest. If I know the coefficient of earth pressure at rest, if I know the unit weight of the soil so, at any depth we can calculate what will be the earth pressure at rest condition will act in a one or in a earth retaining structure ok.

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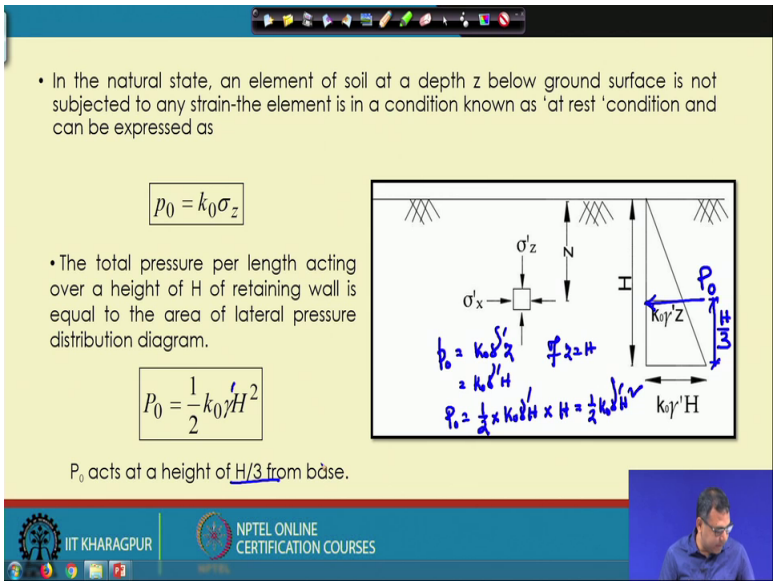

So, we can write the similar thing is explain here that, this is the vertical stress is acting at any element ok; sigma z and this is sigma x, a bar means it is in terms of effective stress ok. So that means, here your p 0 is equal to K 0 into gamma into z or gamma bar into z. So, if it is linearly varying with z because, if z increase because this is constant if it is a homogeneous soil and K 0 is also constant for a soil. So, it is a function of z. So, we can write that at any if your z is equal to H so, we can write K 0 gamma dash into H ok. So, the force that will act P 0, this is small p zero is the stress. So, total force will be the area of this triangle ok.

So, this area of this triangle will be P 0 is the half into this base K 0 gamma dash H into this height H ok. So, finally we can write half K 0 gamma dash H square a square ok that is the thing half k 0 gamma or gamma dash H square. Gamma means there is no water table and gamma dash this in terms of effective stress or we are talking about we are considering the water table effect ok. And remember that this P 0 at a height of H by 3 from the base of the wall or the retaining structure. So, that is the H by 3 from the base. So, this way we can calculate the earth pressure and the earth force due to this lateral earth pressure at rest condition.

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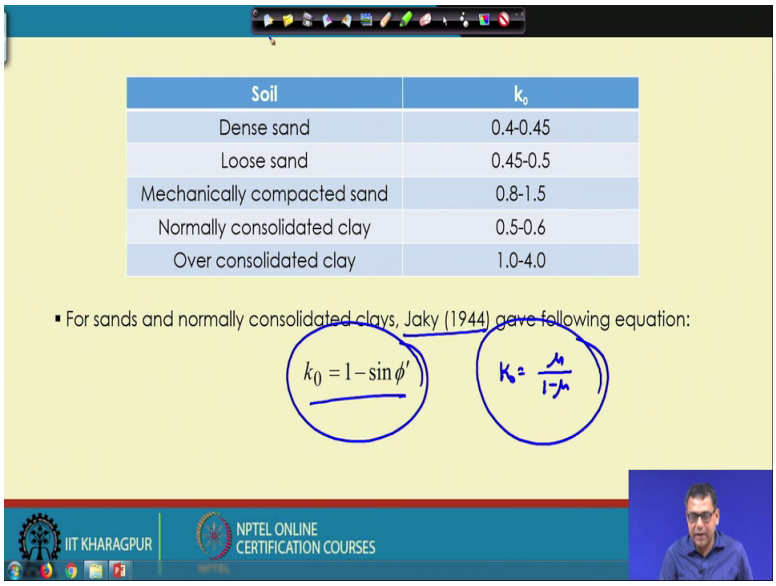

So, these are the some k 0 values which is given for the dense sand 0.4 to 0.45, loose sand 0.45 to 5. This compact mechanically compacted sand mean, it is compacted and 0.8 to 1.5, normally consolidated 0.5 to 0.6, over consolidated clay 1 to 4. And another way one expression is k 0 expressions is you know that k 0 expression is mu 1 minus mu, mu is the Poisson ratio. Another way also Jaky 1944, he has given one expression, that if you know the phi value of the soil you can also calculate the k 0 that is 1 minus sin phi.

So, either you can use these expression or these expressions to know the k 0 value or if you have this table from this table also you can get the k 0 value. If you do not have any mu value or the phi value available, but these if these values are available you can use either this expression or these expression to calculate the coefficient of earth pressure at rest. And once you know the k 0, then you can calculate what is the pressure and the force is acting on a retaining structure earth retaining structure.

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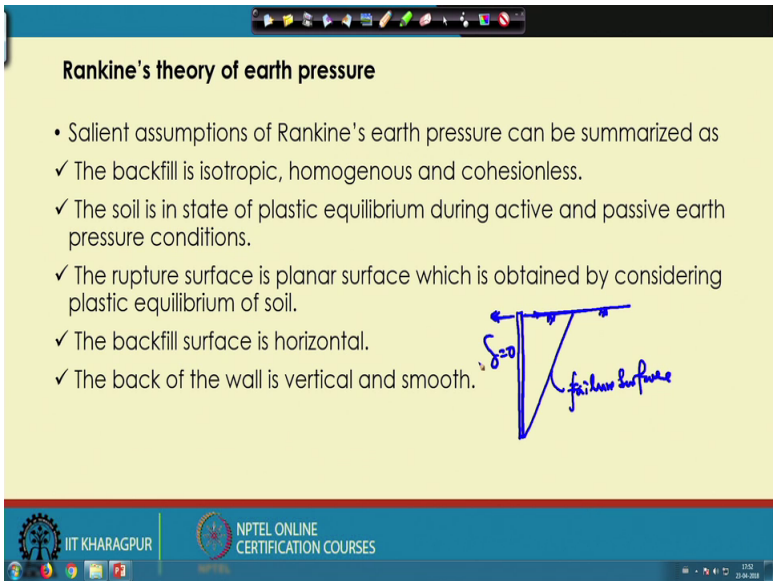

So, next one that will discuss about the active and passive earth pressure. So, there are two theories which are very popular, the first one is the Rankine's theory of earth pressure. So, these are there are some assumptions behind these theories. So, the first assumption is backfill is isotropic, homogeneous and the cohesionless. So, the backfill means the soil which is retained by the earth retaining structure so, that soil is cohesionless ok. Initially earth pressure theory proposed by the Rankine where the soil is cohesionless ok, let me c value is 0 ok. And there is homogeneous, if there is no layer soil its homogeneous and it is isotropic.

And then the soil in is state of plastic equilibrium during the active and passive pressure condition. So, it is in the equilibrium condition and the rupture surface is planar surface which is obtained by considering plastic equilibrium soil of soil. So, rupture surface is planar means if this is your wall so, when this wall moves either this direction or this direction. So, this portion of the soil if suppose if this wall moves this direction. So, what will happen? This portion of the soil will also move and there is and if this wall moves excessive amount then, this soil will also fail because it is moving this soil will also try to fail. So, this failure surface it is assume is planar ok. This is the ground surface through this is the rupture surface or the failure surface.

So, it is as per Rankine it is assumed that this failure surface is linear; that means, if the soil moves in this direction so, this portion of the soil within this wedge will also try to

fail along this surface ok. So, that is the failure surface is linear and backfill this is horizontal ok, this is the backfill this is perfectly horizontal and backfill wall is smooth and is vertical. So that means, this is the wall that is also perfectly vertical and this is smooth, smooth means the friction between the soil and the wall is 0. So that means, here delta value is equal to 0 ok. So, delta value is friction between the soil and the wall. So, that is also assumed as a 0.

So that means, what are the assumptions? Your backfill surface is perfectly horizontal, it is a planar failure surface, wall is perfectly vertical and it is smooth. That means, delta value is 0, friction between the soil and the material is 0 and the soil is in plastic equilibrium and backfill is cohesionless. That means, c is equal to 0 and the soil is also homogeneous ok. So, these are the assumptions against the Rankine's theory.

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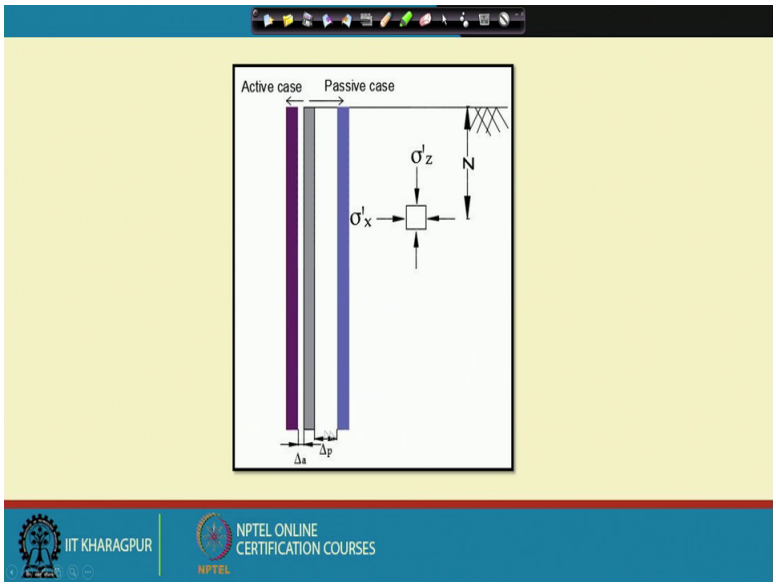

So, again this is the two cases as I have discussed one is active, one is passive. So, you can see this is the deformation of active case is less than the deformation of the passive case.

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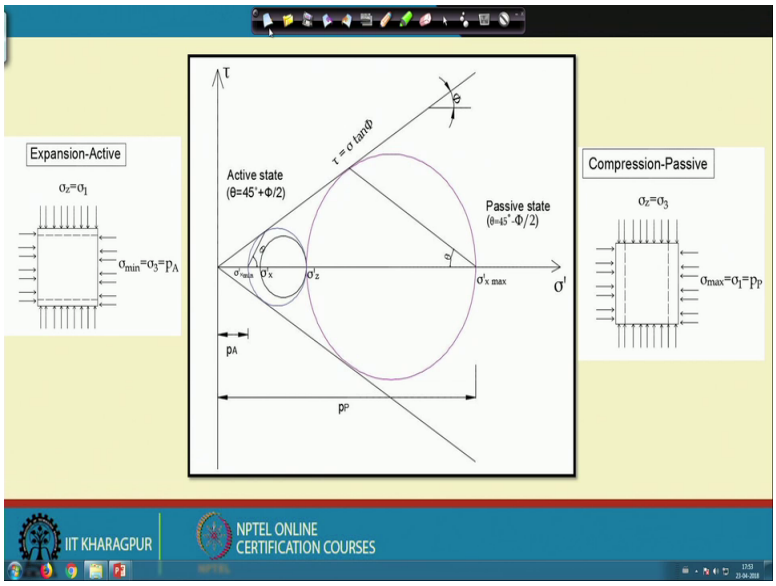

Now, we will discuss about the active and passive cases. So, this is the two cases: one is active, another is passive. So, I will discuss in terms of your Mohr circle, that first we will talk about the active case ok.

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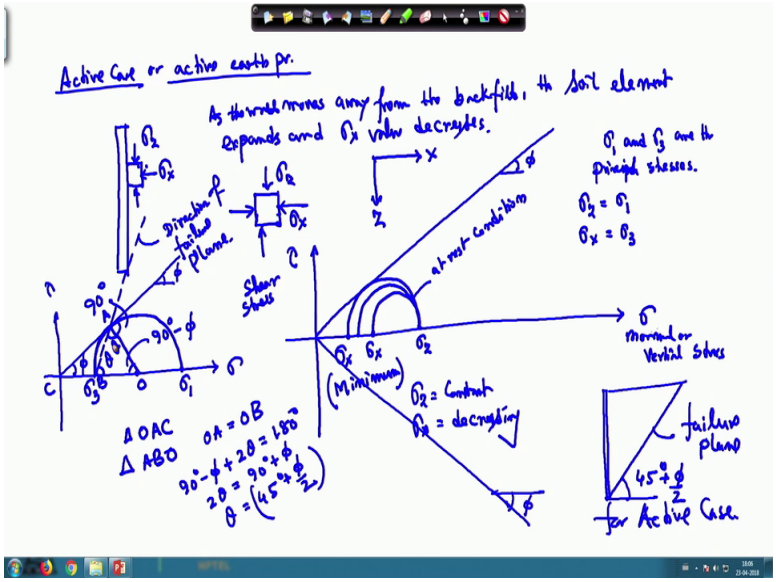

So, it is the active case or active earth pressure. So, suppose this is a wall and you have a element here or you can say that element is on the wall and here this is sigma z, this is sigma x and this is this direction and this direction also there is a limit. Now, when active condition means that as the wall away from the so, this is very important that as the wall

moves away; now if I take the element. So, this is sigma z sigma x because, this is the z axis, this is the x axis ok, this is x, this is z.

So, as wall moves away from the backfill, what will happen this element will expand and the sigma x value will decrease. And, as I mentioned that sigma x mean lateral earth pressure and if it is at rest conditions you more pressure is acting and as you are allowing some deformation so, pressure will be released. So, same thing this pressure will be released. So, now if you look at this figure here it is in active case it is expand. So, this pressure on this wall will release. So, now if I draw a Mohr circle here; so, you are drawing a Mohr circle, this is sigma x and this one is tau or this is sorry, this is sigma vertical stress and this is the normal stress shear stress.

So, this is shear stress, this is normal or vertical stress axis and you can see in this element there is no shear stress is acting ok. So that means, all the vertical stresses are principal stresses. So, what is principal stresses? Principal stresses means a plane, if you are shear stresses are 0 then the normal stresses that is acting on that plane is called the principal stresses. So, these all are the principal stresses because your shear stresses is shear stresses are 0 ok.

So, now at suppose at rest condition you are this element is at rest condition so, this will be the Mohr circle at rest condition. So, this is your, this will be the sigma x and this will be sigma z ok, this is sigma z and this is sigma x. So, and this is the Mohr circle and this is if this is the Mohr coulomb envelop and this angle is phi. Now, this is the Mohr circle at rest condition.

Now, your wall starts moving, if it starts moving at a point your sigma z vertical stress that will not change, sigma stress a sigma z is constant because, at a point your sigma is suppose gamma into z. So, that will not change it is constant, but your sigma x that the lateral stress that will reduce ok. So, next Mohr circle it is because, it is sigma x reduce means your sigma x point will shift towards the origin because, it is reducing and sigma z will remain same. So, the next Mohr circle will be something like this is the.

And this is the sigma x is changing and sigma z because, sigma z is constant and sigma x is decreasing ok, sigma x is decreasing. So, there will be a situation when your Mohr circle will touch this envelop; that means that is the limit because after that because any Mohr circle will not cross this line because, this is your failure envelope. So, if any Mohr

circle touches this line means if there is a failure. So, remember that any Mohr circle will not cross this line.

So, this is so that means, there will be a limit or minimum value of sigma x at which your full active stress or active condition is develop, after that there will be failure. Because, this as I mention it is in equilibrium conditions you are just considering it will not fail, just it is in equilibrium condition. So, the Mohr circle this is the lowest sigma x value that is the minimum sigma x value. So, you cannot get a sigma x value minimum this value because, your Mohr circles touches the line was it cannot go above this line because, any Mohr circle we will not exist above this line remember that.

So, and you can see I have drawn a half circle you can draw the full circle the similar way this will be the phi. So, you can complete this full circle also and this will also touch this line ok, you can complete the full circle. So, this is the sigma x minimum and as I mention so, at this condition all the stresses are the principal stresses. So, I can write in this way the this is my sigma and this is the tau and this is the failure envelope phi. And, this is the lowest sigma x y and writing sigma 3 and this is sigma 1 sigma 1 and sigma 3 are the principal stresses. So, these are all principal stresses.

So, I can write that your sigma z is equal to sigma 1 and sigma x is equal to sigma 3 because, these vertical stresses or principal stresses. So, this is the point I have written. So, if this is the same term O and if I draw a line here definitely it will be 90 degree because, this is the tangent over this point and we are joining this point and the center. So, this will be 90 degree because, this failure lie envelope is the tangent ok. Now, if I join this point and this point so, this dotted line this line will give you the direction of the failure surface or the rupture plane. So, this is the direction of failure plane ok.

So, say these that direction this angle is theta. Now, what is this angle value? Because, you can see that if I am considering O if this is your O, this is C and this is A, this point is A. So, now from triangle OAC I can write this angle is equal to because this is phi, this is 90 degree. So, this angle will be 90 degree minus phi because, if you take this triangle this angle is 90 degree, this angle is phi. So, this will be 180 degree minus 90 degree minus phi so, these are angle is be 90 degree minus phi. Now, if I take this triangle so, this is a B, this point is B.

So, now if I take triangle ABO then, if this is theta this will also be theta because, this is this side and this side are same because, both of the radius of the circle. So, this side as your OA is equal to OB because, both are the radius of this circle. So, this will be the both the angles will be theta. So, I can write that 90 degree minus phi plus 2 theta will be equal to 180. So, your 2 theta will be 90 degree plus phi so, theta will be 45 degree plus phi by 2 ok. So, it means that the failure line will make an angle 45 degree plus phi by 2 with the horizontal.

So, now if this is your say wall and if there is a rupture angle, active stage so, this is your failure plane. So, it will make an angle 45 degree plus phi by 2 this is for active case. So, for the active case your failure angle this theta will make an failure plane will make an angle 45 degree plus phi by 2 with the horizontal ok. So, the next class I will discuss about what would be the stress active stress acting on a earth retaining structure and what do the failure angle, if that is in a passive case ok. And, I will also discuss how we will calculate the passive earth pressure considering or with the help of this Mohr circle.

Thank you.