

**Foundation Engineering**  
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**Lecture - 02**  
**Introduction (Contd.)**

Welcome all. So, in the next part on the lecture 2 I will also continue my previous introduction part, and I will discuss a few terms that will be used for this course. So, next thing that is very important is the relative density so, the in that it is also important for a granular soil.

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**Relative Density**

In granular soils, the degree of compaction in the field can be measured by relative density ( $D_r$ %)

$$D_r(\%) = \frac{(e_{\max} - e)}{(e_{\max} - e_{\min})} \times 100$$

where  $e_{\max}$  = void ratio of the soil in the loosest state  
 $e_{\min}$  = void ratio in the densest state  
 $e$  = in situ void ratio

Relative density (%)	Classification
<15	Very loose
15-35	Loose
35-65	Medium
65-85	Dense
>85	Very dense

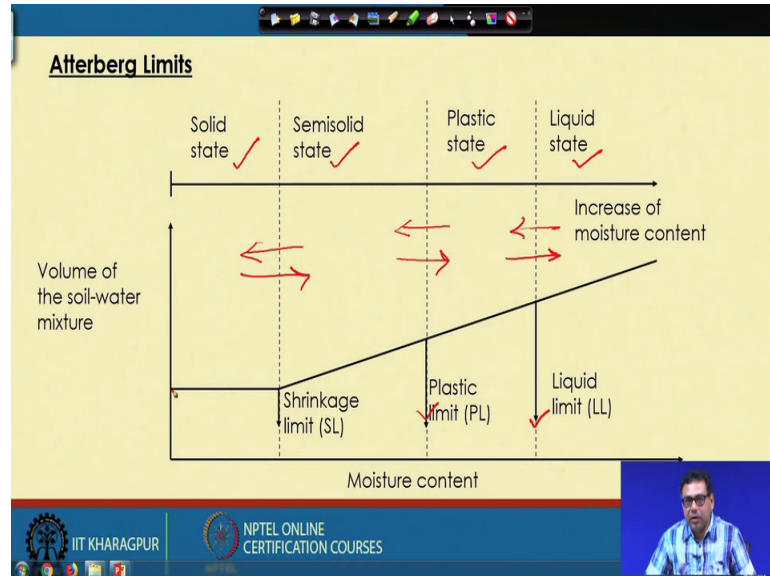
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So, relative density is the degree of compaction in the field can be measured by relative density. So, relative density definition is. So,  $e_{\max}$  minus  $e$ , and the difference between the maximum and the minimum the void ratio and it is generally expressed in percentage.

So, where  $e_{\max}$  is the void ratio of the soil in the loosest state, and  $e_{\min}$  is the void ratio in the densest state and  $e$  is the in situ void ratio. So, based on this relative density, we can classify the granular soil. So, if the relative density is less than 15 we can say the soil is very loose, if 15 to 35 it is loose and if it is greater than 85 then the soil is very dense. In between that we have given other values also.

So, is this that is why the relative density also very important parameter for the granular soil.

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So, next one is the Atterberg limits. So, these atterberg limits are very very important. So, we will use this term. So, there are 3 limits. So, as we know that soil has 4 state. So, if you look at this if you see this picture, the soil has for states that is the liquid state plastic state. So, this is the liquid state, plastic state, semi solid state and solid state.

So, now soil has four state and the junction of each state has a limit for example, that if you reduce the water content in a soil so; that means, soil will change its state from liquid to semi solid then if you further reduce the water content, it will go to a solid if you reduce the water content then soil will go from liquid state to plastic state; if you further reduce the water content soil will go from plastic state to semi solid state and if you further reduce it, then it will go to semi solid state to solid state.

So, liquid, then plastic, then semi solid and solid; Now the that means, the liquid limit is the water content at which soil changes its state form liquid to plastic or vice versa; that means, if you add a keep on adding the water on the on a if solid soil as in solid state if you keep on add on water in the soil; that means, from solid state to it will go to semi solid state then from semi solid state it will go to plastic state, and from plastic state it will go to liquid state.

So, in other way liquid limit is the water content as at which soil changes its state from plastic to liquid. So, that is the liquid limit similarly we have the plastic limit, where the soil changes its state from plastic to semi solid and another one is a shrinkage limit, where soil changes its state from semi solid to solid or vice versa. So, it can be in the both ways.

So, now if this figure is the moisture content and the volume of the soil water mixture if you see that, that if you reduce the water content then soil volume also reduces, but after the shrinkage limit soil volume does not change, if you further if you reduce the water content. So, these states are very important and these limits are also very important these limits will use for our design purpose.

So, next one is the hydraulic conductivity of soil.

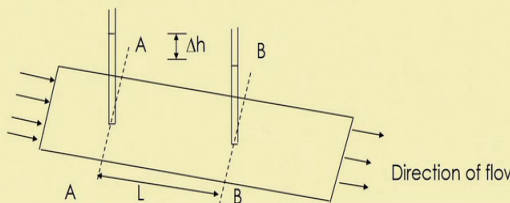
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**Hydraulic Conductivity of Soil**

- Darcy (1856) proposed the following equation for calculating the velocity of flow of water through a soil

$$v = ki$$

where  $v$  = Darcy velocity (cm/sec)  
 $k$  = hydraulic conductivity of soil (cm/sec)  
 $i$  = hydraulic gradient ( $i = \Delta h/L$ )



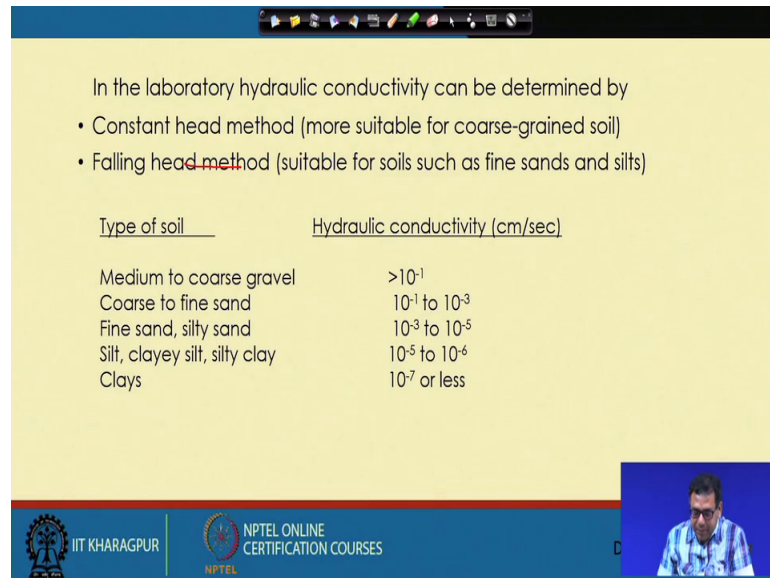
The diagram illustrates a soil sample in a rectangular container. Water is being poured from the left side into the container. Two vertical tubes, labeled A and B, are inserted into the soil. Tube A is at a higher elevation than tube B. The difference in water levels between the two tubes is labeled as  $\Delta h$ . The length of the soil sample between the two tubes is labeled as  $L$ . Arrows on the right side of the container indicate the 'Direction of flow' from left to right.

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So, hydraulic conductivity can be written as we can we know the velocity of flow of water through a soil is equal to  $k$  into  $i$ . So, the  $k v$  is the Darcy's velocity and  $k$  is the hydraulic conductivity of soil, and  $I$  is the hydraulic gradient or that we can write  $\frac{\Delta h}{L}$  the pressure head the head difference, divided by the length of these two points.

So, this is the direction of the flow. So, this hydraulic conductivity is also very important and we have that.

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In the laboratory hydraulic conductivity can be determined by

- Constant head method (more suitable for coarse-grained soil)
- Falling head method (suitable for soils such as fine sands and silts)

Type of soil	Hydraulic conductivity (cm/sec)
Medium to coarse gravel	$>10^{-1}$
Coarse to fine sand	$10^{-1}$ to $10^{-3}$
Fine sand, silty sand	$10^{-3}$ to $10^{-5}$
Silt, clayey silt, silty clay	$10^{-5}$ to $10^{-6}$
Clays	$10^{-7}$ or less

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So, this hydraulic conductivity in the laboratory, we can determine by constant head method, more suitable for coarse grained, soil and falling head method which is suitable for soil as fine sand and silts now the type of soil that we have a table, by which we can get an idea that what is the hydraulic conductivity of different types of soil. So, this is the medium to coarse gravel, it is greater than  $10^{-1}$  centimetre per second, and for the clay which is the list and that value is  $10^{-7}$  or less centimetre per second.

So, these values are the hydraulic conductivity values for different types of soil, we have the coarse to fine sand fine sand, silt silty sand we have silt clay silt and silty clay. So, these are the ranges are given. So, next one that we will start the another properties, that we have is for our effective stress which is also very important. So, we will discuss about the effective stress part.

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**Effective Stress**

- Effective stress can be defined as:  
$$\sigma' = \sigma - u$$
where,  $\sigma'$  is the vertical effective stress  
 $\sigma$  is the vertical total stress  
 $u$  is the pore water pressure
- For dry soil,  $u = 0$ ;  $\sigma' = \sigma$

The diagram illustrates the concept of effective stress. The top part shows a rectangular soil sample with downward arrows representing total stress ( $\sigma$ ) and upward arrows representing pore water pressure ( $u$ ). The bottom part shows two cross-sections of a soil container. The left one has a rigid support (Spring) at the bottom, and the right one has a valve (Water) at the bottom. A graph on the right shows the distribution of stress ( $\sigma$ ) and pore water pressure ( $u$ ) over time, with  $\sigma$  increasing and  $u$  decreasing as time progresses.

So, effective stress is a very important thing and when we calculate the stresses during our settlement calculation, then we will use these effective stress frequently. Now, what is effective stress? Now, effective stress you can see that when this is the these are the soil particles, and in between the soil particles we have water. So, when we apply the stress on a soil then initially the elevates stress distribution among the water and the grains.

So, for example, that means, the stresses are shared by the water and the soil grain so; that means, soil if I apply the stress for example, if we have a container and one valve is attached here and this is a rigid support. So, this support is very rigid and we have a spring here and this is the water is there.

So, here the water and this is the spring, now we are applying a load here on this piston, this is the piston where this is the valve. Now, if the valve is closed, then all the stresses will be taken by the water because our this spring will not deform. So, it will it is not taking any load or stress. So, initially under this condition when you have a container and water, you are not allowed to water to move from this container. So, the initially all the stresses will be taken by the by the water itself.

Now, if I in the next condition the same container now what we are doing that, we are allowing this water to flow that means, we open this valve. The first case this valve is closed now we open this wall and now we are applying this same load here now what

will happen? This water will start coming out from this container and this piston will deform as what is coming out. So, this now this spring will start taking load or stress and what will happen this spring will deform and this total system. So now, this say this piston will come here.

So, this is the amount of deformation of the spring. So, the pose suppose this is the amount of deformation of the spring, this is  $\Delta H$  is the amount of deformation of the spring. So now, what will happen that this spring will start taking stress, and when the total water will dissipate from this container, then the total stress will be taken by this spring only.

So, that is why now if I draw a graph that is this is the time  $t$  and this is the stress. So, initially at  $t$  equal to 0, the spin is not taking any load. So, this is the time versus stress distribution. So, this is the maximum stress or  $p_{max}$ . Now at time equal to  $t$  as the spring is not taking any load. So, the maximum stress the total stress is taken by the water, but at time progresses these stresses are transferred to from water to the spring.

So, now if I draw the time versus stress distribution of the water; So, this is the time verses distribution of the spring, and this is the water. So, we can see that initially the spring which is taking no load, then the slowly slowly start taking the load or the stress and water the stress on the water will reduce as water is dissipated from this container. So, here this spring is representing the soil particle and water is the water in between the void so; that means, if we apply the load on a water and if I do not allow these water to move then the stresses will be taken by initially water.

Then when the water start disappearing form this from this the area so; that means, all these stress will be taken by the soil particle. So, the effective stress is the stress, which is taken by the soil particle itself. So that means, we can write that the effective stress is the total stress minus pore water pressure; that means, the pressure which is or the stress which is taken by the water or the stress which is developed due to the application of this load so; that means, that stress is the pore water pressure.

Pour water pressure is  $u$  is the pore water pressure and for the dry soil as there is no water. So, all the stresses will be taken by the soil particle. So, effective stress will be equal to the total stress. So, we can move to the next part that, how I can determine the stresses at any point or effective stress at any point.

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Unit weight =  $\gamma$  *bulk*

Ground water level

Saturated unit weight =  $\gamma_{sat}$

● A

$$\sigma' = \sigma - u = (h_1 \gamma + h_2 \gamma_{sat}) - h_2 \gamma_w$$

$$= h_1 \gamma + h_2 (\gamma_{sat} - \gamma_w)$$

$$= h_1 \gamma + h_2 \gamma'$$

where  $\gamma'$  is the submerged unit weight of the soil =  $(\gamma_{sat} - \gamma_w)$

*Handwritten notes:*  
 $\gamma$  or  $\gamma_{bulk}$   
 $\gamma$  or  $\gamma_{sub}$   
 $\gamma_{sat}$   
 $\gamma$  or  $\gamma_{sub} = \gamma_{sat} - \gamma_w$

So, these calculations will use number of times during our stress calculation in the foundation design. So, suppose we have a soil and this is the point A, where I want to determine the stress or rather than I should say the effective stress. So, effective stress is this is the sigma bar or sometimes as sigma dash, sometimes we can use sigma bar also sometimes we can you use p dash or you can use p bar also.

So, now the we have a soil layer and where the point A where I want to determine the effective stress it is at a height of h by all at a depth of h 1 plus h 2 from the ground surface and water table is at a height of h 1 from the ground surface. So, what I can write that the first I will calculate the total stress; now the total stress will be the. So, suppose this is the unit weight of the soil or you can say this is the unit weight gamma bulk, sometimes we in this course if nothing is mentioned then if it is mentioned that gamma then you will assume that this is gamma bulk.

So, if gamma is mentioned; that means, gamma bulk otherwise I will mention gamma dry if it is a gamma dry soil or gamma saturated if is a saturated soil or gamma sub that is suppose gamma saturated minus gamma w. So, here nothing is mention is gamma bulk or gamma so; that means, here unit weight is gamma or gamma bulk. So, and the saturated unit weight is gamma sat. So, the total unit way or total stress will be h 1 into gamma plus h 2 into gamma sat minus the pore water pressure pore water pressure will be h 2 into gamma w.

So, I can write that  $h_1 \gamma + h_2 \gamma_{sub}$ . So,  $\gamma_{sub}$  or  $\gamma_{sub}$  is equal to  $\gamma_{sat} - \gamma_w$ . So,  $\gamma_{sub}$  is the submerged unit weight of the soil. So, as I mentioned that the how I will I will mention this, I will show the different unit weight either  $\gamma$  or  $\gamma_{bulk}$ , then  $\gamma_{dash}$  or  $\gamma_{sub}$ , then  $\gamma_d$  or  $\gamma_{dry}$  then  $\gamma_{sat}$  is the saturated unit weight.

So, this is the bulk unit weight, this is the submerged unit weight, this is the dry unit weight, this is the saturated unit weight. So, these are the different units that we use. So, the effective stress we can write that  $h_1 \gamma + \gamma_{sub} h_2$ . So, that is the effective stress at a point a from the ground surface which is at a depth of  $h_1 + h_2$ . So, next one is the compaction is also a very important thing.

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**Compaction**

- Expulsion of air from the void space in the soil mass
- Standard Proctor test is done to find out OMC (Optimum moisture content) and MDD (Maximum dry density).

$V_1$ 
 $V_2$

Air
Air

Water
Water

Soil solids
Soil solids

So, there is two terms one is the compaction one is the consolidation.

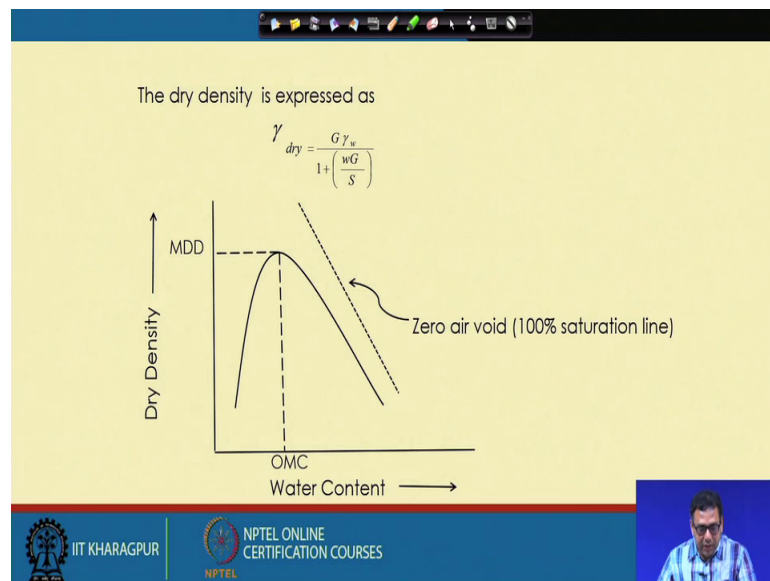
So, basic difference between the compaction and the consolidation in the compaction we are basically eliminating the air voids and in the consolidation we are eliminating the water voids. So, in the compaction we use the standard proctor test to done the compaction and we have the modified compaction also. So, here the we determine the optimum moisture content and maximum dry density.



So, you can see the here in this figure the solid volume in terms of volume, the solid volumes remain same after compaction water also remain same after compaction, but air volume will reduce after compaction.

So, because the here compaction, we are reducing the air voids and consolidation we are reducing the water voids.

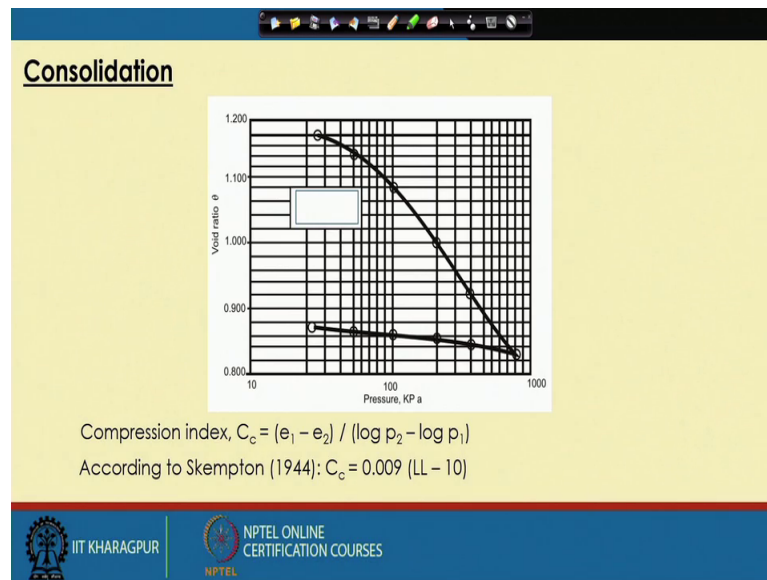
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So, in the compaction part if I if you will if I show you the typical compaction curve; So, this is a typical compaction curve in x axis, it is water content and in the y axis it is dry density. So, we have a curve and from this curve the peak is given the maximum dry density and the corresponding water content is optimum moisture content and this dotted line is 0 air void lines or 100 percent saturated saturation line.

So; that means, 100 percent saturation means there is no air void present in the soil. So, these are the this is a typical compaction curve. So, and this is the our dry unit weight expression, that we are getting from the previous expression that I have already discussed in the first class. So, next one is the consolidation.

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As I mentioned that these two things one is compaction another is consolidation. In the consolidation also we are reducing the volume of voids, but here we are reducing the water voids.

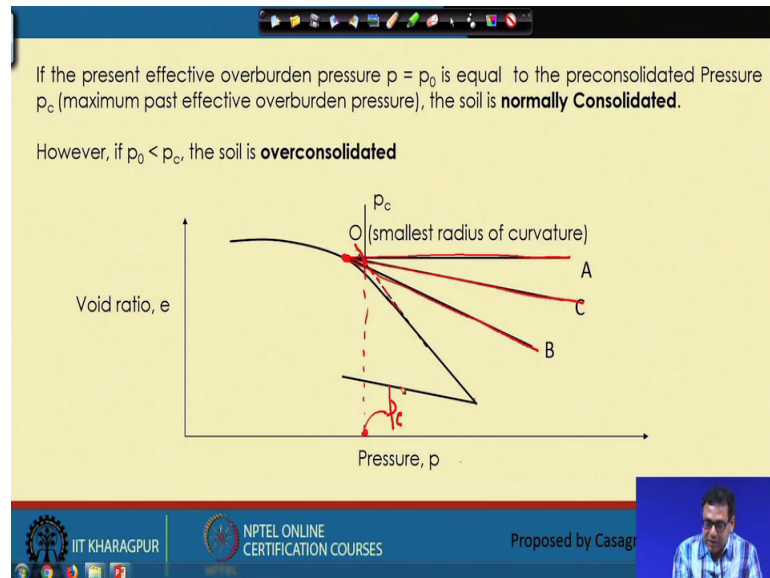
So, here this is a typical consolidation curve. So, this is the, this part the first part is the loading curve and the second part is the unloading curve. So, you can see from this part the first part will get a compression index, which is very very important this compression index will be required for the settlement calculation. So, these compression index is basically the slope of this straight portion of this loading curve, and the compression index is basically the difference between the void ratio this is the void ratio this is void ratio  $e$ .

So, this is the void ratio and the pressure this is the small  $k$ , this is void ratio and pressure curve. So, this pressure it is in log scale and so, this is the difference of void ratio divided by difference of pressure in log scale in log. So, that is the  $e \log p$  curve. So, from there the slope will give the compression in this. According to Skempton also if I know the liquid limit of a soil we can determine the compression index and that is  $0.009$  into liquid limit minus  $10$ .

So, a liquid limit valve I have to put and I will get the compression index or compression index, we can determine from the  $e \log p$  curve will do a consolidation test and from this test data we get the  $e \log p$  curve we can we do it for the loading and we can wait for the

unloading. So, we will get this the curve for loading and unloading. So, the slope of the straight portion of loading curve will give the  $C_c$ .

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So, now a term which is very important is the normally consolidated soil and over consolidated soil. Now, these two terms we will use frequently for our design there now what is normally consolidated soil and what is over consolidated soil. Now if the present effective overburden pressure is equal to the preconsolidated pressure or the maximum past effective overburden pressure then the soil is called the normally consolidated soil.

So; that means, if we that mean the if the stresses that is applied on a soil is more than the stress the soil is previously experienced, then that soil is called as a normally consolidated soil. That is why the present effective stress is equal to is equal to the preconsolidation pressure or equal to or greater than the preconsolidation pressure or the maximum past effective overburden pressure of the soil.

And the over consolidated soil means if the present stress which is applied on a soil is less than the stress this soil is previously experienced, then that soil is called as a over consolidated soil. For example, excavation suppose we during the excavation we remove the soil.

So, now if I remove the soil, then the stress on a soil is released; that means, suppose we are removed removing soil up to height of height of  $h$  so; that means, this  $h$  total stress which is acting on a depth or height  $h$  below the ground level. So, that stress is removed. Now if now if I apply the stress on that point, now that stress if it is less than the previously applied stress on that point then that is called as a over consolidated soil.

So, now how I can determine that maximum past effective overburden pressure or the preconsolidated pressure that is very important, because we should know the soil that on which I am constructing or designing the foundation the soil is normally consolidated soil or the soil is over consolidated soil. So, how that means, we should know that which type of soil it is so, that for to know that, we have to determine what is the preconsolidated pressure or maximum past effective overburden pressure.

So, this technique is or this meter is proposed by casagrande in 1936 now what is the method so; that means, from the test will get  $p_e \log p$  curve. So, this is also in  $\log e \log p$  curve. So, now, the straight portion; So, if this is a curve, this is a loading portion this is the unloading portion in the loading portion if this is the curve. So, we will find a point where the curvature is the smallest radius of curvature.

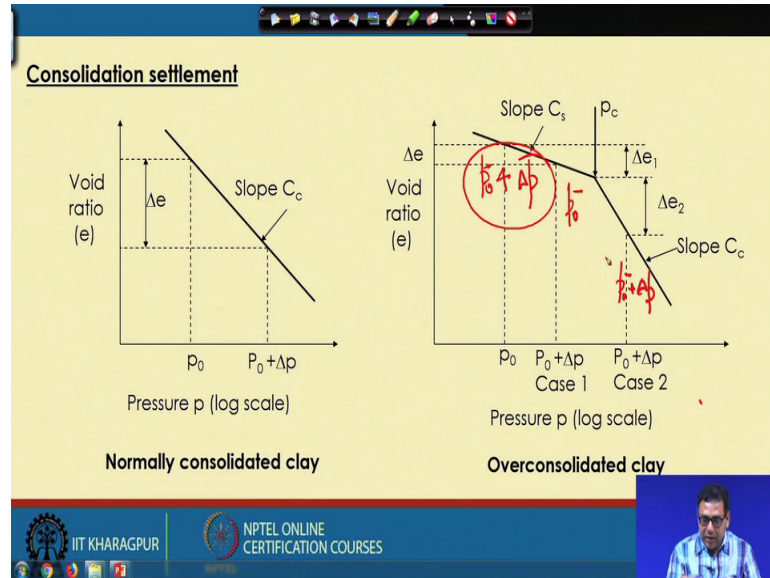
So, this is the point  $O$  is the point where you can see this is the smallest radius of curvature because you can see this curvature is changing, the radius of the curvature is changing for this curve. So, this is the point where radius of curvature is the smallest. So, we identify that point and then we will draw a line  $O A$  which is parallel to the  $x$  axis. So, draw a line  $O A$  which is parallel to  $x$  axis. Then I will then draw a tangent  $O B$  this  $O B$  is a tangent at  $O$ . Then we have angle  $A O B$  now bisect this angle.

Now, you extend this straight portion of the curve, and it will intersect this bisection line. Now this point corresponding to the stress is called the preconsolidation pressure. So, this  $p_c$  is very important we have to identify this  $p_c$ ; So, that when you design we should know whether soil is overconsolidated or normally consolidated.

So, I explain these methods again first we will identify the point  $O$ , which has the smallest radius of curvature then we draw a line  $O A$  which is parallel to  $x$  axis or the pressure axis, then we draw a tangent at point  $O$  this tangent  $B O$  at point  $O$ , when we bisect the angle  $A O B$ , then extend the straight portion of loading curve and the

intersection point between the extension line and the bisecton line and corresponding stress corresponding stress of that point is equal to  $p_c$  or preconsolidation pressure.

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So, now we have this is the two cases. So, this is the normally consolidated clay, this is the overconsolidated clay. So, you can see this is the loading part and the slope of that loading part will give the compression index, and then we have this is the  $p_c$  and then we have a straight portion, and this is the this portion is normally consolidated portion and this portion is the overconsolidated portion.

So, now we have to identify which portion the soil is; So, based on that we can determine the settlement of the soil.

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**Normally consolidated clay**  
Settlement  $S = (C_c / (1 + e_0)) H \log_{10} ((p_0 + \Delta p) / p_0)$

**Over-consolidated clay**

Case 1:  $p_0 + \Delta p < p_c$   
Settlement  $S = (C_c / (1 + e_0)) H \log_{10} ((p_0 + \Delta p) / p_0)$

Case 2:  $p_0 < p_c < p_0 + \Delta p$   
Settlement  $S = (C_c / (1 + e_0)) H \log_{10} (p_c / p_0) + (C_c / (1 + e_0)) H \log_{10} ((p_0 + \Delta p) / p_c)$

where  $C_c$  is swelling index

Handwritten notes:  $S = \frac{C_c}{1 + e_0} H \log_{10} \left( \frac{p_0 + \Delta p}{p_0} \right)$  Effective stress

Diagram: A vertical double-headed arrow labeled 'H' representing the thickness of the soil layer.

Now if you have the so; that means, based on that it has two cases one is normally consolidated clay. So, if the soil is within that range that mean if the present pressure is greater than the  $p_c$ , then the settlement we can calculate that  $C_c$  settlement we can calculate that  $C_c / (1 + e_0) H \log_{10} (p_0 + \Delta p / p_0)$ .

So, this is the expression. So, this expression you are getting from that Terzaghi's one-dimensional consolidation theory, I think these expressions are coming from the soil mechanics part. So, this expression this term  $C_c$  is compression index  $e_0$  is the initial void ratio,  $H$  is the thickness of suppose this is the thickness of soil layer  $h$  is the thickness of soil layer and  $\log$  this is based 10,  $p_0$  is the effective stress or effective overburden pressure and  $\Delta p$  is the additional stress, which is applied due to the external load.

So, I will explain that how these stresses are calculated, how I will calculate  $p_0$  bar how I will calculate  $\Delta p$ . So, when I will determine the settlement of settlement of the soil. So, this is these expressions you just remember this expression, later on I will come back to this expression and I will frequently use this expression to determine the settlement of the soil. So, the in that time I will in detail explain, how I will determine the  $p_0$  bar effective overburden pressure and  $\Delta p$  for a particular foundation.

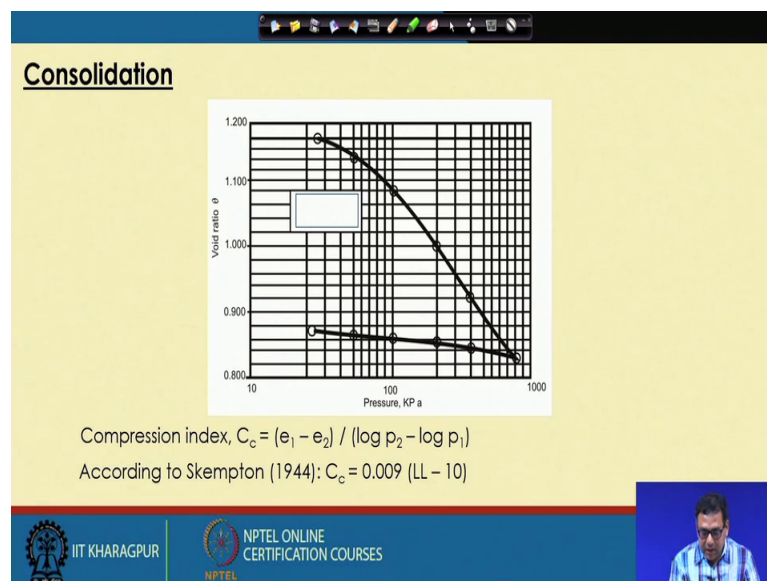
Now, the thing that I am talking about for the over consolidated clay, it has two cases. That one case is that that the soil can be within this range and stresses so; that means,

here the initially  $p_0$  bar is in this range, but when I apply the  $\Delta p$ , then  $p_0$  bar plus  $\Delta p$  will be in this range; another one that  $p_0$  plus  $\Delta p$  both are in this.

For example that that one case is your  $p_0$  and  $\Delta p$  both are within this range, another case your  $p_0$  is within this range, but  $p_0$  plus  $\Delta p$  is in this range. So, there are two cases. So, if there is two cases. So, we can write that this is the first case  $p_0$  plus  $\Delta p$  both are less than  $p_c$ . So, we can get  $C_c$  instead of  $C_c$  we will  $C_s$  where  $C$  is the swelling index that swelling index will get the slope from the unloading curve.

So, we have a loading curve and unloading curve, the  $C_c$  I will get from the slope of the unloading curve. So, for example, if I show you that; so, this is the typical curve. So,  $C_c$  I will get from the state portion slope of the straight portion and  $C_s$  will get from the slope of the unloading curve or that is the swelling index.

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And if the case two  $p_0$  is less than  $p_c$ , but  $p_0$  plus  $\Delta p$  is greater than  $p_c$  then I will use these expression. So, these expressions are given  $p_c$  is the  $p$  consolidated pressure or past maximum effective overburden pressure.

So, these are the equations I will frequently used and then I will again explain how I will calculate these values in the in the during the design of foundation part. So, today I will finish this lecture two here, and then I will go for the shear strength part of the soil in the lecture 3, and we will discuss how the other parameters with the major important design

parameters of the soil, that we will determine from the laboratory test and what are the shear strength of the soil.

Thank you.