

Introduction to Mineral Processing
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Lecture - 09
Particle Characterization (Contd.)

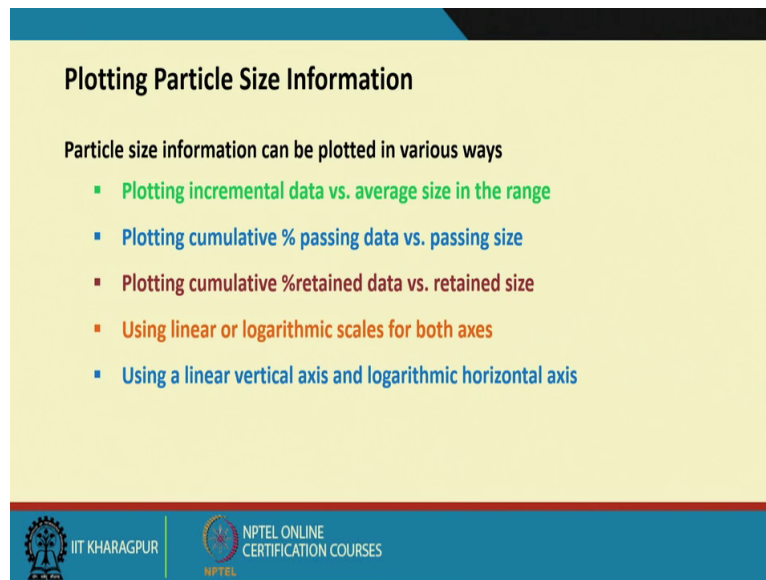
Hello. Welcome back to this course, we are discussing Particle Characterization and specifically the particle size distribution. So, last time where we stopped that we said that for getting a size distribution properly, that information properly to be done. We must use the identical standards. There is another reason also behind that.

That is, when you select the topmost sieve say suppose that is 200 micro meters. So, the next sieve you have to follow a series the various series, the most popular one is the root 2 series. Root 2 series means that if I have started with 200 micro meters, the next finer sieve should be 200 divide by root 2 micrometer or the course the next courses sieve should be 200 multiplied by root 2.

Why root 2? Because, when the particle is passing through a sieve aperture, basically you are looking at the surface area. So, what is the surface area? It has a relationship with the square of the diameter of that particle. So, when I am plotting my information, I have got a sequence that I am discretizing it in a manner that it will give me some information about their surface area distribution also.

So, similarly we have got fourth root of 2 series sieves, we have got tenth root of 10 series sieves also. So, when I am using a root 2 sieves, I must use a route 2 sieves all across all along the stack; that means, all the 8 sieves what will be using they have to be from the same standard and this would follow the same series.

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Plotting Particle Size Information

Particle size information can be plotted in various ways

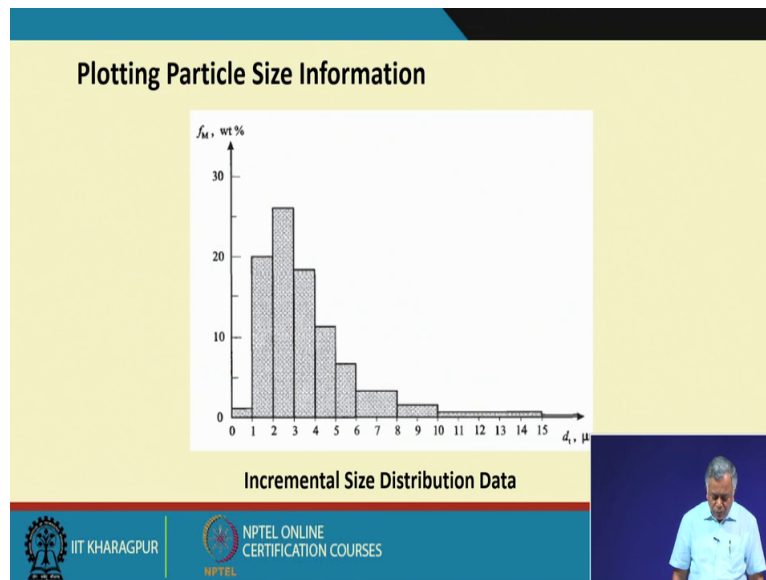
- Plotting incremental data vs. average size in the range
- Plotting cumulative % passing data vs. passing size
- Plotting cumulative % retained data vs. retained size
- Using linear or logarithmic scales for both axes
- Using a linear vertical axis and logarithmic horizontal axis

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Now, once I get that information in a laboratory scale in a laboratory, what do I do with that, how do I get the particle size distribution related information. So, if we plot the particle size information in various manners, we can get to know very important information about the particle size distribution, even from my mine particles or the material of the particles which I am supposed to process. So, we can plot the incremental data versus average size in the range, we can plot cumulative percent passing data versus passing size, that is how much is passing that there can be only 2 probability of the particles when I am using a sieve either they could pass or they could retain.

Plotting cumulative percent retained data versus retained size. Then you can use some linear or logarithmic scales for both axes or using a linear vertical axis and logarithmic horizontal axis. So, they have various ways of plotting the data, but some of the common methods, I will try to discuss with you.

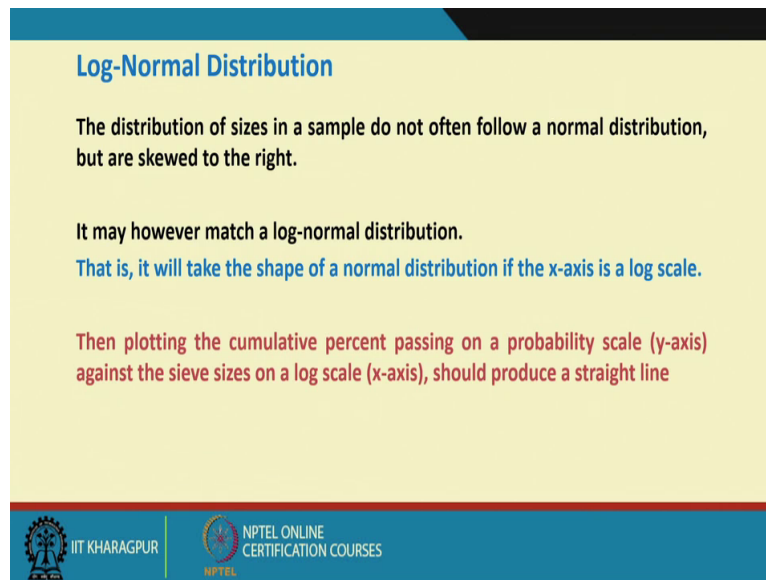
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Now, this is the incremental size distribution data versus your relative weight percent of that. So, what it is showing that if I have that this histogram if I convert into a basically into a say discrete or say differential plot, we see that distribution pattern is skewed towards the left. That means, it has got more relatively more particles, which are reporting into the finer sizes. That means, I have started say suppose from 600 micrometer to 40 micrometer. So, this will give me an information that my the population of the particles, they are slowly they are slightly inclined towards the left; that means, they are having relatively much more proportions in the finer fraction in relation to my course of actions.

Similarly if it is basically skewed to the right, we can say that we have got relatively coarser particle size distribution than the finer one.

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Log-Normal Distribution

The distribution of sizes in a sample do not often follow a normal distribution, but are skewed to the right.

It may however match a log-normal distribution.

That is, it will take the shape of a normal distribution if the x-axis is a log scale.

Then plotting the cumulative percent passing on a probability scale (y-axis) against the sieve sizes on a log scale (x-axis), should produce a straight line

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So, we can this distribution of sizes in a sample do not often follow a normal distribution. Normal distribution means that is the ideal plot, that they are not skewed neither on the left or nor in the right. But normally when we gets this size distribution analysis is done from the mine sample, we hardly get the normal distribution. But it may follow a log normal distribution what does it mean? That if we plot the x axis in a log scale there is a size; then probably you will find that this distribution gets converted into a straight line and when there is a straight line a distribution plot? We can get to know the slope and we can generate many more information. But the very purpose of generating this information there is a particle size distribution, from the point of view of a mineral processing engineer are different than the other industries.

So, what we are interested to know from the particle size distribution? That if I have this particles at a rate of 1000 tons per hour, what is that how much of that quantity is finer than 40 micrometers some of that is 20 percent for discussion sake so; that means, per hour I have to handle 200 tons out of 1000 tons, which are finer than 40 micrometers what do I mean by handle; that means, I have to firstly, that transport that material from a place to another place, which are basically maybe 500 meters apart.

So, how do I transport them this 200 tons of material per hour, which are having sizes less than 40 micrometers, can we transport it through conveyor belt? No because the 40 below 40 micrometer particles, they will have a tendency to be airborne. So, my colleagues from

environmental engineering section they may object and it is not only the environmental related issues, how do I store them, how do I feed them into a conveyor belt and then what will be the losses.

So, I have to think of maybe that in the form of slurry I have to transport them to a distance of 500 meters. So, when I have to transport them in the form of slurry, and I know it is 200 tons per hour. So, I can calculate again that how much of water we require to do that. And pipeline transport means there has to be a head or that is your driving force. So, we need a pump. So, what would be the pump capacity and all this we have to decide based on the information I have generated into a laboratory on particle size distribution only. And now that below 40 micro meters particle why I am transporting a distance to 500 meters? Is it going to be treated into another equipment, whether that equipment can accept this particle size range whether their efficiencies are at best for while dealing with these particles.

So, all sorts of information all sorts of decisions I can take, but that is only 20 percent I have talked about what about the remaining 80 percent which are coarser than 40 micrometers. So, I have seen that by size distribution is telling me that I have got particles from 600 micrometers to 400 micrometers which is only maybe 2 percent of the total; should we really worry about 2 percent material, should we have another bulk material handling system, should we have another processing unit for that?

Most of the cases it is not required if that 600 to 400 micrometers particles are the particles which you want to separate out from the rest. So, you know that the remaining particles which are within 400 micrometers to 40 micrometers, now you know the bulk volume per unit time you have to treat and you have to process. So, you can decide on the likely flowcid we call it flowcid or say flow chart, and for handling the each and every discrete particle size ranges based on the your technology available, based on the applicability of the process, and you can also think of even your likely bulk material handling systems.

So, if I have a plot only based on the discrete sizes, because I cannot have the n number of your circuits, n number of processes for processing my discrete particle sizes, what we do? We also try to convert this information into a cumulative manner. So, in a cumulative plot what we do what does it mean, the cumulative that I will try to explain.

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Rosin - Rammler Distribution

$100 - P = 100 \exp(-bd^n)$
p = cumulative undersize in per cent
b and n are constants
d = particle size

$$\log \left[\ln \frac{100}{100 - P} \right] = \log b + n \log d$$

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So, cumulative means that that we have used 8 sieves for generating the size distribution. Just for example, suppose we have got 600 micrometer, 500 micrometer, 400 micrometer 300, 200 like that we have got 8 sieves down the line. Now we imagine that if I have only one sieve of 100 micrometers, why we should imagine of a one sieve of having 100 micrometer dimension? Because we have got technology to process particles within the size range of 600 micro meters to 100 micrometers, and we have a different technology for processing below 100 micro micrometer particles.

So, now say suppose I have got the 600 micrometers to 100 micrometers. So, I am only interested in knowing how much will be they are in 100 micrometer, which are basically coarser than 100 micrometers and how much is finer than 100 micrometers. Should I do separate tests? No it is not required, because I already have the discrete informations. So, now, the particles which are already reported as coarser than 600 micrometers, they will be definitely coarser than 100 micrometers is not it its natural.

So, now, say suppose I have got 5 percent which your material of the total population is coarser than 600 micrometers, and then 10 percent of the total population is coarser than 500 micrometers, but finer than a coarser than 500 micrometers, but finer than 600 micrometers like that up to 100 micrometers. So, whatever particles have been retained on 100 micrometers, they all basically will retain whatever has been deported into the next courses sieves.

So, I will add them together, I will show you do not worry I will show you some numerical examples that how to do it let me first clear your concept. So, the meaning of cumulative is cumulative weight percent is, that if I have a single sieve out of the total population how much of that will retain on that and how much will be finer that. Suppose 60 percent of the total material you are seeing that it will retain on that, naturally. So, I know that I have got 60 percent particles of the total population which are coarser than 100 micrometers, and I am thinking of setting up a mineral processing plant, which is thousand tons per hour.

So that means, I have to have my unit operations or the technologies to be capable of handling or processing, my mind material at a rate of 600 tons per hour because I said this is 60 percent. Remaining 40 percent whether I have got a technology or whether I have to decide, there whether I can dump it or not even for dumping also you have to transport it. So, as I explained you before that we can design about the; at least that dimensions of my unit operations bulk material handling systems I can think of.

So, this a very very important information and now many times what we do that, when you are plotting this cumulative say size distribution information, like your x axis is the particle sizes and y axis is the cumulative weight percent either retained or passed. So, you get a generally a curve, but many a times, that curve is enough to generate this information what I have explained you just now. But many a times there may be your fluctuations in the size distribution of your mind sample, and you want to use a simulated where I want to use a particle size distribution relation a related information as a variable because that will again modify the models, what will be using for your optimizing the processing parameters.

So, that time in that case we need a bit more comprehensive information. So, because of this reason the many attempts were made that to have to convert this particle size distribution data, which are basically in a form of curve to make it in the form of straight line. There are various models available that, but 2 models I will just briefly explain you I will just show you, which are very popular one is called the Rosin Rammler distribution. These are only trying to linearize your size distribution information, what you have generated into the lab scale c v.

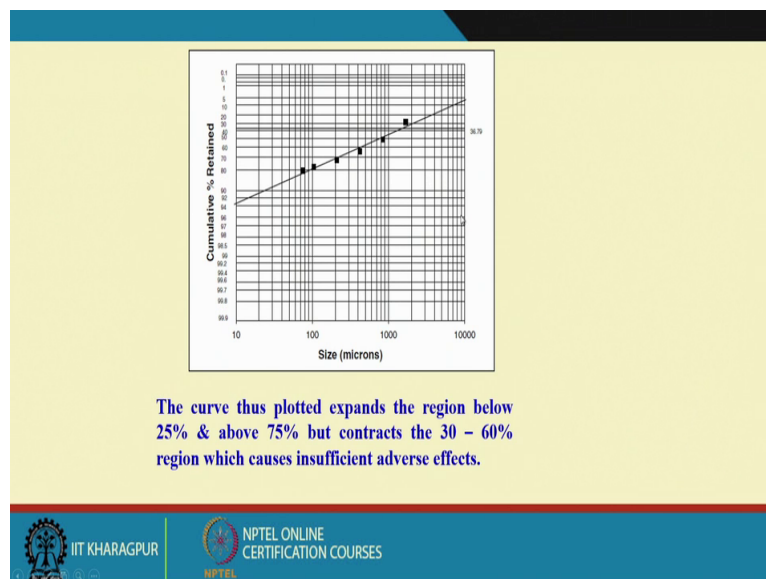
So, the Rosin Rammler distribution says that it is written as $100 - P = 100 \exp(-k d^n)$ that is bd to the power n , what is P ? P is the cumulative undersize in percent if the size of your screen is d ; that means, when you are basically having a cumulative information when I

said that what is the size you must specify. So, that d is specifying that size and this b and n are constants they have got various other meanings, but I do not want to trouble you with all these details because this is an introductory course.

So, what happens now this $100 - P$ is equal to 100 exponentiation of your $b d$ to the power n , if I take logs of this twice, you can practice it is not that difficult. So, what will happen first time when I am taking a log. So, $\log 100 - P$ is equal to $\log 100 + b d$ to the power n . So, that $\log 100$ I can take it out from this side and I can say the $\log 100$ by $100 - P$ and then I take another log of this. So, that is if I take a double log of this equation I can write that \log of $\ln 100$ by $100 - P$ is equal to $\log b$ plus $n \log d$.

So, where your d is the particle size n and b are the constants. So, now, if I plot this particle size that is d as a your versus your say your $100 \ln 100$ by $100 - P$ is plotted against the size, then I should get a straight line why? Now this is having a form of y is equal to mx plus c , that is mx this is c , c is the constant term.

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So, this is the an example of your Rosin Rammler distribution. So, here you see that and in statistics literature this is called a Weibull distribution anyway that is a different story. So, when we plot them into a graph paper this time there is a your we call it even Rosin Rammler graph papers you know.

So, this is the size of the particles and this the cumulative weight percent retained, then you get a straight line, but what is interesting to see here that the this curve thus plotted expands the region below 25 percent and above 75 percent. It expands, but it gets contracted in into the region of 30 to 60 percent region, which normally causes insufficient adverse effects. Because mostly our processes are what we apply there efficiencies are more affected by the relatively coarser sizes or relatively finer sizes particles. Like if you have relatively coarser particles you have a different problem like say suppose if I am using a froth flotation process, where you use some chemicals to make them hydrophobic and then you want to float them. So, if I have more of coarser particles that is difficult to float them.

Similarly, if I have relatively more number of particles in the finer size ranges. So, that be induce viscosity related problem into my slurry. So, this is the information these are the pros and cons of this, but most commonly this Rosin Rammler distribution, we use while dealing with coal particles.

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Gaudin - Schuhmann Distribution

$$y = 100 \left[\left(\frac{x}{k} \right)^a \right]$$

y = cumulative mass % retained on size x
x = screen aperture size
a & k are constants

This plot severely contracts the region above 50%, and especially above 75%, which is a major disadvantage of the method.

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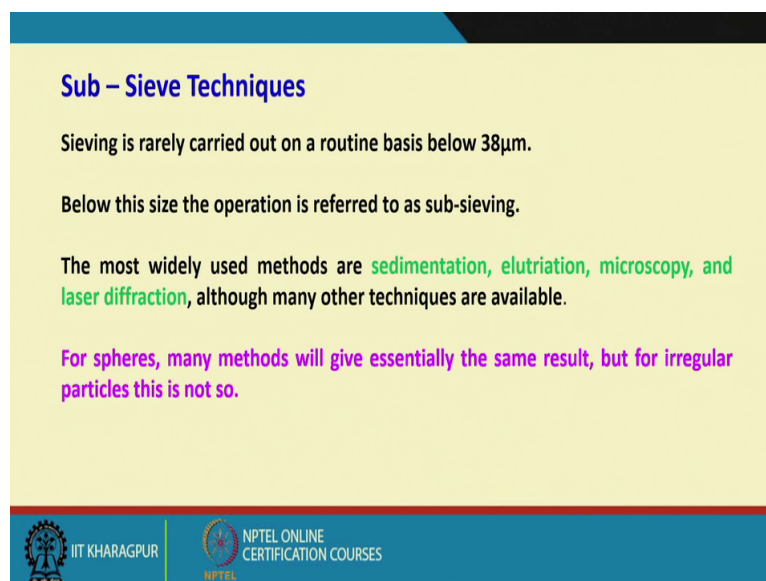
For mineral particles we use another model that is called Gaudin Schuhmann distribution even that is been modified as gates Gaudin Schuhmann distribution like that. So, according to this model this model is y is equal to 100 x by k to the power a. So, where y is equal to the cumulative mass percent retained on size x, that is x is the aperture size and a and k are constant. So, again if I take a log of this, that is log y is equal to and said say log 100 say and then we can convert it into this log form and then we can plot it like your particle size versus

cumulative weight percent passing you know and so, it is $\log y$ by 100 is equal to a $\log x$ minus a $\log k$.

So, we can plot it like that and then we get an information like this and this plot severely contracts the region above 50 percent and especially above 75 percent which is a major disadvantage of this method; that means, when we try to focus this is the cumulative weight percent passing and when you want to focus that, how much of coarser particles are there and this is not giving me the very detailed information above 75 percent passing sizes; that means, we are not getting adequate information about the relative distribution of the coarser particles in my assemblage. Although it has got different merits also, but, let us stop it here for this discussion.

So, there are also other techniques that are for sub sieve techniques as I said that below 40 micrometers it is not generally recommended to go for sieving method or you can you have to use the weight sieving method.

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Sub – Sieve Techniques

Sieving is rarely carried out on a routine basis below 38 μ m.

Below this size the operation is referred to as sub-sieving.

The most widely used methods are sedimentation, elutriation, microscopy, and laser diffraction, although many other techniques are available.

For spheres, many methods will give essentially the same result, but for irregular particles this is not so.

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So, that is why I have seen written that sieving is rarely carried out on a routine basis below 38 micrometer because the data is not reproducible always. Below this size the operation is referred to as sub sieving and what are the techniques we then use? We use either sedimentation we call it elutriation microscopy and laser diffraction techniques. So, interested if you are interested on this, you can just look at the different textbook that what this techniques they deal with and how do they do it.

So, for spheres many methods will give essentially the same result, but for irregular particles this is not so, which is our realistic situation.

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Some approximate factors for a given characteristic size are given below – these should be used with caution:

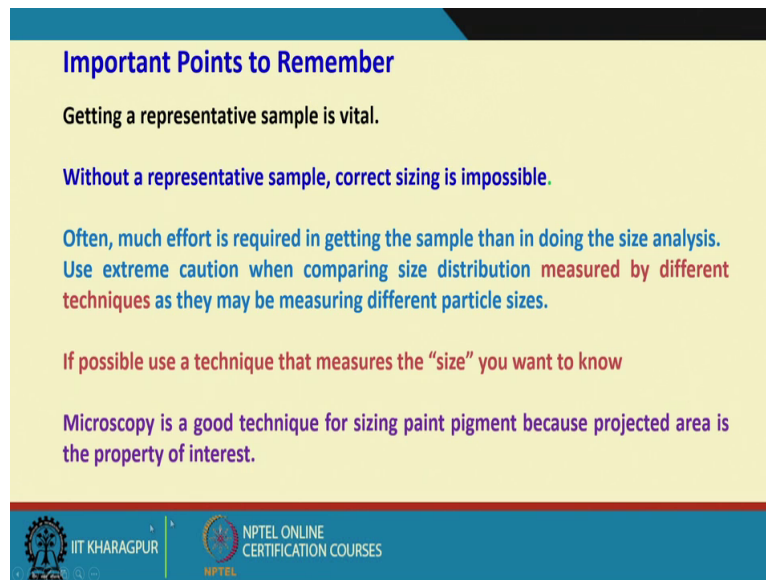
Conversion	Multiplying Factor
Sieve size to Stokes diameter (sedimentation, elutriation)	0.94
Sieve size to projected area diameter (microscopy)	1.4
Sieve size to LASER diffraction	1.5
Square mesh sieves to round hole sieves	1.2



There is a very important thing that, suppose I have got a mined sample. So, it may have particle size ranging from 1 meter to 1 micrometer or maybe 10 micrometer. So, if it is 1 meter sizes I cannot use a sieve probably I have to use a screen or something like that, but when the particle sizes are below 38 micron, we have discussed we have I have already said that we have to use different techniques like sedimentation or all.

So, what in essence I am trying to say that to have a size distribution data for a huge range, I may have to use the different techniques for different discrete size ranges, but I want a complete size distribution information. Normally this is being advocated in literature that these are the some multiplying factor like your sieve size to stokes diameter, you have to multiplying factor is 0.94 like that, I have given some other numbers that is how do I convert them and then you plot it; that means, these are the correction factors, that is either you bring to one equivalent sizes, that is how you have defined the size and then you plot.

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Important Points to Remember

Getting a representative sample is vital.

Without a representative sample, correct sizing is impossible.

Often, much effort is required in getting the sample than in doing the size analysis. Use extreme caution when comparing size distribution measured by different techniques as they may be measuring different particle sizes.

If possible use a technique that measures the “size” you want to know

Microscopy is a good technique for sizing paint pigment because projected area is the property of interest.

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So, important points to remember getting a representative sample is vital, without a representative sample correct sizing is impossible and much effort is required in getting the sample than in doing the size analysis, you may have good sophisticated techniques or say instruments to perform the size analysis. But if your sample is wrong, that is it is not truly representing your entire population, then your entire decision could be in a different direction you know it is not appropriate. Then if possible use a technique that measures the size you want to know. So, first you must be very clear that what do mean by the size, what size exactly you are looking up you are looking for.

So, microscopy for example, is a good technique for sizing paint pigment because projected area is the property of interest, and as I said that sedimentation velocity is a right technique when you are designing a sedimentation basin.

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If fine dust is present in the sample, dry sizing techniques will give a different size distribution to wet sizing techniques because the dust adheres to the larger particles.

In particular, beware of dust blinding screens for dry sieve analysis.

It may be necessary to “deslime” the sample over a fine screen first.

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If fine dust is present in the sample, dry sizing techniques will give a different size distribution to weight sizing techniques because the dust adheres to the larger particles; in particular beware of dust blinding screens for dry sieve analysis. So, what I am trying to say that if you have very fine particles along with the coarse particles, it is better to separate out the very fine particles from that and then you do separately the size analysis for the relatively coarser particles and the dust; otherwise you may have erroneous information.

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If you need the number size distribution use a technique that counts particles.

If you want the mass size distribution use a technique that measures mass directly.

Very large errors can occur from converting from one form of the size distribution to the other.

Never accept size analysis data on face value. Always view the data critically in light of the comments made above.

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If you need the number size distribution use a technique that counts particles, if we want the mass size distribution use a technique that measure mass directly. Very large errors can occur from converting from one form of the size distribution to the other. Finally, I would say that never accept size analysis data on face value, always view the data critically in light of the comments made above.

Thank you very much.