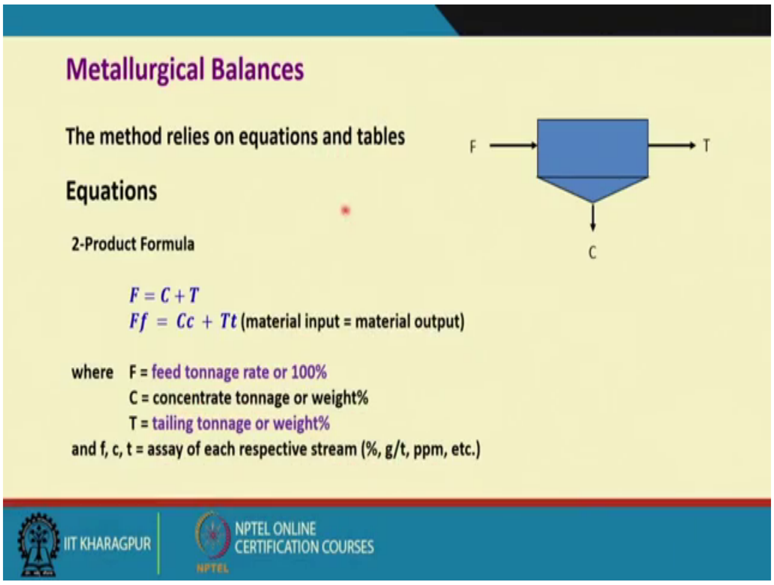
**Introduction to Mineral Processing**
**Prof. Arun Kumar Majumder**
**Department of Mining Engineer**
**Indian Institute of Technology, Kharagpur**

**Lecture – 46**
**Mass Balancing (Contd.)**

Hello, welcome everybody. So, in the last lecture I have shown you that how to calculate the percentage solids at by weight and percentage solids by volume, how to calculate the slurry density and how do I use the concept of dilution ratio. And then we have started discussing about the metallurgical balances; the importance and the precautions we have already discussed. This lecture I would like to show you how exactly we do it ok.

(Refer Slide Time: 00:54)




Normally if you see that your most of the equipment; we look at most of the equipment to use in mineral processing, they are having two products one we call it overflow or and another one is under floor for some cases we say that is over sized or undersized. That is why it is classification of size separation we say some are over sized, some are under sized in case of process related think some something we send it through overflow something we send it your under flow.

So, that is two product system; so, for two product system like this now this metallurgical balances many times whatever equations will be discussing now, you can have it already incorporated into excel spreadsheet. And then you can use it automatically, but for understanding you must know how do you formulate this equations. Suppose this is my unit operation and it is a two product system any process ok. So, here the feed comes feed enters here it does some kind of your improvement and the improved quality that is much more concentrated material with of my desired material I say it that is concentrate.

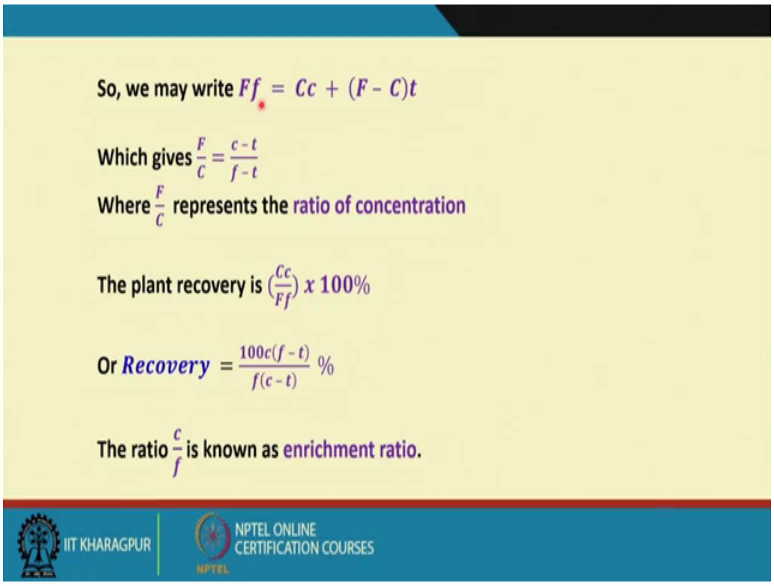
So, I take it out from this opening in may be under flow it may be over flow. And another one which is having least amount of my wanted material we just say that is tailing ok. So, if the feed is represented by capital F, concentrate is represented by capital C and tailings many times we call rejects also. So, that is the tailings that it represented by T. So, first equation we can write because input is equal to output we are saying that is the steady state condition and no accumulation inside. So, I can write the first equation capital F is equal to capital C plus capital T; that is your input is equal to output that is C plus T.

Now, where I can write that F is equal to feed tonnage rate or reducing the number of variables we can take it as equivalent 100 percent or maybe you can say if it is fraction, we can say it is equal to 1 unit or maybe it is 100 unit ok. Capital C is the concentrate tonnage or weight percent, capital T is equal to tailing tonnage or weight percent; that means, if I have F is equal to 100 tons of that material; suppose C is 60 tons. So, 100 minus 60; I can automatically get that is 40 tons; assuming that there is no loss and no your accumulation here that is in steady state.

Now if I say that the assay of my wanted material there is say copper; so, copper content of these feed represented by small f. So, suppose first let me explain you the say suppose I have got 1 percentage copper here. So, we are done some processing here; so, we are concentrating it. So, now, say suppose this assay here is 20 percent of copper. So, what will be the assay of here that I can calculate it back, but before that say suppose here it is 20 percentage and here it becomes 0.02 percent. So, that is why if I write the assay of this copper at feed as small f assay of my copper in concentrate as small c and assay of my copper in the tailing is small t. Because as I said in the very first lecture of this lecture series that you will be losing some of your valuable material along with reject material that is a losses most of the cases it happens like that.

So, if I represent the small f as the assay content of copper in the feed, small c is the assay contained of copper in the concentrate and small t is the assay contained of my copper in the tailings that is why in this language I would said assay of each respective stream because we are calling it they are streams. So, it is in it could be in percentage, it could be in gram or tons, it can be an ppm parts per million etcetera any unit ok, but the unit has to be consistent and you must be very careful about this unit conversions.

(Refer Slide Time: 06:18)




So there what we can write? Now if I want to balance I want to write another equation based on my assay balances this is slightly confused you have to understand it properly.

So, let us start with that is I have got 100 tons of copper ore in that I have got 1 percent of copper. So, that is the small f; so, capital F is 100 small f is 1. So, how much of copper essentially I am feeding into my unit? That is 1 ton of copper I am having there; now capital C that is constant rate is 40 tons, concentrate is 40 tons means that is you have got 40 tons and there the copper percentage or say suppose it will be too much it is I have got 100 tons of copper you have got 1 percentage of assay. So, I have got 1 ton of material there; now say suppose I have got concentrate weight is 4 tons that is your 4 percent of the feed and their my assay is 20 percent; that means, in that 4 tons of copper concentrate; I have got 20 percent of copper.

So, what is 20 percent of 4 ton that is your 0.8 tons? So, you have got 1 ton of feed, 1 ton of copper in the feed out of that I have recovered 0.8 tons of copper in my concentrate.

Where the remaining 0.2 tons have got that has gone to my tailings, but in terms of percentage what it will be that is there I have to know what is the capital T because you have paid 100 tons. So, out of that 100 tons you have got 4 tons in the concentrate.

So, I am left with 96 tons of material that is a total feed material going to the tailings and in that I have got 0.2 tons of copper. So, I have to now convert it that is if I have 0.2 tons of copper in 96 tons of material; what is the relative percentage? It will be around 0.18 or something 0.1818 percentages. So, that is what is called the balancing based on the assay analysis and that is what in general form we are writing that is F f that is 100 tons multiplied by 1 percent that is the f is the assay that is the 1 percent.

So, that will give you 1 ton of copper is equal to this capital C is equal to 4 tons of concentrate multiplied by 20 percent of that. So, that is multiplied by say 20 percent means it will become 0.8 tons; so, that is your 0.8 ton. So, and then how much of your tailing I will left with? That is if I have already given C that is 4 tons that is 100 minus 4 tons I am replacing that capital T with F minus C because we have already written that F is equal to C plus T. So, C is equal to F minus C or we can write C is equal to F minus T like that we can write. So, here we have replace that capital T with F minus C.

So, you have got 100 tons of these F; 1 percentage is smaller. So, you are having total 1 ton of copper here you have got 4 tons of concentrate where you got 20 percent of these copper that is a copper assay is 20 percentage; that means, I am having 0.8 tons of copper in my concentrate So, I had 1 ton of copper out of that I have recovered 0.8 tons in the concentrate. So, that is small t I still do not know, but it should be; so, how much of material I have send there in the tails that is a F minus C that is 100 minus 4 that is 96 tons.

And there I have got in total 1 minus 0.8 that is 0.2 that this small t is not equal to 0.2 tons; it should be in terms of percentage. So, what is the assay of this? So, it is 0.2 by 96 multiplied by 100. So, that is equal to your say that will be the assay of this ok. So, that is the assay content of that; so, I hope you are clear about this F f is equal to capital C to c small c plus F minus C into t.

Now, what we can write that is if you rearrange it that is F f is equal to capital C into c plus F t minus C t. So, now, F t I bring it here; so, I can write capital F into small f minus t plus if I take C capital C as common. So, this part will be a small c minus t. So, if I write F by C is equal to c minus t divide by f minus t ok. So, this capital F by capital C is equal to small c minus small t divide by small f minus small t.

So, these are based on assay and this is the tonnages. So, it is the whole lot of material; so, that is the what is the tonnage I am producing and that I am converting it in terms of assay. Why we are doing it? Now many times when you have your process control steps; maybe you have got a assay analyzer; like you have got a copper analyzer installed in the feed stream as well as in the tailing stream as well as in the concentrate steam.

So, when I have the data for assay for split stream, tailing stream and concentrates stream by using this we can get back to know that what is the capital F and what is the capital C that what is the ratio of that F by C or maybe we can calculate it back how much tonnages we are producing. So, by knowing some of the unknown parameter; we can get this now you see that in that equation in that case how many unknowns are there we have got F C T and you have got small f c t.

So, you have got six unknowns now when you are setting it F is equal to 100 percent; that means, I am reducing the total number of unknowns to 5 ok. Out of that if I know the values of three unknowns; I can easily get back to the two unknown values we can calculate it that is the entire exercise you are doing. Because you do not have to measure all the 5 parameters; if we can measure only 3 parameters we can get back to the values of other 2 parameters.

And that is why we are doing all this; so, this F by C; it represents it is called as the ratio of concentration. So, is a why we use this as the ratio of concentration means that is you have got your 1 percent it is like your analogous to the concept of reduction ratio, whatever we have used it for combination that is you have got 1 percentage copper in your feed and you are improved it to 20 percentage. So, the ratio of concentration is F by C that is how much of improvement you have there and what is the weight of your material that is you have got 100 tons of F out of that only 4 tons you are recovering is as concentrate.

So, ratio of concentration is basically 100 by capital C. So, that is 100 by 4 the example I have given the ratio of concentration is 25 is to 1. So, what is that it is saying that by improving your copper grade from 1 percent to 20 percent with this process, you are not only improving the grade from 1 percentage to 20 percent, but also you are reducing the quantity of material for further processing or maybe for transportation purposes that is reduced by at a ratio of 25 is to 1; that means, now I have to transport only 4 tons of material in place of 100 tons of material of course, I will be losing that 0.2 tons of material in my tailings ok.

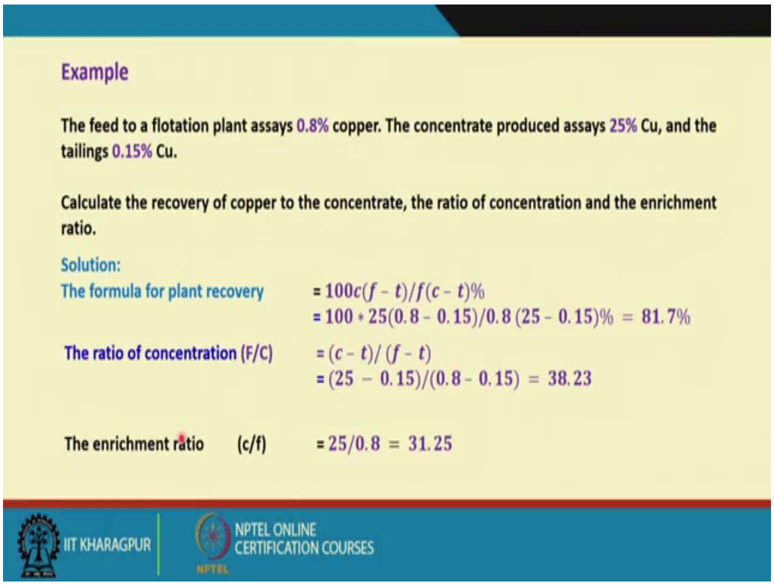
So, that will give you the what type of bulk material handling system have to use and then what is the advantage of doing this; how much of saving will be there in the transportation cost like. So, on you know there is an another term that is called the plant recovery. So, plant recovery is defined as capital C multiplied by c that is basically you are trying to do that is how much of copper I had in my feed, out of that how much I have recovered in the concentrate. So, if you get back to that example of 1 percent; so, you have got 1 ton of copper in my feed, out of that you have recovered in the concentrate 0.8 tons. So, what is your recovery plant recovery is 0.8 by 1 into 100; so, that is 8 percent.

So, planned recovery is capital C into small c divide by capital F into f into 100 percent so; that means, your recovery is 80 percent. So, 20 percent of the available copper for which you have invested for mining, you are losing it as a tailings. Now you can take a decision that whether I try to recover further copper from my tailing or I can or I am happy while losing that much of copper I do not worry because that does not have my that does not give me the economical benefit ok.

So, this is what the recovery term is coming or we can write the recovery is equal to because F by C is this. So, c by f is equal to f minus t by c minus t. So, I can write that is your f minus t by c minus t or by replacing C by F and then you are left with your 100 c by f. So, that is the recovery is equal to 100 small c into small f minus t divide by small f into c minus t. So, beauty of this is that that is if I know the assay content of the three streams even without knowing the tonnages, I can calculate the recovery by applying this formula.

So, these are the conversion we are will be using the ratio c by f that is small c by small f is known as enrichment ratio. So, what is the c here now in this example? It is 20 percent what is the feed assay? It was 1 percentage so; that means, the enrichment ration is 20 sorry I miss quoted that that was not analogous to that is capital F by C was not analogous to reduction ratio; this is analogous to enrichment ratio is similar to your reduction ratio; that is what is that level of improvement I am having in my concentrate grade or assay; that is it is 20 times the quality has been improved ok. Because from 1 percentage; I have concentrate it I have improved it to 20 percent.

(Refer Slide Time: 20:04)




Now, let us see that how we can apply this equations example that is a problem that is feed to a flotation plant assays 0.8 percentage copper; I am giving it from that proper example. Feed to a processing feed to a floatation plant assays 0.8 percentage copper, the concentrate produced assays 25 percent copper and the tailings 0.15 percent copper. So, these are all assay contents of three different steams that is the feed stream, concentrate stream and the tailing stream. Now I need to know I need to calculate the recovery of copper to the concentrate what is the recovery of copper to the concentrate? The ratio of concentration and the enrichment ratio; so, how do I use it? So, the formula for plant recovery was that is 100 c into f minus t divided by f into c minus t that percentage these are all assay content

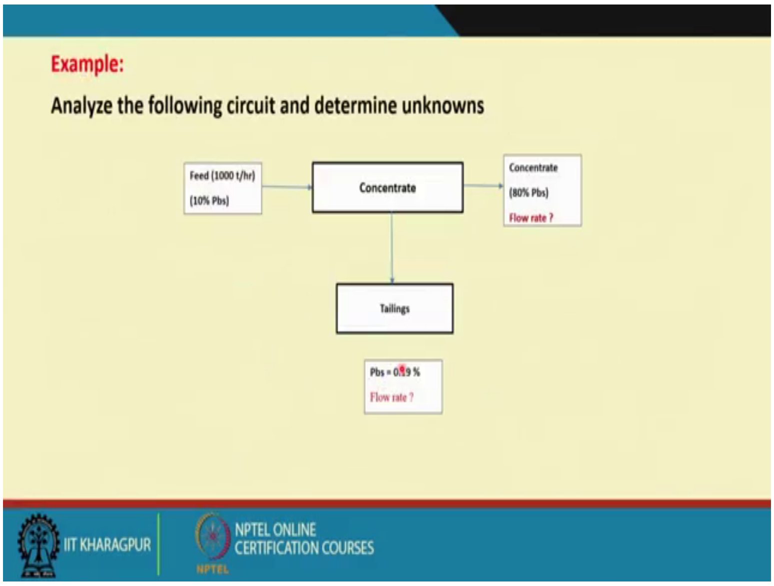
So, I put these values of the assays and then I get a value of plant recovery of 81.7 percent. So, this is the beauty, but as I said that it is greater to have clear cut concept that how we are getting it. So, better to derive it from the basic mass balance equation f is equal to c plus t and ff is equal to Cc plus Tt or otherwise you have to remember this formulas, but for practicing engineers I would suggest that you hook up all these equations into a smaller say if you into a simple excel spreadsheets and then you can calculate it based on the known values whatever you are getting ok.

The ratio of concentration is defined as capital F by C which I can convert it in terms of assay content as small c minus t divide by f minus t; now I can simply put this values. So, if I remember the equation or if I know how to derive the equation based on the grades or capacity on the assays, I can get the ratio of concentration is 38.23 that is F by C ok.

So; that means, you are saying that your ratio of concentration is F by C 38.23; that means, you have to now transport 1 by 38 of your weight of the material what you have already mine. So, that gives you your idea about that how many how much material I have to handle per unit time or how much of transportation cost I should be a for this. The enrichment ratio that is small c by f is 25 by 0.8 because these are the values is given; so, it is 31.25.

So, this example demonstrates that how do I apply the formulas that is the most of the operating plant these days, you have got on stream analyzer assay analyzers. So, normally you get the values of your assay contents a from each stream.

(Refer Slide Time: 23:56)

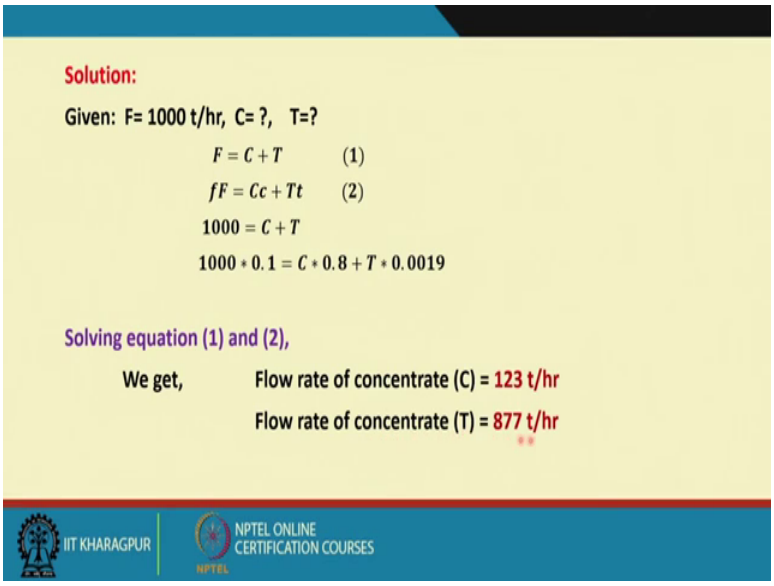



Now, you can apply this formulas to get all these values; now I give another example this called as simple flow chart; that is you have got the following circuit and determine unknowns that is have already given the known values here and I ask you to do or calculate the unknown values. What did it is telling that here feed is basically 1000 tons per hour and it has got 10 percent Pbs that is your galena we call it as galena as I this says should be capital sorry there is a topographical mistake.

And this is not concentrate this is your concentrated is again typed sorry for that. So, this is a feed at 1000 tons per hour and you have got in this at 10 percent Pbs; that is the assay content of this. Now you have got a unit operation, we call it concentrator. Now it has got two products one is concentrate and one is tailings; in concentrate you have got 80 percent Pbs and in tailings the Pbs is 0.19 percent.

So, I need to know what is the mass flow rate here and of the concentrate and what is the mass flow rate of the tailings. So, by using the equation whatever we have discussed we want to calculate back all these unknowns.

(Refer Slide Time: 25:36)

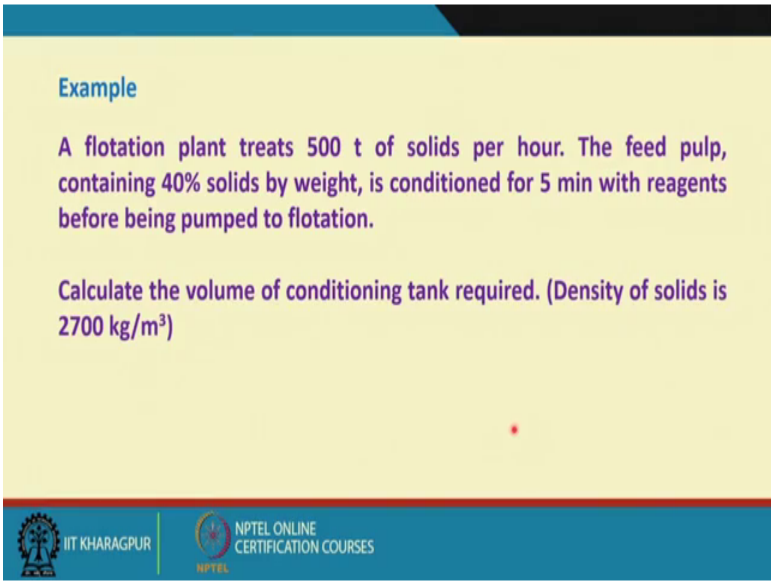



So, given is F is equal to 1000 tons per hour I need to know the capital C, I need to know the capital T right. So, first thing I can write that is F is equal to C plus T the capital C plus T and I can also write capital F into small f is equal to capital C into small c plus plus capital T into small t. So, here the f value is given; so, I get one equation 1000 is equal to capital C plus capital T; I do not know the values of C and T. And here I have the value for capital F is 1000 and here the small value of f is 10 percent.

So, when the small value f is 10 percent. So, I can convert it into your fraction that is your f has to be in fraction; so, that is your 0.1 ok. So, then capital C I do not know small c is given as your small c is given as 80 percent so; that means, it is 0.8 fraction and this will be your Pbs is 0.19 so; that means, it is 0.0019. So, that is equal to 1000 into 0.1 is equal to C plus C multiplied by 0.8 plus T into 0.0019. So, this is another equation I have got; now can we not solve bit for C or T what we can do that is here.

So, what we can write that is equal T is equal to 1000 minus C. So, if I put that value T replace that value of T here. So, it is 1000 minus C into that; so, by simplifying this we can get a value of C. Similarly if I replace the value of your C is equal to 1000 minus T; so, I replaced it here. So, 1000 into 0.1 is equal to 1000 minus T into 0.8 plus; this we can get a value of this. So, we can get by solving this two equations the flow rate of concentrate that is capital C is equal to 123 tons per hour. I request all of you to recheck it and to your practicing purposes we must do it otherwise you will not be able to answer this questions properly in your exam and flow rate of your say flow rate of the tailings or that is also type of. So, we get flow rate of concentrate that is capital C is equal to 123 tons per hour and flow rate of tailings that is equal to 1000 minus 123; it is 877 tons per hour right.

(Refer Slide Time: 28:51)

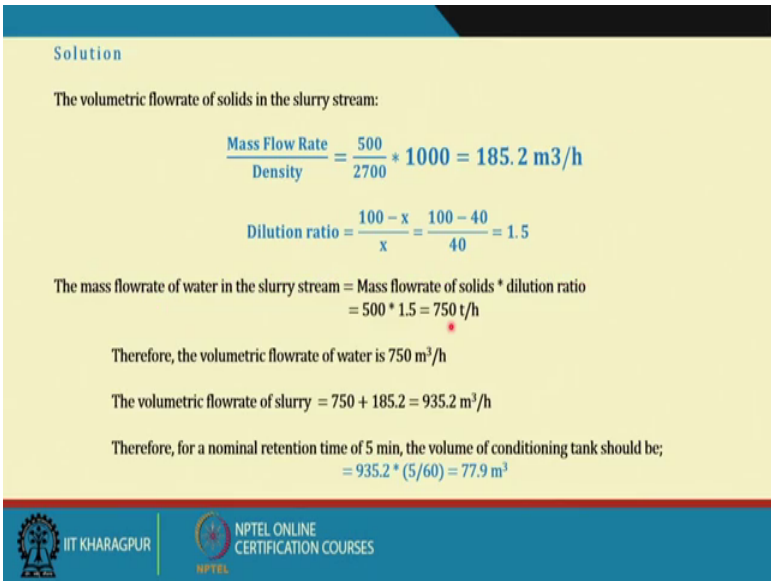



So, this is how this is another example show you that how to apply this equations. There is another example that is a flotation plant treats 500 tons of solids per hour ok; the feed pulp containing 40 percent solids by weight is conditioned for 5 minute with reagents before being pumped to flotation. So, a flotation plant you do not have no worry about what is flotation plant right now.

So, there is a mineral processing operation; it treats 500 tons of solids per hour, but for that you need a preparation circuit that is before I feed it to the plant I have a separate tank where I prepare the slurry for further processing. So, there the feed pulp it has got 40 percent solids by weight is conditioned in that tank with some chemicals that is the reagents for 5 minutes before being pumped to this cell.

So, what I am saying that you have got 500 tons of solids per hour you have to feed it to a unit, but before I feed it I have a feed pulp containing 40 percents solids by weight and you are conditioning it, you are putting it into a vessel into a tank having a resistance time of 5 minute before being pumped that slurry to a pump to the flotation cell. So, calculate the volume of conditioning tank required; so, what is that volume of conditioning tank is required if the density of the solid is 2700 kg per meter cube let us try to solve it.

(Refer Slide Time: 30:56)




So, how do I do it? Now the volumetric flow rate of the solids in a slurry stream because I need to know the volume, but the data is given in 500 tons 40 percents solids by weight.

So, I have to convert them into volume otherwise I cannot get to know the volume. So, the volumetric flow rate of solids in the slurry stream, is mass flow rate divide by density. So, I need to process the 500 tons of solids per hour; so, in terms of volumetric flow rate of solids what that volumetric flow rate of solid I want to process per hour first you have to convert it. So, it is 500 divide by the density of that solid is 2700 and it is tons. So, I have to convert it into meter cube per hour.

So, multiplied by 1000; so, that will give you 185.2 meter cube per hour. So, that is your volumetric concentration of solid that is your what say your volumetric flow rate of your solids that is 185.2 meter cube per hour. Now you are saying that you need to condition that solids into a tank at a 35 percent or a 40 percents solids by weight. So, first let us calculate what is the dilution ratio; that means how much of water I need to add ok? So, what will the volumetric your flow rate of my water that is what I want to know?

So, the dilution ratio now we see that how we are using the dilution ratio; so I want 40 percents solids by weight; so, that x is equal to 40; so, it will be become; so, dilution ratio is the ratio in between water weight of water divide by weight of solids, the solids fractions is 40; so, this is 60; so, 60 by 40 will give 1.5. So, now, if I multiply this factor weight the dilution ratio that should give me the mass flow rate of water in the slurry stream. So; that means, mass flow rate of solids into dilution ratio is equal to 500 into 1.5 because I want to process. So, that is the volumetric flow rate, but that is the 500 tons per hour of solids I want to process.

So, what is the mass flow rate of water in the slurry stream? It is 500 into 1.5 that is 750 tons per hour as because the density we can assume for water is 1000 meter 1000 kg per meter cube. So, you can simply convert this into a volumetric flow rate of water is 750 meter cube per hour. So, now, I know the volumetric flow rate of solids is 185.2 meter cube per hour and the volumetric flow rate of water is 750 meter cube per hour. So, the volumetric flow rate of slurry will be 750 plus 185.2 that is equal to 935.2 meter cube per hour. Now you need a retention time of 5 minute; so, for a nominal retention time of 5 minute the volume and conditioning tank should be 935.2 that is per hour.

So, that you basically I have to convert it into the 5 minute; so, that is 5 by 60, so that will give you your 77.9 meter cube. Because your do retain this much of material for 5 minutes not for an hour. So, that is per hour; so, I have to calculate it back that is 935.2

into 5 by 60. So, the volume of the tank should be 77.9 meter cube; this is a this is a classic example of how you can use the concept of dilution ratio, how you can use the conversion in between your volumetric fraction from mass fractions and that is how you can get to know that is what will the requirement for your different equipment that is your tank and all these even conditioning tank.

So, this is how we can use the mass balancing equations for getting the different meaningful data from your plant. We will continue this in the next lecture till then.

Thank you very much.